Verzögerung der Magenentleerung, eine Verspätung der Dickdarmfüllung und eine typische Aenderung der Kurven, welche die Gesamtlänge der Dünndarmschatten angeben, zur Folge gehabt, so dass im Prinzip genau dasselbe gefunden wurde, wie nach Labyrinthexstirpation bei normalen Tieren.

Zusammenfassung.

Der Reflexbogen für die Störungen der Bewegungen des Magen-Darmkanals, welche nach Labyrinthexstirpation bei Katzen auftreten, verläuft hauptsächlich oder ganz über die Nn. vagi; werden beide Nn. vagi vor der Labyrinthexstirpation durchschnitten, so übt letztere keinen deutlichen Einfluss mehr aus auf die Magen-Darmbewegungen.

Eine Ausschaltung der sympathischen Innervation vom Magen bis zum Coecum durch Durchschneidung der postganglionären Fasern auf ihren Verlauf, hebt den Einfluss der Labyrinthexstirpation nicht auf.

Physics. — *On the jump in the expansion coefficient of liquid helium in passing the lambda-point.* By W. H. KEESOM. Supplement No. 75a to the Communications from the KAMERLINGH ONNES Laboratory at Leiden.

(Communicated at the meeting of February 25, 1933.)

Summary. A relation is derived between the inclination of the lambda-curve in the p,T-diagram, the jump in the specific heat, and the jump in the expansion coefficient. This relation is used to calculate the expansion coefficient of liquid helium II at 2.19° K. The value obtained is in harmony with the results obtained by KAMERLINGH ONNES and BOKS in their measurements on the density of liquid helium.

§ 1. *introduction.* In the measurements on the anomaly in the specific heat of liquid helium under its saturated vapour pressure, made in collaboration with Miss A. P. KEESOM[1]), it appeared that in the afterperiod the galvanometer in the measurements above 2.19° K. had quite a different course from those below 2.19° K.: comp. Fig. 1 of Comm. No. 221d. The explanation which readily presents itself for this phenomenon is the following.

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[1]) W. H. KEESOM and Miss A. P. KEESOM. These Proceedings **35**, 736, 1932. Comm. Leiden No. 221d.

The liquid helium in those experiments was heated from above, cf. Comm. No. 219e § 2[1]). According to the measurements made by KAMERLINGH ONNES and BOKS [2]) below 2.19° K. [3]) the expansion coefficient of liquid helium is negative. As a consequence the upper layers, which are heated first, become heavier, so that convection currents arise, which promote a rapid temperature exchange in the liquid. Above 2.19° K., according to the same measurements, the expansion coefficient of liquid helium is positive, so that temperature exchange goes on much more slowly then.

However, in Comm. No. 221d, this explanation was not accepted because a thermodynamical reasoning seemed to lead to the conclusion that the density maximum of liquid helium could not coincide exactly with the jump in the specific heat. During a discussion with my colleague Prof. EHRENFEST it became evident that in the thermodynamical deduction alluded to a sign had been taken erroneously. If this is corrected the objection just mentioned does not exist, so that the explanation given above becomes valid. The explanation given as an alternative in Comm. No. 221d then becomes superfluous.

The thermodynamical considerations alluded to lead to an interesting confirmation of the way in which KAMERLINGH ONNES and BOKS presented the results of their measurements on the changes in density of liquid helium in this domain. Moreover the relations deduced may be useful in the discussion of the results of an experimental survey of the isochores of liquid helium to be published before long. So they may be communicated in this paper.

§ 2. To begin with it should be remembered that the question whether the change in the specific heat of liquid helium, when it passes from the condition II to the condition I — either under its saturated vapour pressure or at constant volume or pressure — occurs abruptly or in a small temperature interval, is still an open one. Further experiments on this point are being prepared. So far it has appeared that in each case this temperature interval is very small, under the saturated vapour pressure certainly smaller than 0.02 degree, very probably smaller than some few thousandths of a degree.

