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# PERSPECTIVES ON ANALYTIC PHILOSOPHY

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*To H. A. J. F. Misset*

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#### PERSPECTIVES ON ANALYTIC PHILOSOPHY

In this publication I shall take up certain vital connections of a cognitive nature, viz. the logico-epistemological and historical connections between the following themes:

- the disregard of/disrespect for *individual human lives* as ends in themselves,
- obstacles to the development of a *climate* as well as a *language* encouraging critical discussion as an instrument for the resolution of *conflicts of avowed opinion*,
- the history and logic of the old philosophico-mathematical notion of *infinitesimals*.



## 1. OSSIFICATION OF (ANALYTIC) PHILOSOPHY

As a preliminary I want to draw attention to the very real danger of ossification of the “subject” of analytic philosophy. Great care must be taken lest analytic philosophy becomes (or: remains) a self-contained, almost closed milieu, with its own holy books and high priests – just as in most other philosophical schools, especially those of Marxism and some other “continental” philosophies. Let us hope to see the day when analytic philosophy as a particular philosophical pursuit and scholarly discipline, with its own staff and its own curricula, will have disappeared entirely from the continental academic scene – together with its present competitors, such as idealist and materialist dialectics (in the Hegelian sense of “dialectics”), Neo-Thomism, Calvinist philosophy, Husserlian and existential phenomenology, and what have you. Such curricula and academic policies surely should be aimed at making themselves superfluous, which in the case of analytic philosophy means, among other things, that one should try to imbue the majority of tomorrow’s philosophers with the spirit and skills of keen analysis so as to make them satisfy *also* the standards of goodness developed in analytic philosophy, whichever topic they choose to write about and whichever ideals they are moved by.

This requires that analytic philosophers and logicians rethink the philosophical situation. If they wish their own canons of scholarship and know-how to permeate philosophy as a whole, then it is self-demolishing to demand that linguistic and logical form *always* be studied as if *only* for their own sakes. It is one thing to *reculer pour mieux sauter*, and quite another thing to classify *every* conceptual leap from the ivory tower as an act of academic indecency. To think that it is, is, however, the fashion in contemporary academic philosophy. That fashion developed after, and in spite of, the examples set by Jeremy Bentham, Bernard Bolzano, John Stuart Mill, Bertrand Russell, Leonard Nelson, Otto von Neurath and E. W. Beth, to mention only some philosophers of the past who are reckoned as analytic philosophers. Arne Naess and John Passmore are contemporary examples.

## 2. REMEDIES

Here are some conditions (probably necessary, possibly together sufficient) for a future successful penetration of analytic methods into culture at large and of the subsequent abolishment, on rational

grounds, of analytic philosophy as a separate academic discipline. Before this can happen analytic philosophy must again, and preferably without delay,

I. become openly and clearly attuned to the *vital problems*<sup>1</sup> of men, women and other sentient beings as it was in the works of Bentham, Bolzano, Mill, Russell and Leonard Nelson,

II. forget its non-critical attitude to the “natural” (read: traditional, inherited) languages – which should remain the proper field of positive linguistics (though the overlap will remain considerable) – and be brought closer to philosophical logic and other normative studies, as well as to the philosophy of mathematics,

III. be brought under the perspective of *public (intersubjective) debate*, i.e. under the perspective of critical discussion aimed at verbal resolution of conflicts of opinion and carried out by means of *suitable parts of* ordinary language, and therefore, in order to expose the many-sorted roots of anti-argumentational attitudes,

IV. become enriched with a *historical dimension*, since, as Russell said,

“ordinary language is shot through with the fading hues of past philosophic theories”,

which is certainly true, whether some language structures are innate or not.

This requires that logicians, analytic philosophers and historical linguists join forces to investigate the intricate subject of *philosophers' languages*, in the hope of disclosing the assumptions responsible for those features of “natural” languages which stand in the way of verbal resolution of conflicts of avowed opinion. The study of philosophers' languages should lead us, within one or two generations, to

V. a different, rigidly systematic and hence highly improved understanding of the *logical foundations* of those *philosophical systems* that dominate the minds of contemporary men and women, including our own.

### 3. SOME VITAL PROBLEMS

We said that analytic philosophy must again become attuned to vital problems. Now human beings, and sentient beings generally,

<sup>1</sup> Here, and perhaps only here, I agree – and always have done – with Herbert Marcuse, who writes that the epistemological problems of how to find the broom or identify the taste of a pineapple are not the primary problems of philosophy (1964, Ch. 7). As I shall show below, some rules of identification (e.g., of the limit of an infinite series) may have a philosophical importance that Marcuse never dreamt of.

have many urgent problems. The majority of these problems are not of a philosophical nature and cannot fruitfully be dealt with by philosophers. But some of the most urgent ones (still) are. As the most urgent and in this sense the most important <sup>2</sup> *philosophical* problems facing a professional philosopher today I would list (*not* in order of increasing importance or urgency):

(1) the philosophical roots of anti-argumentational attitudes, together with the linguistic obstacles to the construction of suitable instruments for the resolution of conflicts of opinion (and possibly: of conflicts of interest),

(2) the design and use of philosophy and theology, more generally: of systems of thought and of ethics, as instruments for the oppression of the females of this world (subjection by cognitive mutilation or corruption), in order to prevent males from mental breakdown in a harsh and complicated world; this point is more urgent – though not more important – than the similar point with respect to the use of thought systems for the oppression of lower-caste and lower-class males *since philosophers (males) are already aware of and at work on the latter* but systematically neglect the former,

(3) the attitudes, assumptions and modes of thought, partly expressed in and propagated by means of philosophical theories, permitting the suffering and low quality of life of domestic and laboratory animals (at a later moment in history I would add: and of animals generally); this topic, too, is about to lose its philosophical urgency due to recent important publications (John Passmore, Peter Singer, G. M. Teutsch), so that in this whole field only (2) remains (in academically accepted philosophy),

(4) the common belief, backed by a number of philosophical systems, in the existence of some deep-level general principle that can be invoked in defense of the act of killing some individuals in the pursuit of goals said to pertain to the interests of some totality, large collective, ideal or abstraction.

Here I shall deal only with the last item on this list. I could have started with quotations from philosophers on the virtues of war. This I shall not do. I could also start with references to the literature published in the nineteen-thirties, 'fourties and 'fifties on the impact of philosophy upon thought and action in Hitler's Germany and before. I shall not do that either, but shall take my starting point

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<sup>2</sup> Obviously: in my opinion. If explicitly added, this phrase would therefore have the rhetorical effect of a slight excuse for my immodesty in selecting these four topics, and the situation does not call for modesty as much as for honesty.

instead in an article that appeared in the *Süddeutsche Zeitung* in 1977:

“Natürlich kann man nicht eine gerade Linie von Nietzsche über Heidegger zum abkippen in die SS-Ideologie ziehen, gar als Verbindung von Ursache und Folge. Ebensovwenig ist Marx für die zwanzig Millionen Menschenopfer haftbar zu machen die Stalin auf dem Gewissen hat, noch Marcuse jetzt für die Mörde in der Bundesrepublik.<sup>3</sup> Gleichwohl ist es notwendig, herauszuarbeiten, warum jene Gedankensysteme nicht bloss zu faszinieren vermögen, sondern sich auch leicht als Vehikel zur Gewalttat und geistige Perversion eignen. *Eine* Erklärung dafür ist wohl darin zu suchen, dass jene Systeme totale, nicht mehr diskutierbare Welterklärungen anbieten, dass sie also geschlossene, dogmatisierte politische Heilsgemeinschaften sein wollen – und weiter, dass sie nicht kontemplativ und deskriptiv sind, sondern zur direkten Tat und zur Aktion motivieren.” (Heigert 1977 – italics mine, E.M.B.).

*Eine* Erklärung: indeed not the only one, for there are other explanations, more specific than this. The fact that these philosophies are total philosophies, dogmatically closed hence inaccessible to rational argumentation, cannot alone explain why they are or are taken to be connected – by terrorists, by defenders of national wars, and others – with sacrifice, again and again, of human individuals. For a total and dogmatically closed philosophical system is just as possible in connection with the outlook that other individuals are sacred. So we obviously shall have to look deeper than that.

Furthermore: sheer destructiveness caused by frustration is one possible explanation of terror, but not of the coincidence between terror and the attempts to back it by these philosophical systems. The link between frustration and destructiveness is a psychological one. Being philosophers we cannot offer our services here.<sup>4</sup> Our object of study cannot be terror, and it cannot be terrorism, but it can be the cognitive aspect of terrorism, or the *rationalization* or *justification* of terrorist activities: whether christian, fascist, stalinist or contemporary, whether masculine or feminine, and whether German, Irish, Italian or Japanese or of any other origin. Similar remarks can be made about war.

<sup>3</sup> Nor, we may add, Sidi Carnot or James Watt for the sufferance and deaths caused by the industrial revolution, or Rutherford for the atom bomb. (The analogy is admittedly halting, since Carnot's, Watts' and Rutherford's work did not contain the pretense to an ethics or workable politics.)

<sup>4</sup> Except, in the case of female terrorists, indirectly: by studying the gender-specific frustrating effects upon females of the philosophical and theological instruments of oppression (oppression through *relative self-appraisal* by philosophizing males; see my (1972), part II: 'Praktijk'). These effects may be direct (in the case of intellectuals) or indirect, through the effects of these philosophies 1. on individual human minds and 2. on social structures.

From another part of the world than Heigert, the Nigerian author Wole Soyinka, in an interview with *Vrij Nederland*, Aug. 12, 1978 (vol. 39), expresses his regret

“that the greatest scoundrels, that is to say those who commit crimes against mankind in Africa, justify their behaviour by an appeal to [*vanuit*] Marxism”

(translation from Dutch back into English by E.M.B.).

The Irish statesman Conor Cruise O’Brien sees his book (1978), written on the background of his experience with the Congo and with Northern Ireland, as an attempt to

“dismantle the legitimation structures” (p. 12)

expressed or assumed by advocates and defenders of violent acts. He writes:

“It is not contended in these essays that the legitimation of violence (force) is always wrong. It is contended that *the legitimation of violence (force) as a lesser evil* in any particular circumstances is a profoundly serious matter which has to be capable of being defended on rational grounds, in relation to those circumstances, if it is to have any moral force” (p. 12, italics mine – E.M.B.).

The legitimation structures that are actually used are of many different kinds. O’Brien mentions legitimation/justification “by play on the emotions, by obliquity, by scientism, by appeal to tribal self-applause and atavistic resentments”, all of them in his opinion lacking in moral force.