In the meantime, for considering the changes which several quantities undergo in passing the lambda-point or the lambda-interval, it will be convenient, and admissible in connection with the accuracy reached in the experiments, to treat the change in the specific heat and the related changes in other quantities as discontinuities. It is in this sense that,

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[1]) W. H. KEESOM and K. CLUSIUS. These Proceedings **35**, 307, 1932. Comm. Leiden No. 219e.

[2]) H. KAMERLINGH ONNES and J. D. A. BOKS. Rep. and Comm. Fourth International Congress of Refrigeration, London, 1924. Comm. Leiden No. 170b.

[3]) Temperature corrected to the 1932-scale, cf. Rapports et Communications VIe Congrès International du Froid, Buenos Aires. 1932. Comm. Leiden Suppl. No. 71d.

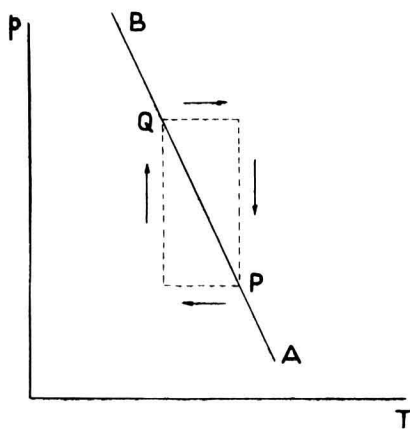in harmony with Comm. No. 221*d* § 4*b*, we speak of the lambda-point, corresponding to the temperature at which this discontinuity (at a definite pressure, or under the saturated vapour pressure) occurs, and of the lambda-curve as the locus of the lambda-points, e.g. in a *p*,*T*-diagram.

Fig. 1.

§ 3. *Deduction of a relation between the jump in the specific heat and the jump in the expansion coefficient.* Let *AB* in Fig. 1 be the lambda-curve, *PQ* an element of it. In going round a small rectangle as indicated by the arrows the total change in entropy must be $= 0$. This gives, considering that there is no latent heat in passing the lambda-curve:

$$\left(\frac{\partial S}{\partial T}\right)_{p_{II}} dT + \left(\frac{\partial S}{\partial p}\right)_{T_{II}} dp - \left(\frac{\partial S}{\partial T}\right)_{p_{I}} dT - \left(\frac{\partial S}{\partial p}\right)_{T_{I}} dp = 0.$$

From $\left(\frac{\partial S}{\partial T}\right)_p = \frac{c_p}{T}$, $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$ follows:

$$\left(\frac{dp}{dT}\right)_\lambda = \frac{c_{p_{II}} - c_{p_I}}{T\left\{\left(\frac{\partial v}{\partial T}\right)_{p_{II}} - \left(\frac{\partial v}{\partial T}\right)_{p_I}\right\}} \quad \cdots \cdots \quad (1)$$

In this equation $(dp/dT)_\lambda$ means the value of that differential quotient taken for the lambda-curve. In introducing the relative expansion coefficient $\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$ we may write:

$$\left(\frac{dp}{dT}\right)_\lambda = \frac{\triangle c_p}{Tv \cdot \triangle \alpha} \quad \cdots \cdots \cdots \quad (2)$$

§ 4. Equation (2) enables us to calculate the jump in the expansion coefficient from the inclination of the lambda-curve in the *p*,*T*-diagram and the jump in the specific heat. For doing so we take, at $T = 2.19$: $(dp/dT)_\lambda = -80.8$ atm./degree $= -81.9 \times 10^6$ dynes/cm² degree from the measurements of KEESOM and CLUSIUS[1],

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[1] W. H. KEESOM and K. CLUSIUS. These Proceedings **34**, 605, 1931. Comm. Leiden No. 216*b*.

$\triangle c_p = c_{p_{II}} - c_{p_I} = 1.9$ cal/g. degree $= 7.95 \times 10^7$ erg/g. degree from the measurements of W. H. KEESOM and Miss A. P. KEESOM, Comm. No. 221d,

$1/v = 0.1462$ cm^{-3} from the measurements of KAMERLINGH ONNES and BOKS, Comm. No. 170b. We find

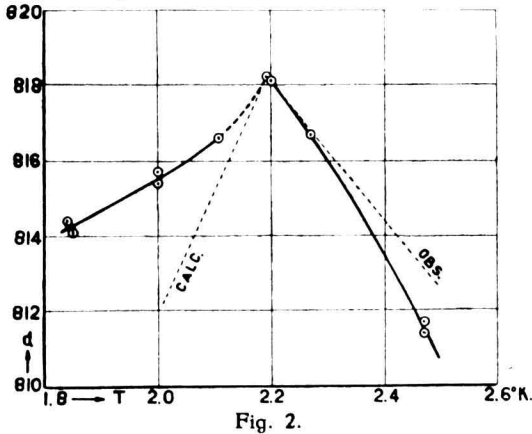$$\triangle a = a_{II} - a_I = - 0.0648 \text{ degree}^{-1}.$$

If further we derive from the last mentioned measurements

$$a_I = 0.0222 \text{ degree}^{-1}, \text{ we find}$$

$$a_{II} = - 0.0426 \text{ degree}^{-1}.$$

Fig. 2 represents the results of the measurements of KAMERLINGH ONNES

Fig. 2.

and BOKS on the density of liquid helium for the temperatures ranging from 1.8 to 2.5° K. $\odot$ are the measured values. They are united by a curve which consists of two branches, which meet at the lambda-point under a finite angle, and are relative to helium I and helium II respectively. By obs the tangent to the curve uniting the measured points for $He\,I$ is indicated. The tangent to the helium II branch, indicated by $calc$ has been drawn according to the value of a_{II} we calculated above.

Fig. 2 shows that this value is very well in harmony with the results obtained by KAMERLINGH ONNES and BOKS.

There is now every reason to presume that Fig. 2 gives a better representation of the density of liquid helium as a function of temperature than Fig. 1 of Comm. N°. 190b [1]) did. Instead of a jump in the density supposed there as possible, a jump in the expansion coefficient alone remains as the only singularity in the density-temperature curve.

§ 5. In the calculation given in § 4 we, without justification, took the values of c and a, which are valid for the liquid under its saturated vapour pressure, instead of c_p and $a_p = \dfrac{1}{v}(\partial v/\partial T)_p$. The justification can be given as follows.

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[1]) These Proceedings 31, 90, 1928.

Let the index s indicate that a quantity is to be taken for the liquid under its saturated vapour pressure p_s.

As from the absence of a transformation heat $He\ II \rightarrow He\ I$ [1]) follows that at the lambda-point there is no jump in the vaporisation heat [2]), and as there is not a jump either in the volume of the liquid (§ 4), we may conclude that $(dp/dT)_s$ at the lambda-point has one single value [3]).

From $c_s = c_p - T\left(\dfrac{\partial v}{\partial T}\right)_p \left(\dfrac{dp}{dT}\right)_s$ in connection with equation (1), it follows that

$$(\triangle c_p - \triangle c_s)\,(dp/dT)_\lambda = \triangle c_p \,.\,(dp/dT)_s. \quad . \quad . \quad . \quad . \quad (3)$$

In the same way from

$$\left(\frac{dv}{dT}\right)_s = \left(\frac{\partial v}{\partial T}\right)_s + \left(\frac{\partial v}{\partial p}\right)_T \left(\frac{dp}{dT}\right)_s$$

in connection with the relation [4])

$$\left(\frac{dp}{dT}\right)_\lambda = - \frac{\triangle\left(\dfrac{\partial v}{\partial T}\right)_p}{\triangle\left(\dfrac{\partial v}{\partial T}\right)_T}$$

it follows that:

$$(\triangle a_p - \triangle a_s)\,(dp/dT)_\lambda = \triangle a_p \,.\,(dp/dT)_s \quad . \quad . \quad . \quad (4)$$

As [5]) $(dp/dT)_s = 0.13$ atm/degree against $(dp/dT)_\lambda = -80.8$ atm/degree (§ 4), according to equation (3) and (4) $\triangle c_s$ can be taken for $\triangle c_p$, and $\triangle a_s$ for $\triangle a_p$, for the accuracy intended in § 4.