It seems to me that the situation is more complicated yet than O’Brien surmises. For of these, McNamara-like “scientism” pretends to be precisely a legitimation structure “on rational grounds”. There are, then, unacceptable (to O’Brien as well as to the present author) legitimation structures that concern students of forms of “rational” thought, “rational” here being used in the non-evaluative sense of referring to or otherwise pertaining to moods of cognition and ratiocination. This kind of “scientism” (which O’Brien discusses in a later chapter) has been dealt with in a first-rate manner by A. Rapoport (1964). There are, however, legitimation structures that pertain directly to attempts at philosophical system-building or to modes of thought that are akin to these systems. Such modes of thought are found in certain theologies as well.<sup>4a</sup>

Since Christian terror became uncommon, philosophical justifications and rationalizations of terror (and of manslaughter generally,

<sup>4a</sup> I cannot in this paper broach the question of whether or not cognitive analogies can be found with the modes of thought that inspired the “scientism” of the larger number. See in this connection Naess 1943.

though with the exception of national wars) come less willingly from Christian pens, and for the last hundred years have been made mainly by appeal to philosophies and modes of thought which are recommended by the philosophers mentioned by Heigert, the author of the article I first quoted from. He quite clearly recognizes that his own characterization of these philosophical systems as totalitarian and dogmatically closed amounts to no more than a very partial explanation at best of their efficacy in justifying manslaughter, and closes with the following words:

“Was aber erst noch zu beginnen hat, ist eine beharrliche, genaue und mutige Auseinandersetzung im geistigen Umfeld des Terrorismus.”<sup>5</sup>

In the spiritual field (fields) by which terrorism – in general: political manslaughter – is surrounded: here philosophers do have a specific and quite professional obligation, and a serious one. Philosophers also have other obligations. I shall not deal with all of them here.

As Heigert suggests, the systematic investigation of the anatomy of the surrounding spiritual field (as he calls it) is still only in its beginning. More correctly, work in this direction is only sporadic. It requires types of philosophical activity which as yet belong to no clearly recognized philosophical discipline and that is neglected by *every* great contemporary philosophical school.<sup>6</sup>

I have come to think that a mere systematization (axiomatization) of the overtly ethical assumptions of philosophical systems, as carried out by Naess in the case of Gandhi’s thought (Naess 1958), will not take us to the deepest layers, and that the investigation should be undertaken as a study of the *logical foundations of philosophical systems*, revealing the professed or tacitly implied assumptions of a syntactic, semantic or otherwise semiotic kind, together with elementary mathematical assumptions about *number*.<sup>7</sup> In other words, I have come to think *that one ought to investigate what is taken to be tautological, rather than what is regarded as ethical principles*.

The remainder of this paper will be devoted to the following thesis:

*The perspectives mentioned under II, III, IV and V above (Section 2) are connected in the closest possible way and are of crucial importance in connection with the goal mentioned under I*

<sup>5</sup> “What still has to get started is a persistent, accurate and courageous analysis in the spiritual field surrounding terrorism.”

<sup>6</sup> If the expression “le trahison des clercs” has an appropriate use in any connection at all then it has one in this one.

<sup>7</sup> Notions of space and time come in so indirectly in this connection that I do not mention them on a par with logical and numerical assumptions.

(to which belongs an understanding of the cognition of persons who maintain that the sacrifice of individual lives for “the totality” of large collectives or for abstract (“concrete”) ideas or ideals is positively defensible).

This thesis hinges on a certain outlook on what is philosophically important about human languages.

#### 4. THE PALIMPSEST THEORY OF SYNTACTIC FORMS

The utility of metaphor in communicating an opinion or a hypothesis in a rough but often expedient manner is well known. In order to communicate my outlook on the current set of ways of expressing oneself I shall make use of a metaphor coined by the linguist Peter Maher. Maher describes a so-called “natural” language as the sum total of inscriptions on a palimpsest, i.e. a parchment on which several texts have been written and subsequently wiped out in order to make room for a new text; however, the elimination of the old text is never complete so that the most recent one has to be deciphered on the background of layers of old inscriptions.

Maher applies this only to the theory of meaning of certain nouns, verbs etc. It seems to me that a vastly more important – and totally neglected – aspect of the palimpsest outlook on ordinary language (*die tradierte Sprache*, as the Germans aptly say) is a palimpsest understanding of present-day syntactic forms and of the elements from which they are constituted. As against an exclusively Chomskyan outlook as well as against an exclusively Montaguean outlook on what is culturally important in the study of “natural” languages I recommend the palimpsest theory of the current set of syntactic forms and their uses. (Since this name, “the palimpsest theory of syntactic forms”, contains a metaphor it already suggests the theory we need in rough outline.) Such a theory is absolutely indispensable to workers in the theory of argumentation and to people who are concerned with argumentation analysis of given texts.

Bertrand Russell, co-creator of contemporary mathematical (“modern”) logic, already held this palimpsest theory of “natural” language, as did the famous linguist Edward Sapir who was his contemporary. A particularly felicitous formulation of Russell’s philosophy of language and its relation to human thought is to be found in a rather unknown book that he published in 1959. I quoted it above and shall do it again:

“... ordinary language is shot through with the fading hues of past philosophic theories” (Russell 1959, p. 309).

Maher quotes Sapir as saying:

“Language and culture . . . are obviously not the direct expression of racial psychology and physical environment, but depend for their existence and continuance on the forces of tradition. . . . A grammatical system tends to persist indefinitely . . . . The forms of language will in course of time cease to symbolize those of culture; and this is our main thesis . . . . The forms of language . . . more accurately reflect those of a remotely past stage of culture than *the present ones* of culture itself” (Sapir 1949 (1912), p. 101f.)

The culturally most important example of this is found in many present uses of “generic” terms. Through “the mechanization of the world picture” (Dijksterhuis) we have lost contact with the model or models behind the genuine historical generic uses of terms and of articles, as behind several other systematically related linguistic forms – “as”, “as such”, “insofar as”, as well as many uses of modalities (cp. my (1974a)). – The original semantic models have been lost, to the point that they are now extremely hard to retrieve. As a result we are left with a syntactic inheritance which has no semantic counterpart in the ontologies that are rationally accepted by most speakers (to say nothing of the listeners), except by paraphrase. I shall give a couple of examples in a moment.

This insight of Sapir’s and Russell’s has been suppressed, forgotten or overlooked not only by grammarians but by analytic philosophers and logicians as well, due in large measure to a false or correct interpretation of the writings of Wittgenstein. For a non-linguist, at least, it is hard to see precisely how Russell’s and Sapir’s outlook could be superimposed upon or otherwise integrated with the Chomskyan philosophy of language and with syntactic theories of the TG type. The situation seems somewhat more favourable vis-a-vis Montague’s approach, which admits of an interpretation as a theory which is *in part* a regimentation, i.e. normative, rather than *merely* descriptive. It regiments where it leaves something out and says so – for instance where it *expressis verbis* leaves out one sense of a many-sense expression or linguistic form, while describing another. For the contention that his theory does leave out important senses of current expressions we have Montague’s own words, in a statement which is as important as it is neglected, and which in my opinion does him great honour:

“ . . . I have made no attempt to capture the ambiguity, felt strongly in this sentence [*a unicorn is an entity such that . . .*], according to which the indefinite article *a* may sometimes have the force of universal, as well as the more usual existential, quantification” (Montague 1970, p. 397).

It seems to me that on the strength of this remark the palimpsest

*outlook* on syntactic forms may be attributed to Montague as well as to Russell, although in his philosophical work Montague did even less with it than Russell did in his. Neither of them attempted a systematic exposition, which is a task that should be dealt with by historical linguists in co-operation with logicians.

The unpacking of the metaphor, “the palimpsest theory of syntactic forms”, could take the following form, to start with:

*Thesis.* Investigators of common language in the twentieth century, and among them students and promoters of argumentation and argumentative linguistic forms, must be prepared to frequently find generally accepted syntactic forms (forms that are accepted by “the native speaker”) that are left-overs from earlier periods, which were dominated by semiotic assumptions more or less foreign to those of our time,

the explanation being that – as Maher puts it – “syntactic and paradigmatic structures linger on after the disappearance of the circumstances that motivated their surface form” (o.c.). I shall put this somewhat differently, with no reference to “circumstances”:

*Axiom.* Syntactic forms and their constituents are eliminated from ordinary as well as from philosophical language more slowly than the semiotic and ontological systems they belong to and in terms of which they can be defined.<sup>8</sup>

I shall speak of the *time-lag* or *delay* of structural elements and of accepted syntactic forms in a language at a certain moment of time, *relative to* the set of accepted ontologies and semantic bases at that moment of time.

The phenomenon of syntactic time-lag is very conspicuous in the language of some twentieth-century continental philosophers. Edmund Husserl, for example, must be read with this in mind, particularly where his “logical investigations” are concerned. Unfortunately this has never been done consciously and consistently. There are, among philosophers, some paradoxical cases:

*Some philosophers even employ generic sentences and other expressions belonging to the same linguistic system (vernacular) for the purpose of amending or even of rejecting the very ontology in terms of which such expressions were originally understood – Jean-Paul Sartre is a clear-cut example; or for the purpose of rejecting the fundamental semiotical assumptions behind such linguistic systems, such as the assumption of a necessary basis in interior monologue –*

<sup>8</sup> Hempel and Oppenheim’s celebrated book, unfortunately no longer available, *Der Typusbegriff im Lichte der neuen Logik*, should also be studied from this point of view, i.e. its linguistic implications should be spelled out (as I have tried to do in my (1971, 1974)).

as Jürgen Habermas tries to do. Neither of these philosophers has understood the language he inherited.

It should be obvious that as long as this phenomenon of syntactic time-lag has not been widely studied and as long as it is not quite generally recognized, the ideal of a workable and widely acceptable normative theory of argumentation remains a chimaera.<sup>9</sup>

In the works of a great many older philosophers, however, the same sentence type is used in a genuine manner, i.e. in accordance with the ontology or logical model of the author. Georg W. F. Hegel quite explicitly states that in “judgements of reflection” the singular sentence form is to be used instead of the universal one:

“statt *Alle Menschen* ist nunmehr zu sagen: *der Mensch*” (*Wissenschaft der Logik*, Lasson-edition, II, p. 292).

However, as long as there are no clear definitions of the meaning – in – critical – use of expressions as “*der Mensch*”, or “*Man*”, “*the/a state*”, and so forth, one cannot employ them in a “generic” sense in critical discussion and wish for a rapid flux of opinion at the same time.<sup>10</sup> It seems reasonable at this moment to expect several other sentence forms and expressions to fall into the same category as *unsuitable or impossible as ingredients of instruments for effective or dynamic resolution of conflicts of opinion by means of language*.

The common retort against studies of this type, viz. the retort that *important and serious conflicts of interest* cannot be resolved by means of language, has the following answer, in four parts:

(i) The theory of argumentation primarily studies *conflicts of opinion* not *conflicts of interest*,

(ii) conflicts of opinion are usually components of conflicts of interest,

(iii) some important and serious conflicts of interest have been settled, and more have been softened, because conflicts of opinion associated with these conflicts of interest were approached with verbal dialectics,

(iv) by improving the instruments (viz., the part or parts of language selected for the purpose, together with the dialectical system,<sup>11</sup> a number of future important and serious conflicts of interest – if not all – may be softened – and perhaps sometimes solved – by means of verbal dialectics applied to the accompanying conflicts of opinion, if and where such components are present.

<sup>9</sup> Examples of philosophical uses of sentences showing syntactic time-lag will be given and discussed in a later paper. Concerning Hegel’s use of contradictions and of “contradiction”, see my (1970).

<sup>10</sup> A detailed discussion of this is contained in my (1971), (1974).

<sup>11</sup> Here: critical-dialogical system; cp. Barth and Krabbe 1979 (1978).

The above will suffice to explain the importance of the following question:

*What is (was) the semantic correlate of generic abstracts – used so profusely in the writings of the philosophers mentioned for instance by Heigert or Soyinka – according to the view of the originators of the logico-linguistic tradition(s) to which they belong?*

In order to answer this question it turns out to be necessary (which is not to say that it is also sufficient) to study the history of notions of “the infinite” and “infinitesimals”. We shall have to go back in time beyond those philosophical distinctions which were introduced in the middle of the last century by, among others, Hahnemann’s and Hegel’s contemporary, Bernard Bolzano, and that are so natural to present-day philosophers of the analytic schools, but which are totally unknown to the thinkers mentioned by Soyinka and Heigert.

By taking up the philosophico-mathematical history of the notion of infinitesimals, I am carrying out a programme I suggested in (1977).

## 5. FIRST DILEMMA: INFINITESIMAL LOGIC OR ARGUMENTATION?

### 5.1 *The Cusanus tradition: infinitesimal entities, of the nature of mind*

The oldest line concerning *infinities* in European philosophy is that whose most characteristic post-medieval representatives were cardinal Nicholas of Cues (Cusa) – theologian, philosopher of nature and philosopher of mathematics, the astronomer Kepler, and the philosophers of the German idealist school and their contemporaries: Samuel Hahnemann, founder of homoeopathic medicine, the philosopher Georg W. F. Hegel, the psychologist Gustav Theodor Fechner. I shall call this tradition “the Cusanus tradition”, for short. Since the official overthrow of the outlook in this school on infinite sets, on series and limits by the Cauchy-Weierstrass mathematics, its adherents have concealed and even tried to suppress the presuppositions about *infinities* in their own systematic thought. As a consequence the following two questions, at least, could for a long time not be answered at all:

(i) What is meant by the expression: “das Umschlagen von Quantität in Qualität” – “the shift from quantity to quality”?

(ii) Why did Samuel Hahnemann – and why do some of his contemporary followers – sometimes dilute medically active matter down to concentrations like  $10^{-24}$  and below?

Writing in the 15th century Nicholas of Cusa defines the infinitesimal or the infinitely small, “the minimum”, as

(C) *that* which cannot be (made) smaller

(Minimum autem est, quo minus esse non potest, *D.d.i. I*, 11), and similarly infinity or the infinitely large as *that* which cannot be (made) larger, thereby presupposing that (in some sense of “is”) there *is* something which cannot be (made) smaller (or greater) – an assumption which, as we shall see, both Leibniz and Newton rejected. The existence or being of something which is smaller than everything else and hence *the smallest* or *Absolutely Small* follows readily when “Aristotle’s Principle of the Absolute” (Beth 1956, p. 9–14) is applied to the relation *y is smaller than x*. According to Nicholas of Cusa, *the smallest* is “the principle”:

“Sicut igitur unum est principium numeri, ita est pondus minimum principium ponderandi, et mensura minima principium mensurandi. Vocetur igitur pondus illud uncia et mensura petitem: Nonne sicut per unum numeratur, ita per unciam ponderatur et per petitem mensuratur? Sic etiam ex uno est numeratio, ex uncia ponderatio, ex petito mensuratio. Ita et in uno est numeratio, in uncia ponderatio, in petito mensuratio” (*Idiota de sapientia, liber primus*, ed. Baur, p. 7. 7–9).

In English:

“For just as One is the principle of Number, in the same way the minimal weight is the principle of weighable things, and the minimal measure [*mensura* – E.M.B.] is the principle of measurement. For that weight is called ounce, and [the, a] measure petit . . .”

*De docta ignorantia* is still more interesting for our purpose:

“Maximum enim, cui non opponitur minimum, necessario omnium est adequatissima mensura . . . Est igitur adaequatissima et praecisissima omnium essentialium mensura infinita essentia” (I, 45).

“Relinquitur ergo quod infinita linea sit ratio finitae lineae. Ita maximum simpliciter est omnium ratio. Ratio autem est mensura. Quare recte ait Aristoteles in *Metaphysicis* primum esse metrum et mensuram omnium, quia omnium ratio” (I, 47).

In English:

“For the Largest/Greatest, to which the Smallest is not opposed, is necessarily the most adequate measure of everything. . . . The infinite essence/being is therefore the most adequate and precise measure of all essences/beings.”

“Consequently the infinite line must be the Reason of the finite line. In the same way the simply Maximal is the Reason of everything. Aristotle therefore in [his] *Metaphysics* correctly holds the First to be the measure of everything, since it is the Reason of everything.”

Notice Nicholas' expression "the (a) First" – *primum* – which I shall make use of later on.

(Concerning the quotation from *Idiota de sapientia*: Lorenz says

"das kleinste Gewicht [ist] Prinzip des Wägens" (Lorenz 1926, p. 32),

and Bohnenstädt:

"die kleinste Gewichtseinheit [ist] Begründung des Wägens" (Bohnenstädt 1954 (1936), p. 45).

The Shorter Oxford English Dictionary (1967 (1956)) reports that "ounce" (from "uncia") was used in the middle ages for "a small quantity" generally, and in 1526 was said figuratively of "imponderable things". Similarly "inch" (also from "uncia") was employed in transferred and figurative use for "the least amount or part" (of space, time, material or immaterial things). I conclude that "uncia" and "petit" (an old unit for measuring the volume of fluids) in the passage quoted are to be given a partly metaphorical/figurative reference. – For Nicholas of Cusa the principle of measurement is nothing else than the principle of all measureable things – an infinitesimalistic variant of operationalism.)

Bohnenstädt explains that

"Es kommt dem Cusanus im *Idiota* vor allem auf die Natur des Geistes an. Geist is 'mens'; Cusanus bringt (*ähnlich wie Albertus und Thomas*) 'mens' mit 'mensura' zusammen. So hat die 'mens' als Grundfunktion das 'mensurare', von dem hier wägen, zählen, messen als Spielarten aufgezählt werden" (o.c., p. 94 n. 10; italics mine – E.M.B.).

For Nicholas, the Maximum and the Minimum coincide (as they are aspects of the Superlative) and this imponderable mental entity/notion is a (the one and only) transcendental Absolute:

"Supra omnem igitur rationis discursum incomprehensibiliter absolutam maximitatem videmus infinitam esse, cui nihil opponitur, cum qua minimum coincidit. Maximum autem et minimum, ut in hoc libello sumuntur, transcendentis absolutae significationis termini existunt" (de docta ignorantia, ch. IV, 23–27).

So Cassirer warns us against misconceptions:

"Das Maximum ist kein Grössenbegriff, sondern ein rein qualitativer Begriff; es ist der absolute Grund des Seins, wie es der absolute Grund der Erkenntnis ist" (1962, p. 21).

But Cassirer has missed the connection with the history of the calculus, and therefore he has missed the connection with the doctrine of the "Umschlagen" of quantity (or extendedness, extension) into Quality (or intensiveness, intensity, intension). For – as Paul Wilpert writes in the preface to his German translation of *De docta ignorantia* – a systematic commentary to the works of

Nicholas of Cusa from the point of view of the history of ideas was never written.

The assumption of something that can be called (the) Absolutely Small concerning an arrangement of quantities by means of the relation  $x$  Is Smaller Than  $y$  would not be strange at all if “quantity” could always be taken to mean either non-negative number or positive integer, so that  $x$  and  $y$  could be thought of as positive numbers or integers; for then the number zero or the number one would satisfy the description. However, the terminological and conceptual distinction between ‘number’, ‘numeral’, and ‘quantity’ or ‘magnitude’ which is taken for granted in large circles today was introduced into (part of) European philosophy in the nineteenth century.<sup>12</sup> The philosophical technical terms “quantity” and “magnitude” formerly covered numbers (in the then current meaning of “number”, which was extremely vague), but in addition everything to which a number can purposefully be ascribed at all, i.e. all measurable things.<sup>13</sup> As generated by the APA when applied to the *smaller-than* relation, the “infinitesimal”  $S$ -entity which tops an infinite series of decreasing  $S$ -entities, quantities or magnitudes, has *exactly the same (fundamental) nature (S-nature)* as the other elements in the series and so is a number (carries the properties of numbers) only if the other elements are numbers. If they are line segments, then *the infinitesimal* will not be a number but a

<sup>12</sup> Yovel (1973, pp. 256–259) holds that Hegel uses “number” for numerical expression and “quality” for number (in the Frege-Russell sense of “number”). Hegel regards the number  $\frac{2}{7}$  as “the good infinite” of its own decimal expansion 0,285714 . . . , the infinite but never completed series of decimals (or rather: of their numerical expressions) being a “bad infinite”. The number itself – the good infinite – is called a quality. “Il semble que Hegel ait besoin de cette idée générale pour plusieurs raisons systématiques . . .” (Yovel, l.c.). – See also Hegel’s Polish contemporary, Hoëné Wronski (1814), a mathematician belonging to the Cusanus tradition.