Moreover it follows from (3) and (4), that equation (2) remains exact if we change $\triangle c_p$ into $\triangle c_s$ and take $\triangle a_s$ for $\triangle a$.

§ 6. The equations (3) and (4) of § 5 can be derived as special cases from a more general relation.

G may be a function of p and T which suffers along a λ-curve

[1]) W. H. KEESOM and K. CLUSIUS. These Proceedings **35**, 307, 1932. Comm. Leiden No. 219e § 8b.

[2]) Cf. Rapports et Communications VIe Congrès International du Froid, Buenos Aires, 1932, No. 9 § 3b. Comm. Leiden Suppl. No. 71e § 1.

[3]) Cf. Rapp. et Comm. etc. No. 4 § 3. Comm. Leiden Suppl. No. 71d § 3.

[4]) For the proof of this relation see P. EHRENFEST, Proceedings of this meeting. Comm. Leiden Suppl. No. 75b. The relation can also be deduced in the way as was done for equation (1) in § 3, considering that there is not a jump in v in passing the lambda-curve.

[5]) Comp. Rapp. et Comm. etc. No. 4 Table II. Comm. Leiden Suppl. No. 71d Table I.

(Fig. 3) a discontinuity of the second order [1]), so that along that curve

$$G_{II} - G_I \quad \triangle G = 0, \quad \ldots \ldots \quad (5)$$

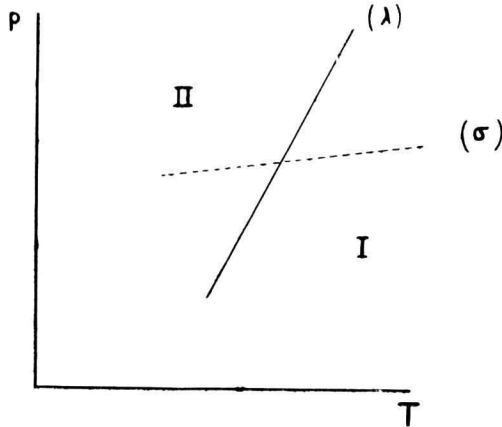whereas the differential coefficients of G make a jump.

Fig. 3

Let (σ) be a continuous line intersecting the λ-curve.

From $\left(\dfrac{dG}{dT}\right)_\sigma = \left(\dfrac{\partial G}{\partial T}\right)_p + \left(\dfrac{\partial G}{\partial p}\right)_T \cdot \left(\dfrac{dp}{dT}\right)_\sigma$ follows

$$\triangle \left(\frac{dG}{dT}\right)_\sigma = \triangle \left(\frac{\partial G}{\partial T}\right)_p + \triangle \left(\frac{\partial G}{\partial p}\right)_T \cdot \left(\frac{dp}{dT}\right)_\sigma .$$

If we let (σ) coincide with (λ), it follows from (5):

$$0 = \triangle \left(\frac{\partial G}{\partial T}\right)_p + \triangle \left(\frac{\partial G}{\partial p}\right)_T \cdot \left(\frac{dp}{dT}\right)_\lambda .$$

From these two equations we derive:

$$\left\} \triangle \left(\frac{dG}{dT}\right)_\sigma - \triangle \left(\frac{\partial G}{\partial T}\right)_p \right\{ \cdot \left(\frac{dp}{dT}\right)_\lambda + \triangle \left(\frac{\partial G}{\partial T}\right)_p \cdot \left(\frac{dp}{dT}\right)_\sigma = 0 \quad . \quad (6)$$

By taking for (σ) an element (s) of the saturated vapour pressure curve, and choosing $G = S$, or $= v$ respectively, the equations (3), or (4) respectively, follow.

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[1]) Cf. P. EHRENFEST. Proceedings of this meeting. Comm. Leiden Suppl. N⁰. 75b.