<sup>13</sup> Compare the historian E. T. Bell: “The academic Greek mathematicians were at ease with number only when it was geometrized into ‘magnitude’ – a vague concept whose tenability they seem never to have questioned. Thus number was supposed to be apprehensible through form, the opposite of what the Pythagoreans first held and of what a majority of mathematicians have believed since Descartes” (p. 183). . . . “From the calculus of Newton and Leibniz to that of Lagrange there is no indication that analysts were aware of the necessity for an understanding of the real number system. Nor is there in the next stage, that of Cauchy. Even as late as 1945 ‘quantities’ occurred frequently in the writings of professional analysts with no explanation of what a ‘quantity’ may signify” (p. 291). . . . “‘Magnitudes’, as we have seen, were replaced [by Dedekind, Cantor and Weierstrass] by ‘numbers’, and geometrical intuitions were driven out to make room for those of traditional [read: bare, without mention of ‘quantity’ or ‘quality’ – E.M.B.] logic. Nebulous ‘quantities’ persisted in the analysis of some” (p. 294).

line segment;<sup>14</sup> if they are decreasing degrees of intensity of a sensation, *the infinitesimal* will again not be a number but an intensity – *the lowest* – of a sensation (of the same kind) – which is the assumption behind the mathematical psychophysics of Fechner. Much as if what we today would call (the notion of) the (physical) dimension(s) of a magnitude were to be approached by contemplation of smaller and smaller instances of this magnitude – a strange assumption indeed.

However – so the philosophers of the Cusanus tradition said – “the quantitative aspect” of *the* concluding, or infinitieth, term of an infinite series was zero; and they added: “as a quantity” it had disappeared. Since zero itself, being a number, was – for them, on their theory of number – *ipso facto* a quantity, it follows that for them *the* infinitely small could not simply be zero (as Newton, Euler and many others would have it). In other words: any infinite series of decreasing or increasing (or otherwise linearly ordered) terms has of necessity an extreme or “maximum”, which itself is a member of the series; we would say: it has an upper *bound*.

‘The infinitesimal’ was not merely a mathematical notion (as in contemporary non-standard analysis). The idealist philosopher Hermann Cohen, writing in 1883, held that

“In Kant’s time, the identity of the intensive and the infinitely small was generally assumed” (p. 14, my translation – E.M.B.).

Before I got acquainted with Cohen’s book, independent investigations based on quite different material (cp. my (1977)) led me to assume that this is the case: in Kant’s time, the identity of the intensive, or merely qualitative, the powerful, and the *infinitesimally small* was very commonly though not universally assumed. In 1819, a German chemist named F. F. Runge wrote:

“Geleitet von der Analogie und von dem aus der Naturphilosophie geschöpften Prinzip, *dass die dynamische Wirkung an Intensität sich steigert, je mehr die Masse abnimmt*, habe ich usw.” (quoted from Snelders 1973, p. 178, italics mine – E.M.B.).

It seems to me that it is not quite correct to ascribe this assumption especially and exclusively to the Naturphilosophen and that it had a wider acceptance. However that may be, that the dynamic effect increases as to intensity when the mass decreases, or: that the intensity of the dynamic effect can be increased *only if* the mass is being decreased – precisely this was assumed as a principle,

<sup>14</sup> Probably more appropriate still is the following formulation: if they are ideas of line segments, then the infinitesimal will be the idea of a line segment (of the smallest possible); and so on. Different writers in the Cusanus tradition may have to be interpreted differently in this respect.

without any attempt at a justification by other principles, by Samuel Hahnemann (see above). I discovered Cohen's book (thanks to mr. V. Sánchez Valencia) and this crucial quotation from Runge (which Snelders does not seem to do anything with at all) several months after the publication of my (1977), hence they come as strong corroborations of the theory which is developed there and which is further developed in the present paper.

Cohen, however, also ascribes the identification of 'the intensive' and 'the infinitely small' to Kant himself, for which I can as yet find little or no direct justification, and he also says that in his correspondence with other scholars Leibniz, too, mentions this principle in passing, but I have been unable to verify this.

In his *History of the Calculus*, Boyer locates the coming into being of "the notion of *intensive* magnitude as opposed to *extensive* quantity" to Germany (Boyer, p. 178). "In the 17th century the Intensive seems primarily to have meant the non-extended", Hermann Cohen wrote, and he personally wanted to keep it that way, while identifying the relation of extended to non-extended – called "intensive" and "infinitesimal" – with the relation between *anschaulich* and merely thinkable (or perhaps we should say: between merely *anschaulich* and thinkable). Hence, *the* powerful (potent) Infinitesimal is the non-extended, thinkable but not observable.

As we shall see, Leibniz' notion of a powerful *monad* (simple substance, unit substance) and his notion of an *infinitesimal* were distinct notions. In the nineteenth century, Cohen, however, like Hegel had done before him, again fuses the two: *the infinitely small* is, for him, nothing other and nothing less than "the analytical expression of the monad" (p. 54), hence they may be identified (p. 92).

Cohen therefore can say that the square, or second power/*Potenz*, of a differential or infinitesimal  $dx$ , i.e.  $dx^2$ , is *simpler than*  $dx$  itself; this means: more nearly monadic, hence (still) more forceful (hence: more intense, of a higher degree of intensity). The square of the infinitesimal  $dx$  is *latent in*  $dx$ , so he says (p. 92; cp. the Renaissance notion of being *occultly present in* something). The differential or the differential quotient is understood in idealistic philosophy as "the intensive *origin* of movement" (p. 95, italics mine).

This means that a velocity is seen as the (nearest) *origin* of a movement and acceleration – hence force – as the (nearest) *origin* of velocity. If we assume that Cohen's own relational concept ( $x$  Has  $y$  as its (Nearest) Origin) is a transitive one, then it follows that force is the origin of movement – precisely as taught by Aristotle (and denied by Newton). It means, furthermore, that

velocity is subsumed under the general category of intensity. The latter assumption is also characteristic a.o. of the Hopi language (cp. Whorf 1956).

In 1789 (<sup>2</sup>1796) an important terminological shift is inaugurated: L. Bendavid introduces the term "Qualität" for non-measurable "magnitudes", at the same time as he retains the earlier vernacular of infinitesimals as "intensive", and even uses the expression "intensive property" for the infinitesimal (Cohen, p. 113). Hence the infinitesimal is now called "a quality". This can be studied for instance in the works of Hegel which were written shortly afterwards – but just before Cauchy, in the 1820's, abolished the notion of infinitesimal terms as logical constants, substituting for it a notion of infinitesimals as variables (differentials). To Hegel they still are logical constants, to be spoken about in the definite grammatical singular – *das unendlichkleine (So-und-so)* :

- (H) "Indem nun *das Unendlichgrosse oder -kleine ein solches ist, das nicht mehr vermehrt oder vermindert werden könne, so ist es in der Tat kein Quantum als solches mehr*" (*Wissenschaft der Logik*, Lasson-edition, I, p. 242, my italics – E.M.B.).

From this he concluded, as the other thinkers in the Cusanus-tradition did, and as his contemporary the physician Samuel Hahnemann did, not that there was no such entity as "the infinitesimal", but that in "the infinitesimal" nothing more – *and nothing less* – was retained than the property characterizing the terms of the series. He does this in the terminology of Bendavid:

"Das unendliche Quantum . . . ist einfach [i.e., a monad – E.M.B.], und daher nur als Moment; es ist eine Grösse-bestimmtheit in *qualitativer* Form; seine Unendlichkeit ist, als eine *qualitative* Bestimmtheit zu sein" (p. 244f.).

Hermann Cohen says, coming to Hegel, that

"Hegel . . . made, one might say, a criticism of the [Leibnizian-Newtonian] concept of the infinitesimal the basis of his logic" (p. 118).

In order to verify this statement it suffices to read Hegel's work.

Every interpretation of Hegel's logic or ethics that tries to bypass his philosophy of mathematics is false.

Boyer sums up the situation in the philosophy of mathematics of the Cusanus tradition as follows:

"idealist philosophers, following Kant, were . . . unwilling to accept the bare [not potency-oriented] formalism of Cauchy and Weierstrass in the realm of the calculus . . . [They] attempted . . . to interpret the differential [ $\Delta x$ ] as having an intensive quality resembling the potenti-



I refrain from trying to characterize the notion, mentioned by Lovejoy, of continuous or infinitesimal gradation (cp. Lovejoy 1936, p. 59ff). That is to say, the symbol “ $\oplus$ ” will have to remain undefined.

This muddled conception, suggesting the “ideal type” of converging series, was already present when, in the seventeenth century, the differential or infinitesimal calculus with its quotient-like expressions “ $dy/dx$ ” entered the philosophical scene. That development required amendment of the old conception. Thus we see that Hegel often, though not everywhere, ascribes Aristotle’s potentiality to the differential *quotient*,  $dy/dx$ , which he takes to mean: the relation between two (non-variable) infinitesimals  $dy$  and  $dx$ , rather than to each separately. This quotient, *Verhältnis*, opposition, polarity, or “contradiction” is Hegel’s *gebrochene Mitte* (cp. van der Meulen 1958). It cannot be denied that Hegel’s opinion here not only is in harmony with Cusanus’ philosophy of polarity but with the dichotomizing approach to logic in *Ramism* as well (cp. my (1970)).

After the publication of Taylor’s theorem in 1715, faraway (powerful, near-infinitesimal) “terms” in the older conception were compared to faraway (near-infinitesimal) terms in a Taylor-expansion (infinite, without remainder!) of a function of real numbers. This is reflected e.g. in the following quotation from Hegel, where he rejects the idea – which he ascribes to Newton and others – of infinitely small *magnitudes* (here: quantities, as against Qualities):

“According to this same [idea], they [the infinitesimal entities] are to be such that not only can they be disregarded in comparison with finite magnitudes, but also their higher orders in comparison with lower orders, as well as products of several [infinitesimals] in comparison with a single one” (*Wissenschaft der Logik*, I, p. 259; my translation – E.M.B.).

## 5.2 *Leibniz and the mathematician’s opponent*

Leibniz, who shares with Newton the honour of having fathered the infinitesimal calculus (as it was formerly called), was less tied to the Platonic-Aristotelian finalistic principle of “the Absolute” – the APA – than Cusanus seems to have been. He thought that things and substances are usually compounded from more simple, in the sense of less compounded, substances, and in this case did believe that there must be ultimate – here in the sense of absolutely uncompounded – substances; he called them monads. These constructs may be understood as the result of applying (Plato’s and) Aristotle’s Principle of the Absolute to the relation  $x$  Is-compounded-of  $y$  (in combination with  $z, u, \dots$ ).

Leibniz cannot, however, have regarded the Aristotelian principle as universally and logically true, he did not take it as a tautology, and he did not apply it indiscriminately. For he says:

- (L1) "... given infinitely many terms [in some linear order, E.M.B.] it does not follow that there must also be an infinitesimal term" (Loemke (ed.), p. 514),

where "infinitesimal" seems to mean simply "infiniteth and last". This is a clear rejection of Cusanus' statement ((C) above). Concerning the special cases of infinitely many line segments of decreasing size he holds, however, that one may often profitably speak and think *as if* such sets did have an upper (or lower) bound, *the* infinite and *the* infinitesimal line segment in the series, so that

"... even if someone refuses to admit infinite and infinitesimal lines in a rigorous metaphysical sense and as real things, he can still use them with confidence as ideal concepts *which shorten his reasoning*, similar to what we call imaginary roots in the algebra" (o.c., p. 543, italics mine – E.M.B.).

When we remember that this was written before the modern distinction between number and magnitude (hence also between infinite numbers and infinite magnitudes), this might be taken to mean that Leibniz approved of infinitesimal numbers though not of infinitesimal magnitudes (non-numerical measurable entities) or of any other "infinitesimal" entities. But Leibniz also says something else, namely that

- (L2) "the mathematicians' demand for rigor in their demonstrations will be satisfied if we assume, instead of infinitely small sizes, sizes as small as are needed to show that the *error* is less than that which *any OPPOSITION* can assign, and *consequently* that *no error* can be assigned at all" (o.c., p. 584, italics and capitals mine – E.M.B.).

What I want to draw attention to here is that Leibniz at this point characterizes what would count as a *rigorous* proof in terms of an interaction between an opponent and a proponent of a thesis, much as in classical mathematical thought. In his characterization of rigorous proof the notion of "infinitesimals" has dropped out of the picture. That of an opponent, i.e. a user of language who systematically expresses a critical attitude (takes the *contra-position*, as I call it) to a proposed mathematical result (statement), has re-entered the philosophical scene, *compensating for the absence of the former notion*, that of 'infinitesimals', and – in Leibniz' opinion – even surpassing it in its capacity of yielding a *rigorous* proof procedure. This shows another side of Leibniz than that which

Arthur Prior capitalizes upon when ascribing to him a logical outlook which he – Prior – calls Egocentric.<sup>15</sup>

The quotations are from letters written by Leibniz in the years 1688–1702. They demonstrate a remarkable cognitive distance from the “visionary” Cartesian idea-ist philosophy of the nature of mathematical proof,<sup>16</sup> though not necessarily of the nature of mathematical entities. In any case, in expressing his belief in the *possibility* of doing away with infinitesimals altogether Leibniz is offering a clear conceptual alternative to the Cusanus tradition. The development of a calculus without infinitesimals was brought to its successful conclusion so far (and *pace* Abraham Robinson) by another German thinker, Karl Theodor Wilhelm Weierstrass, during the second half of the nineteenth century. The mental Infinitesimal was eliminated via, first, the introduction of the notion and terminology of the Limit (of an “infinity” of terms) and, second the formulation of a *rule of identification* for such a limit if there is one (cp. Note 1):

(1) An infinite series of numbers  $S_1, S_2, \dots, S_n, \dots$  is said to have, or to converge to, a limit  $S$ , in symbols:

$$(i) \lim_{n \rightarrow \infty} S_n = S$$

if and only if:

from some term onwards each term in the series (“an arbitrary term”, or “a given term”) differs from  $S$  by less than a “given” real number  $\varepsilon$ , however small that number may be.

The variable “ $\varepsilon$ ” suitably abbreviates “error”. In order to facilitate comparison with the Cusanus tradition, “(real) number” should be replaced by “quantity”:

from some term onwards, each term in the series differs from what is said to be the absolute limit of the series,  $S$ , by less than the small given quantity  $\varepsilon$ .

This – relational – explanation (explanans) of what is meant by

<sup>15</sup> I do not doubt Prior’s interpretation. However I insist upon recognition of the fact that Leibniz *also* had it in him to formulate the logic of the “infinitesimal” calculus in terms of critical debate. So, when Prior says: “I have constantly referred to the *speaker*; but no such person is mentioned within Egocentric itself (Prior 1977, p. 38), then – *a fortiori* – in Egocentric itself no mention is made of *the listener* or of *the opponent*. Hence we are forced to say that *the logic of Leibniz’ philosophy of mathematics* was *not* written in Egocentric. Egocentric is, it seems to me, (even) much more characteristic of many other philosophers (e.g., of those in the Cusanus tradition, Descartes) than it was of Leibniz, heir to Eudoxos, Euclid and Archimedes.

<sup>16</sup> An eminently readable account of which is found in Hacking 1975.

(i) does not contain any expression referring to “infinity”, nor does it contain any reference to or assumption about the existence of “infinitesimal” non-Archimedean differences at all.

The explanation can be made more easy to handle (“more precise”) by formulating it in internationally known well-defined symbols. However, since the explanation is a relational one, *this requires a language form in which one has recourse to multiple application of the syntactic operations called universal and existential generalization*, pertaining to real – hence Archimedean – numbers. The language form of first order Fregean predicate logic has the feature of allowing for this. The reformulation consists in the following well-known definition, formulated in a language of that form (the references to opponent and proponent being added by the present author):

$$(2) \quad \lim_{n \rightarrow \infty} S_n = S \quad = \quad \text{Df} \quad (\varepsilon)(\exists N)(n)[n > N \rightarrow |S - S_n| < \varepsilon]$$

$\uparrow \quad \uparrow \quad \uparrow$   
 opponent's choice  
 $| \quad |$   
 proponent's choice  
 $|$   
 opponent's choice

The variable “ $\varepsilon$ ” suitably abbreviates the word “error”.

The symbol “ $\infty$ ”, however, does not occur in *explanans* at all. The word “given” in the phrase “a given number  $\varepsilon$ ” that occurred in the first formulation offered in explanation of (1) means that the mathematician who proposes a statement of the type (1) is to be given (to be informed about) this  $\varepsilon$  *by his/her opponent*, as the error that this particular opponent is prepared to tolerate. *The logic at work here is a completely practical one.* It can be unpacked in a “dialectical” or double-focussed Beth-Lorenzen analysis of the procedure of a proof of (1). I shall show it in a dialogical tableau of the kind that was developed by Paul Lorenzen, as enriched with Naess’ dialogical notion of a “precization” of a locution<sup>17</sup> (See p. 29).

Notice that an infinitesimal ( $dS$ ) is not mentioned at all. (– I have used the letter “ $S$ ” in this diagram, but it is hard to choose between “ $S$ ” and “ $\sigma$ ” – cp. Section 5.1 – for the purpose of bringing out the complex relation between the vague old conception and the modern theory. This will be felt strongly in the application below.)

The limits – if any – of series of increment-quotients  $\Delta y/\Delta x$  with

<sup>17</sup> Cp. Naess 1966. Naess’ translator A. Hannay introduced a new verb, *to precizate*, into English, as a translation of the Norwegian *å presisere*. It is a verb for a dialogical speech act, on a par with *to criticize*, *to defend*, etc.

Opponent	Proponent	Theses
(The opponent concedes the arithmetic of real numbers)	$\lim_{n \rightarrow \infty} S_n = S$	$T_0$
Prec.? (I.e., can you express more clearly just what your claims are?)	$(\epsilon)(\exists N)(n)[n > N \rightarrow  S - S_n  < \epsilon]$	$T_1$
Assuming that the opponent understands the claim "for all": $\epsilon_0$ ? (I.e., do you defend this for $\epsilon_0$ ?)	$(\exists N)(n)[n > N \rightarrow  S - S_n  < \epsilon_0]$	$T_2$
? (I.e., <i>how</i> do you defend that?)	$(n)[n > N_0 \rightarrow  S - S_n  < \epsilon_0]$	$T_3$
$n_0$ ? (where $n_0 > N_0$ ) (I.e., do you defend this for $n_0$ ?)	$ S - S_{n_0}  < \epsilon_0$	$T_4$
? (I.e., <i>how</i> do you defend that?)	(The proponent now has to show, given the arithmetic of real numbers, that $ S - S_{n_0} $ is indeed smaller than $\epsilon_0$ ; if this can be done for every conceivable opposition, then the proponent's initial thesis $T_0$ is mathematically true.)	

decreasing values of the variable  $\Delta x$  is a special case: if

$$y = f(x),$$

then the meaning of a statement of the following form:

$$(ii) \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = g(x)$$

is defined as follows: for every value of "x",

$$(3) \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = g(x) =_{\text{DI}} \underset{\substack{\uparrow \\ \text{opponent's choice}}}{(\epsilon)} \underset{\substack{\uparrow \\ \text{proponent's choice}}}{(\exists \delta)} \underset{\substack{\uparrow \\ \text{opponent's choice}}}{(\Delta x)} \left[ \Delta x < \delta \rightarrow \left| g(x) - \frac{\Delta y}{\Delta x} \right| < \epsilon \right]$$

(Observe that one defines the meaning-in-critical-use of a whole statement form, and not of the *limes*-term in isolation.)

The logic of a proof of the correctness of (ii) can be shown in a dialogue tableau. However, this time I shall change the "quanti-

riers” – references to second-order properties – into “instructors”, i.e. operators instructing the debaters about possible moves.

Opponent	Proponent	Thesse
(As above)	$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = g(x)$	$T_0$
Prec.?	$\varepsilon^o \delta^P \Delta x^o \left[ \Delta x < \delta \rightarrow \left  g(x) - \frac{\Delta y}{\Delta x} \right  < \varepsilon \right]$	$T_1$
$\varepsilon_0?$	$\delta^P \Delta x^o \left[ \Delta x < \delta \rightarrow \left  g(x) - \frac{\Delta y}{\Delta x} \right  < \varepsilon_0 \right]$	$T_2$
?	$\Delta x^o \left[ \Delta x < \delta_0 \rightarrow \left  g(x) - \frac{\Delta y}{\Delta x} \right  < \varepsilon_0 \right]$	$T_3$
<p><math>(\Delta x)_0?</math>, where <math>(\Delta x)_0 &lt; \delta_0</math>  (I.e., I have chosen a value of <math>\Delta x</math> which is indeed smaller than your <math>\delta_0</math>; do you defend it in this case?)</p>	$\left  g(x) - \frac{\Delta y}{\Delta x} \right  < \varepsilon_0$	$T_4$
<p>? (I.e., why should I believe this?)</p>	<p>(The proponent now has to show, given the arithmetic of real numbers, that <math>\left  g(x) - \frac{\Delta y}{\Delta x} \right </math> is indeed smaller than the proposed tolerance (acceptable error) <math>\varepsilon_0</math>; if this (etc.).)</p>	

(A prenex instructor  $i^o$  means that the opponent may freely choose a value of  $i$ , and a prenex instructor  $i^P$  means that the proponent has the right to do so.)

In the history and philosophy of mathematics there are two, and no more, competing paradigms concerning mathematical analysis, of which one is older than the other. There is, first, the view of Cusanus, Kepler, Hegel and Cohen, and indirectly of Samuel Hahnemann and Gustav Fechner, which goes back to certain Aristotelian assumptions and still earlier periods. Second, that of Newton and of Leibniz where he speaks about what constitutes mathematical rigour of proof, the view to which Eudoxos, Euclid, Archimedes, Euler, Lagrange, Cauchy, Bolzano and Weierstrass have contributed. In its fully-fledged form the latter view can be formulated in two-role argumentational terms, as we have just seen. Hence a philosopher who wants to entertain an opinion about

current systems of philosophy will have to side with either of these two outlooks,<sup>18</sup> and, if he is honest, he will have to take the consequences of his choice. Which is to say that he is confronted with a dilemma – he will have to choose between the notion of “occult” powerful infinitesimals and the possibility of critical discussion. He cannot have it both ways.

### 5.3 *A model for generic sentences: infinitesimal logic*

Returning now to the problem that I formulated at the end of Section 4:

What is (was) the semantic correlate (“meaning”) of a “generic” abstract in the philosophies mentioned by Heigert and Soyinka?<sup>19</sup>

– we realize that we shall have to split up this question into two separate ones:

- (I) What is/was the semantic correlate of a generic abstract as used by thinkers (of any rank) who take the logic of German idealism as a reference frame and who employ the same linguistic forms as those we find in the Cusanus tradition (even though some of their spiritual fathers – e.g., Marx – were explicitly critical of some uses of some generic terms – e.g. “der Mensch”)?

This question has no straightforward answer. We said at the end of Section 4 that the phenomenon of syntactic time-lag probably includes a large percentage of their uses of generic abstracts and

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<sup>18</sup> The mathematical work of Abraham Robinson, creator of so-called non-standard analysis, has had no impact on the *philosophical* world. It is viewed with skepticism by most mathematicians, so it is doubtful whether it ever will. More importantly, Robinson’s work is definitely not a revival of the Cusanus tradition and not even of the “pro-infinitesimals” part of the Leibnizian philosophy of mathematics. As Freudenthal aptly expresses it:

“There is more metaphysics in Leibniz’ [and his contemporaries’ and predecessors’ – E.M.B.] speculations on Calculus than is usually known, e.g., attempts to understand the relation of body and soul by an analogy with that between a magnitude and its differential. The genetic theory of preformation which asserted that the new creature has been preformed in its progenitors and particularly the whole of mankind in Adam, led to the idea of the differential of a genus, from which the genus developed as its integral” (in *Discussion*).

<sup>19</sup> We are of course not here posing – not to speak of answering – the quite general question of the meanings of generic terms in *any* school and in any period.

other semantically intractable forms, which is to say that in order to interpret them in a manner true to the intentions of their author one will have to read these abstracts somehow anaphorically or else to assume that they can be eliminated by paraphrase.

I believe, however, that many authors – and among them such diverse thinkers as Friedrich Nietzsche and Karl Marx, in spite of certain writings of theirs that perhaps may be taken to demonstrate the beginnings of a logico-linguistic awareness – were at best only half emancipated from the monadic, one-role infinitesimal logic of the mainstream German idealists with which they grew up, and that their thought as well as their language is “shot through with the fading hues of past philosophic theories”. I also believe that the importance of this has not been recognized and not been studied, though many writers have pointed to the fact itself. Schumpeter, for example, says, writing about Karl Marx:

“Pure philosophy of the German kind was his starting point and the love of his youth . . . It is no wonder that his German and Russian readers, by bent of mind and training similarly disposed, should seize primarily upon this element and make it the master key to his system” (1950, p. 9).

(Here Schumpeter might well have added readers from still other regions who also have, or had, a similar “bent of mind” and a more or less similar training.) This is

“a mistake and an injustice to Marx’s scientific powers”,

although

“he enjoyed certain formal analogies which may be found between his and Hegel’s argument. He liked to testify to his Hegelianism and to use Hegelian phraseology”.

But

“nowhere did he betray positive science to metaphysics” (p. 10).

The logician E. W. Beth has suggested (1965, part I, esp. p. 9–14) that the situation as to the right interpretation also of Marx’s thought may be *vastly more complicated*.<sup>20</sup>

I believe, furthermore, that this, understandably, holds to an even much stronger degree of *the vast majority of his followers*. Hence I recommend as a working hypothesis the assumption that in the circles that are directly or (very) indirectly influenced by

<sup>20</sup> I only know of one reaction to this suggestion by a Marxist, viz. the reaction of Zeleny (1970, p. 45f.), who tries to reduce the importance of Beth’s analysis by emphasizing the difference between “fixed” and “fluent” *Wesens* (read: Absolutes).

these authors, such *non-eliminable occurrences* of generic abstracts (or concretes) – “the State”, “Society”, “the Revolution”, and even “Man” – abound, occurrences that can neither be rescued by reference to earlier parts of discourse nor by paraphrase, however elaborate. For these occurrences which – *ex hypothesi* – are due to the influence of infinitesimal logic on the authors’ minds and on their language, we shall have to turn to the following question:

(II) What is/was the semantic correlate of a generic abstract in the philosophies of conscious or unconscious adherents to infinitesimal logic?

(The conscious adherents include the Polish mathematician Wronski, Fichte, Hegel, Hahnemann, Cohen, and – presumably – Fehner.) The answer to this question is:

The semantic correlate (or semantic value, as we would call it today) was either an “intensive” infinitesimal or it required for its interpretation the assumption of such infinitesimals, ontologically as well as epistemologically and logically.

#### 6. MENTAL, INTENSIONAL, ARGUMENTAL

To the inner language that was assumed as semiotically basic by all “idea-ists” (term used by professor Hacking), P. T. Geach has given the name “Mental”. What Prior calls “Egocentric” (above, Section 5.2) and ascribes to Leibniz is a logic for Mental, but not the only one. Another, related logic for Mental, culturally more important today, is the logic of intensive, powerful infinitesimals and infinities or near-infinities. Neither logic is oriented towards argumentation, neither can sustain argumentation.

When therefore, as often happens, philosophers of the idealist and Marxist schools talk with disdain about the possibilities and value of public debate and even oppose emphasis on the conditions for critical verbal communication, this should not come as a surprise. The philosophies in which they are raised and the language in which they are trained are founded on a semiotics of inner monologue (Mental), viz. the semiotics of non-expressed, one-role activities of Reason. Unfortunately most adherents of idealist logic and even some Marxists are more interested in polarity of gender than in polarity of logical roles. Since they are not yet emancipated from the semiotical, logical and linguistic background of idea-ist semiotics, infinitesimal logic and Mental, their very own use of generic abstracts prevents effective discussion.<sup>21</sup>

<sup>21</sup> This is the – professed – background also of the early works of Jürgen Habermas, which explains the irritation caused in logico-analytic circles

As a counter-argument in a rational discussion with idealists or with Marxists, in the sense of an argument *against their argument* against the desirability of discussion and of promoting argumentative attitudes, this is not a permissible argument. To regard what I have just said as such an argument would be to commit a fallacy of "looking to the source". It is not a counter-argument in a current discussion but a *thesis* – which may or may not start a new discussion – a thesis in which a certain connection is posited. To understand this connection is necessary if one wants to pursue the goals mentioned in the present paper.

Mental and its logic should not be identified with the post-Fregean development in intensional logic (due to C. I. Lewis, Kanger, Hintikka, Kripke, Montague and others). For a professional philosopher this remark is superfluous (or ought to be) but it may be of some use to non-philosophers. Let "Intensional" be our name for the language defined by Montague for the purpose of describing and analysing languages, "natural" as well as artificial. Intensional is not Mental, its logic resembles neither the Leibnizian Egocentric nor infinitesimal logic. It does not encourage "dialectical fideism" (Ong 1958, p. 56, my (1974), p. 397). Self-predication of abstracts is not assumed, hence – as Montague himself said, though in other words – in Intensional the traditional generic statements do not occur. There are no infinitesimals in its models, *a fortiori* no intensive or powerful qualities, infinitely small or large, and generating the individuals. There is no field of force. The old rational continuum that was supposed to carry a field of force is replaced by a grammar and ontology of discrete categories. A come-back of the notion of a rational field of force would therefore require a (re-)combination at least of (i) intensional models with (ii) non-standard analysis and (iii) a propensity theory of probability.<sup>22</sup> No such combination is in sight at present, as far as I can see.<sup>23</sup>

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by his recommendations, in a language which clearly reflects this background, of verbal discussion and of a definition of "Rationality" in terms of discussion and discussability.

<sup>22</sup> See in this connection: Ivo Schneider, 'Why do we find the origin of a calculus of probabilities in the 17th century?', preliminary draft, p. 22, about the continuum of qualitative levels especially in skeptical philosophy.

<sup>23</sup> Cp. Cocchiarella 1976 on the possibility of hierarchies of natural kinds in the framework of a variant of contemporary intensional logic. – The "method of idealization and concretization" in terms of which L. Nowak tries to reconstruct the pattern underlying the thought of Karl Marx (and which in one interpretation is trivial to any physicist), contains no explicit reference to a field of force. Second, Nowak does not (yet) discuss the significance of that method for the semantics and syntax of Marxian and Marxist *language*, i.e. for the propagation of Marxist patterns

Let "Argumental" be our name for a language with such properties that it will encourage and sustain discussion, due to its qualities as an instrument for the resolution of conflicts of opinion.<sup>24</sup> It will have to allow the construction of systems of *formal dialectics*. We do not know today how Argumental will look when it is fully grown. Clearly it will have to grow out of suitable parts of the "natural" languages. It will have to incorporate Fregean or post-Fregean logic, for which – thanks to Paul Lorenzen and others – systems of formal dialectics are available. It will be free from traces – "fading hues" – of past contra- or anti-argumentational assumptions and attitudes. We cannot avoid, however, that it will contain precipitations of very general contemporary outlooks as to the ingredients and structure of the world. Hence it cannot be a permanent language but will undergo revision as the ontological outlook evolves in the company using it. Similarly it will probably never be *one* language but a number of languages with a relatively simple common nucleus.

Argumental is not Mental, but it is not Intensional either. At the present moment the relation of Argumental to Intensional is not clear, due in the first place to the fact that the precise purpose of the latter, as of any other part of linguistic theory, is itself not too clear. Argumental shall, however, have to incorporate a great many elements and figures which are analysed semantically today on an "intensional" basis. Hence some – I would say: considerable – influence from the one field of research upon the other is unavoidable.

Observe that a necessary condition for the philosophers of the Cusanus tradition is that the expression "infinitesimal" be understood as linguistically monadic (unary): single magnitudes, later: single numbers, are said to be, or not to be, infinitesimal. Observe also that in the Cauchy-Weierstrass theory the term "infinitesimal" is no longer applied to numbers individually (e.g., not to values of functions) but to a function as a whole, e.g. to a series. Furthermore, that it is now treated as a relational predicate, more specifically: as linguistically and logically triadic; for a function (series) is said to be, or not to be, *infinitesimal for a certain variable at a value* (argument) of that variable. The infinitesimal concept of the

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of thought, or of what is taken to be Marxist patterns of thought. Third, Nowak does not (yet) discuss the pattern of Hegel's thought, and believes, as far as I understand, that it is possible to understand Marx without so doing. Cp. Nowak 1977.

<sup>24</sup> The question of the efficacy of uses of language for the purpose of dealing with conflicts of *interest* (as against conflicts of opinion) will not be dealt with here.

Cauchy-Weierstrass tradition is, then, as Thiel points out (1976), a *relation* between a totality, an individual concept, and a number.

Hence the antagonism between the two ways of looking at mathematical analysis is not merely one of egocentric, or idealist idea-ist, *versus* argumentational semiotics, it is at the same time intrinsically tied up with the distinction between monadic and polyadic thought. Leibniz, it seems, was too deeply impregnated with the idea of a monadic basis for logic to be able to set the notion of infinitesimals out of his head.<sup>25</sup>

Argumental will be free from all tropes and figures and all linguistic or logical assumptions that do not permit the construction of systems of formal dialectics (the expression is taken from Hamblin 1970).

## 7. SECOND DILEMMA: INFINITES-INFINITESIMALS OR INDIVIDUALS AS ENDS IN THEMSELVES

### 7.1 *The problem of logical firsts*

As was said above, the main belief in this connection that Leibniz did not propagate but which characterizes the Cusanus tradition in logic, metaphysics and the philosophy of mathematics, is the belief that, given any magnitude, i.e. a property that presumably can be had in different degrees, such as:

- (more-or-less) Human
- (more-or-less) Free
- (more-or-less) Perfect
- (more-or-less) Great
- (more-or-less) Small,

there is an – the one and only – infinitesimal, or infinitieth, term/value of the predicate, *than which nothing is more human* (free, perfect, great or small).<sup>26</sup> Leibniz, it seems, did not hold or propagate the belief that “the intensive” is “the infinitely small”, which thinkers in the mainstream of the idealist tradition (to which

<sup>25</sup> “In Egocentric, two-place predicates . . . disappear into modalisings of propositions” (Prior, o.c., p. 39). I.e., egocentric modalisings were used in part as a *compensatory device* (from a modern point of view), which to some extent made up for the absence of polyadic (or: polyadically interpreted) predicates. The parametric style introduced by Schönfinkel may be regarded as a solution to Leibniz’ problem of how to tuck away relata, though such relata-parameters are not considered today as determining a “modality” (such as a set of possible worlds), as Leibniz would have it.

<sup>26</sup> This means that, *at least* in discussions of idealism, what Beth calls “Aristotle’s Principle of the Absolute” – the APA – may be called “the Principle of the Infinitesimal/infinitieth term”.

the *Naturphilosophen* as well as Samuel Hahnemann may be counted) may – unconsciously – have derived from the former assumption in conjunction with the thesis that “the intensive” is the non-extended. The ensuing contradiction that “the (infinitely) Great is (the) infinitely small” (Cusanus, *De docta ignorantia* I, 11, 12, 45) can be responded to in different manners, but to the Fichte-school, which emphasizes (affirms<sup>27</sup>) rather than avoids flagrant contradictions, this caused no problem at all. The main point of interest for them is that by “going to infinity” or approaching it, by suitable means, one reaches or approaches the most powerful, “true” version of that very characteristic that defines the series. The infinitesimal, or infinitieth, is the bound to which the series is pointing. Going on – according to some method or other – from one term to a more remote one of the same kind one will, at least conceptually, by and by approach the *terminus ad quem* (Boyer) – the infinitesimal. At that stage the “quantitative aspect” of the terms is *aufgehoben*: the limit is not Euler’s simple zero but (a carrier of) the *logos* of all the terms that preceded it in the series. It is, to use a term taken from von Freytag-Löringhoff, their *logophore*. Or, as George Berkeley put it almost two hundred years earlier: “the ghosts of departed quantities”.

Many idealists, and among them Cusanus and Hegel, held the variant of this theory of “magnitude” according to which “the intensive” is not necessarily “the infinitely small” but rather “the infinite” generally, i.e. on either side of the spectrum of “magnitudes”. Hence a “leap from quantity into Quality” will also take place in the direction of magnitudes or *masses growing beyond the countably large*, and not merely in direction of the infinitely small. Hence Hegel says quite generally:

“das *Prinzip* der Philosophie [ist] der *unendliche* freie Begriff” (Hegel, *Wissenschaft der Logik*, II, p. 476, italics mine – E.M.B.).

But the infinitely small tends to be given a prominent place in his logic.<sup>27a</sup> The interpretations of the Cusanus-Hegel framework in twentieth century political philosophies return as it were to Nicholas of Cusa and favour the extremely large.

Hegel blames Newton, and Cohen, seventy years later, does the same, for his unphilosophical mode of thought, complaining that

“out of fear of the *occult* qualities . . . Newton refrained from taking *pure* concepts as his basis” (Cohen, o.c., p. 13, italics mine – E.M.B.).

<sup>27</sup> See W. Becker on “Fichte’s principle of affirmation” (Becker 1972, p. 10 ff.).

<sup>27a</sup> Cp. Schumacher 1973, p. 85–92.

In his discussion of Dühring, the idealist Cohen refers to “that concept of [intensive] *degree*” which Dühring obviously rejects and which, Cohen says, is not a new concept at all (as Dühring seems to have held) but

“the old classical prototype of the infinitely small, the *First-borne* [*Erstling*] of that creative metaphysics which through its “suppositions”<sup>28</sup> has enriched the present foundations of *extensive* mathematics and which has given it more depth by the erection of the notion of *intensive Reality*” (o.c., p. 121, my italics – E.M.B.).

Like Newton, Dühring, too, is blamed quite generally for his “trivial rationalistic understanding of the infinitesimal, which is opposed to Reality” (o.c., p. 120f.).

I shall dwell for a moment on the underlying notion which is called by the name “Principle” (Hegel), “basic” (Cohen), and “Erstling” (Cohen).

By the *logical firsts* of a system of logic I shall understand the *basic* carriers of properties and relations in its model(s), the basic logical subjects by reference to which all other logical subjects are conceptually constructed and defined. A system of logic has one or more categories of logical firsts, the most common such category in “analytical” logics being the category of (possible) individuals (persons or things). Not all who reject (possible) individuals as logical firsts, or who deny that they are the only ones we need, jump into the Cusanus tradition and look for the basis of logical thought in infinitesimals. Some, such as Leibniz, recommend that the logical basis be taken in general characteristics (of individual persons or things) that do not have the properties of infinitesimals. In our century mixed bases (though without infinitesimals) for logical theory have become a possibility thanks to the ideas of Alonzo Church.<sup>29</sup> Rejection of the “dialectical” basis in infinitesimals does not force one to take human beings and other individuals as given and their names as undefinable for *all* purposes in logical theory, nor does it imply that they are not. Hence there is not a theoretical *dichotomy* between infinitesimal logophores and individuals as *logical firsts*, for there always have been and still are other possibilities. Individuals and infinitesimals are two extremely different and incompatible kinds of logical firsts, but they are not the only conceivable ones.

<sup>28</sup> Combine this with the discussion in my (1974), Section XII-5.

<sup>29</sup> There are modern, normative *epistemologies* – not to be identified with systems of logic – which require that the conceptual basis be taken in sense-data, or occurrences of general characteristics. And there are descriptive epistemologies in which the structural network of the model as a whole is emphasized as determining the lay-out of someone's thought, but this concerns *meta-philosophy* rather than normative logic and ethics.

## 7.2 *The problem of ethical firsts*

Furthermore, the idealist tradition characteristically identifies the concept of being logically first with the concept of having value, hence identifies (a subset of) the logical firsts with basic values, or ends in themselves (ethical firsts). This conceptual identification is foreign to analytic philosophy. In this part of philosophy there are several notions of value, but none of them is identified with the notion of what comes *logically* first. However, there is a very strong tendency in analytic philosophy to regard individuals, in the sense of sentient beings, as values in the sense of ends in themselves, and often even as the *only* ends in themselves. That is to say, in analytical circles the concept of a logical first and the concept of an end in itself tend to be co-extensive in arguments about human affairs, even though the concepts will differ and even though they usually belong to separate theories. Observe that in our time it is usually not its capacity for rational thought which makes that an individual is regarded as an end in itself, but whether or not it is a *sentient being*:

“The question is not, Can they *reason*?, nor Can they *talk*? but, Can they *suffer*?” (Bentham, 1789, Ch. 17).

The only *politically* serious philosophical competitor to the idea of sentient individuals as ends in themselves, so natural to most analytic philosophers, is the “dialectical” paradigm of infinite or infinitesimal logophores as the *termini ad quos*, for knowledge and for action. For this reason I think that, when focussing on the relevance of philosophy for contemporary politics, one can speak, without much inaccuracy,<sup>30</sup> of a second dilemma: that of infinitesimals *versus* sentient individuals as ends in themselves. As *die schöne Seele*, Nietzsche’s notion *Der Übermensch* presupposes the same palaeo-logical constant: maximal (infinitesimal) Man, the human Quality. That Nietzsche ascribed other characteristics to it than earlier philosophers had done is another matter.

When – as sometimes happens – the philosophy of Nicholas of Cusa is described as an important element in bringing about enhanced human selfrespect and respect for other human beings,

<sup>30</sup> Perhaps: with no inaccuracy at all. In fact I strongly suspect that philosophers who, though they do not belong in the “dialectical” (Hegelian or Marxist) fold, deny that individuals are the only ends in themselves, entertain assumptions which depend on a view of ‘quantity’ and ‘quality’, of degrees, of series and of infinitesimals, and of number generally, which are related to those in the Cusanus tradition, both historically and systematically. I suspect that this holds whether they call themselves Platonists, neo-Thomists, or something else. For there are no other clear paradigms to hang on to in this matter.

then in the light of the above this may not seem understandable at all. Nevertheless it is easy to explain. In this and related philosophies human minds are regarded as the gateways to (near-) *Anschauung* of the infinite(simal) Absolute(s), and *therefore* they have a central epistemological value and deserve our respect.

Relatively to some yet older philosophies this view may be a step forward indeed and a help even to Dick and Tom, but it is a view which does *not* imply and which is *not* compatible with the view that each individual is an ethical first in the sense of an irreplaceable end in itself. If a live individual is valued primarily as a possible gateway to (near-)acquaintance with the Absolute(s), then, if one living individual is killed (eliminated), as long as there are millions of others it is possible to maintain that total disaster has not taken place.

## 8. THE ABSENCE OF CONSTRAINTS

### 8.1 *The absence of programmatic constraints*

I return once more to the article in the *Süddeutsche Zeitung*. Only a die-hard idealist could hold the theory that politics, wars and murder are simply *caused* by the systems of thought that were fed into the actors. Our author does not hold this theory. He realizes, as most people do, that to speak or think of the relation of a philosophical system to the atrocities – or the good deeds – committed by some of its adherents in terms of cause and effect would be a mistake. For to produce a “cause” for an event would be to produce a sufficient set of conditions for it to take place, but merely studying and entertaining the ideas of, say, Friedrich Nietzsche does not suffice to produce actions of the kind with which Nietzsche’s writings have been associated. The author of the article in the *Süddeutsche Zeitung* realizes this but is at a loss to find a more adequate description, and so leaves the problem open.

Before I offer another description, let me sum up some important conclusions which were drawn above. First, we have said (in Section 5.3) that those European *philosophical systems* which have been invoked in attempts at legitimizing (justifying) political killing – variants of German idealism, Marxism – are either based directly and quite clearly on infinitesimal logic or were written under the influence mainly of philosophies that are so based. Second, that in the philosophies based on infinitesimal logic the semantic correlate of generic terms – “the State”, “Society”, and “Man” – either was or required the assumption of “intensive” infinitesimals or infinities. Third, that in these philosophical systems the postulated intensive infinitesimals or infinities were, and are, regarded

as ends in themselves, as the *termini ad quos*, both for knowledge and for action, and that human or other individuals, having extension (spatial coordinates), are not. According to many adherents of these philosophies – Walter Kaufmann and J. N. Findlay may be good examples – it does not follow logically from the tenets of these systems of thought that the lives of individual human beings may be sacrificed for some (postulated) intensive infinity/infinitesimal. It is possible, they must then say, to interpret the texts of these systems in such a way that the relative unimportance of individual human lives does not follow.

For the time being this may be accepted. (I shall return to the conditions for such interpretations in a moment.) The crucial point is, however, that neither does it follow from these systems of thought that individuals may *not* be thus sacrificed. Such a statement does not belong to these systems, not even by implication, whereas infinitesimal logic (dialectical logic, with or without stress on “polarities”), which localizes basic values in infinitesimals and infinites, does belong to them, explicitly, or by implication, or by semantic presupposition (cp. Section 8.2).

These philosophies are characterized by a complete *absence of such programmatic (systemic, or immanent) constraints* as might prevent interpretations in which the lives of individuals are relatively unimportant (“for” History, or The Race, or The Nation, or The People, or Society, or The Revolution, or The Labour Class), in the sense that they may be exterminated if the theoreticians deem that the potential infinitesimal/infinite will thereby become actual or at least come much closer. It is not a part of the Hegelian, Nietzschean, Wagnerian, Heideggerian, or Marxist programmes to see to it that this does not happen. The protection of human lives against absolutistic claims and measures is neither an explicit part of their systems nor is it one by implication, nor is it one by pre-supposition.

This fact – and I maintain that this is certainly a fact – is what we have to concentrate upon, and not that according to some thinkers innocent interpretations of these infinity-based philosophies are possible.

The absence of systemic constraints gets its full importance in combination with another feature of the systems of thought we are concerned with here – perhaps with the exception of the works of Nietzsche. This is *the claim to systematic definitive completeness in principle* in the matters dealt with, i.e. completeness as to what is of *fundamental* importance in (the structure of) a *philosophy of human life and affairs*. With the exception of Nietzsche, the authors of these systems are understood by friend and foe to make *at least*

this claim (and are frequently understood to make even wider claims, concerning the inorganic sciences as well). They certainly do not refer to other thinkers for fundamental principles which they themselves do not formulate, except in order to refute them.

This claim to definitive completeness leaves no room for serious revision of any one principle and it leaves no room for the addition of one or more new basic principles. These systems are, as one often says, dogmatically closed.

One necessary condition for me to accept e.g. the “innocent” Hegel-interpretations of Kaufmann and Findlay as historically correct is that it be admitted *and emphasized* that Hegel’s system is (then) *morally irrelevant and morally useless*, hence incomplete. In readers who do not carry certain moral constraints within themselves already the claim to completeness which is made for some of these philosophies quite understandably induces an interpretation according to which human lives (in the biological sense) are indeed relatively unimportant (as compared with Man, the State, the People, History, the Race or the Revolution) since they are *relatively* worthless (value-less).

Whoever does not carry within him- or herself the corresponding emotional constraints can therefore indeed *legitimize* the killing of individuals by referring to that Quality (infinitesimal) or Totality (infinity, “near”-infinity) in the service of which the action is carried out; to the relative moral unimportance of the extended and the countable; and to the claim to moral completeness (and superiority) in principle. In the absence of systematic constraints on the interpretations, the spirits called up by the philosophies of Hegel or Heidegger, Nietzsche or Marx have nothing to fear from their theories when they use them to defend manslaughter.

The “legitimation structure” depends, historically and systematically, on a certain position in the philosophy and history of mathematical analysis, the “infinitesimal calculus” – as discussed above.

To historians of philosophy and other students of intellectual history I ask the following questions:

Can a coherent formulation of the foundations of Friedrich Engels’ version of theoretical “Marxism” be given on the basis of a clear rejection of infinitesimal logic? What about Vassilij Lenin’s version of “Marxism”, where it verges on ethics and the theory of action? These are questions that must be approached by all future students of Marxism who have any respect for the historical determination of human thought.

*In which measure and in which respects* did Karl Marx, Friedrich Nietzsche or Martin Heidegger personally depend on *the various*

*tenets of infinitesimal logic in their cognitive and literary practice?* This is as yet unclear and ought to be subjected to careful investigation by hundreds of specialists among historians of philosophy. However, as far as Marx is concerned, whatever the outcome of that investigation will be of little importance for the precise problem that has occupied us here. For we know that Marx did admit to some inspiration from the very structure of Hegel's logic, *hence – by presupposition – from the Cusanus tradition in the philosophy of mathematics*, without which Hegel's logic is inconceivable. Whether Marx himself understood this or not is of course completely irrelevant.

Marx, in some sense inspired by and in some sense dependent on Hegel's system of logic, did not formulate the necessary systematic constraints on the interpretation of his work. Therefore, as I said above, the spirits that Karl Marx called up have nothing to fear from his theories when, rightly or wrongly, they interpret them as legislating manslaughter as a political instrument.

Analogous statements hold of Friedrich Nietzsche, Richard Wagner and Martin Heidegger – among others.

## 8.2 *The absence of external constraints*

In practice, the absence of immanent constraints may be compensated for by constraints deriving from sources external to the philosophical system in question.<sup>31</sup> The Christian and Jewish axiom "Thou shalt not kill" is one possible such theoretical (as against merely emotional or legal) external constraint. Other possibilities are the philosophies of thinkers like Jeremy Bentham (compare the quotation above). Still another possibility is to add certain principles taken from Buddhism. Only if (but not necessarily: if) part of some other system of thought is accepted in which individual beings are taken as so many absolute values, can one arrive at a revised system which offers no possibility for legitimation of manslaughter, so that individuals who, alone, in groups, or in armies, kill other individuals will have to survive spiritually without the soothing effects of legitimation by appeal to philosophical or religious thought.

In the nineteenth- and early twentieth-century Germany, philosophical systems which were based on or which indirectly depended

<sup>31</sup> It will in fact be logically sufficient for this purpose to add the statement: "This philosophy *does not sanction* (legitimate, justify) the killing of human beings under any circumstances". It is not necessary to explicitly forbid anything in order to make *legitimation* (of manslaughter for instance) *logically* impossible. Most readers of a philosophical text will, however, need more words than these.

on infinitesimal logic dominated the philosophical, if not the scientific scene completely. Types of ethics and social thought as those of Bentham, Bolzano or Mill could have offered an antidote but could not get a firm foothold. They could not be even partially absorbed by the public at large, since the basic principles of their philosophies cannot be organically conjoined to the German systems and often contradict them.

Therefore, as Christendom lost its grip (in some measure, I suppose, due to the writings of Nietzsche), non-innocent interpretations of the infinity-infinitesimal-based German philosophy (including parts of philosophical Marxism, such as the axiom of the Leap from quantity into Quality) could take hold of large numbers – millions – of individual human minds. In the twentieth century there was in Germany a complete absence of *generally accepted* external constraints of a *philosophical* nature.

Analytic philosophy as a philosophical “school” came into being around 1900, as a reaction to infinitesimal/infinity-based normative thought, but it has roots in all former attempts to do without and to outgrow the assumptions of the latter. In Germany, before the second world war infinitesimal/infinity-based normative thought was outgrown only by a modest minority (except, perhaps, among people in the exact sciences) and in the ‘sixties experienced a tremendous public revival. The call for a return to this conceptual pattern in the writings of Herbert Marcuse and very many others who were brought up in this tradition must be accepted, and regretted, as a natural back-lash, caused by the humiliations that were brought down upon German intellectual culture in and through the Nazi period and the second world war. The back-lash was supported by everyone whose intellectual world was associated in some way or other with that culture or its logic.

This burdens analytic philosophers once more with duties of the most serious kind. From these duties no honourable escape is possible.

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