Functioning and teachings of adaptive logics

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Abstract

This paper concerns some formal systems, viz. adaptive logics, that display a specific flexibility in the meanings of logical terms. Both the flexibility that occurs within the systems and the question as to how we may arrive at such systems is discussed. Both, it is argued, are relevant to bridging the gap between logic and argumentation.

1. Aim of this paper

In the present paper, I report on some formal systems, viz. adaptive logics, that display a specific flexibility in the meanings of logical terms. I shall discuss both the flexibility that occurs within the systems and the question as to how we may arrive at such systems. Both, I maintain, are relevant to bridging the gap between logic and argumentation.

I shall start by examining an opposition that underlies the alleged opposition between logic and argumentation and is more fundamental. Neglecting the underlying opposition may result in a misguided approach to the relation between logic and argumentation. I shall show how the underlying opposition brings us to the problem of the flexibility of terms, including logical terms. At that point I start my story on adaptive logics.

I set out for the modest task to discuss one of the many aspects of the logic-argumentation opposition. It is not all-embracing and perhaps not even central. Still, it seems to me that it is fundamental in that the opposition cannot be overcome if one does not crack this nut.

2. Monologism and Plurilogism

Monologism is the doctrine of the one true logic. Few logicians maintain that the true logic is available right now, but many believe that it exists and may eventually be fully described. It may contain a wide variety of so-called logical terms: connectives, quantifiers, modalities, etc. All of them, however, should belong to one single system. As seen from monologism, the interpretation of a logical term, say, in a natural language, concerns the mapping of this term on a term *of* the logical system. An alternative is *plurilogism.* It allows not only for a variety of logical terms, but also for a variety of mutually incompatible logical systems. According to this view, there is room for contextual meaning in a very extreme sense (for some clarification and consequences, see my (1985), (1992a) and (1992b)). Here language is flexible in a totally different sense. The flexibility does not concern the interpretation of a term with respect to a formal system, but the choice of a formal system from a variety of alternatives. Even this description is not fully accurate, because the appropriate alternative may not be known; so, the interpretation of a term may require one to devise a new formal system. According to plurilogism, we do not only use the same terms to express different meanings (in natural language, in thinking and communicating), but we jump from one logical universe to another.

The majority view among formal logicians always was monologism. Not that everyone agreed about the true logic. Many classical logicians even opposed extending classical logic with non-extensional logical terms. Relevance logicians, especially Anderson and Belnap, and students of them, argued at length that some classical inferences are not correct (in the one true logic). Australasian paraconsistent logicians (Routley/Sylvan, Priest, etc.), argued that even most relevant logics are too rich and too classical to be sensible candidates for the true logic.

Most people from the argumentation tradition adhere to plurilogism (although they hardly ever put the matter in these terms); so do a variety of linguists and many literature theorists (who face concrete problems of text interpretation).

Quite often the opposition is underestimated. No one doubts that there is a variety of (technically respectable) logical systems. Actually, a battle had to be fought to arrive even at this meagre agreement. First intuitionistic logic and modal logic became tolerated. Later followed relevant logics and paraconsistent logics, both having a much harder time. This tolerance, however, is merely passive: philosophical fights about who is right have almost stopped, some results are carried over from one approach to the other, but the philosophical disputes are not settled. The parties consider each other as technically interesting curiosities. If monologism is correct, however, then either the relevance view or the classical view is mistaken; and similarly for other comparisons. (Just as one cannot at the same time be a classicist and an intuitionist with respect to mathematics.) The situation is even worse: there is not and cannot be an agreement on what the precise distinction between rival logics is; the metalanguages available to the several parties result in different descriptions of the same logics (see my (1990)).

Natural language may be approached from both the monologicist and the plurilogicist tradition. In my view, the present most impressive attempt to do so from the first tradition is actually a very heterodox one: the approach defended in Graham Priest's (1987). It is impressive because it tackles the presumably most fundamental theoretical problem of natural languages, viz. self-reference and the semantic paradoxes, as well as the paradoxes and limitative theorems (Gödel, Church, Löb, etc.) of classical mathematics. The result is a coherent system that contains its own metatheory in a way comparable to natural language. Priest's actual logical system is rather poor, but it may almost as easily be supplemented with new logical terms as classical logic. It has one major theoretical drawback: it does not enable us to adequately express that we *reject* a statement — this is argued at length in my (1990).

Logicians and argumentation theorists start at opposite ends of a continuum. At the bottom, there is the exactness and accuracy, but also poverty and rigidity of formal languages. At the top, there are the rich and relevant contributions from argumentation theorists, that leave room for flexibility and interpretation, but usually lack formal strictness and mathematical accuracy.

Precisely because both sides seem to come closer to each other, a warning is needed: the opposition between monologism and plurilogism should be taken seriously. The continuous enrichment of a monolithic logical system will not enable us to solve the problems dealt with by argumentation theorists. If monologism is correct, the (many times implicit but nevertheless) standard doctrine behind argumentation is simply mistaken. That is quite possible, of course, but it seems to me that the present evidence points in the opposite direction. I expand a bit on this in the next section.

3. Some problems for monologism

According to monologism, the logical terms that occur in natural languages may be ambiguous and perhaps even vague, but the underlying logical concepts, the meanings of the disambiguated terms, are stable and fixed. At the conscious level, these meanings may be discovered (in the literal sense) and this may require time and analysis. At the unconscious level, the true logic must always have been there, not (only) in some platonic heaven, but in people's minds, even if they had no theory about it.

Outside of the domain of logic, a similar point of view would sound completely outdated. At least to anyone who has some knowledge about the evolution of natural languages. Also to anyone who has some insights in the history of the sciences, especially where active thinking is concerned (creative understanding, creative problem solving, etc.). New concepts are created, not 'discovered'. They are the products of the evolution of our theories about the world, much more than of the world itself. They contain heavy interpretations, many of which turn out to be completely on the wrong track with respect to reality (as it is seen in later periods). Given that this is the common view for non-logical terms, what is so special about logical terms, that their meanings should be stable and fixed, even within the human mind (at the unconscious level)?

The reason cannot be that logical terms are not 'referring'. For neither are mathematical terms, and we all agree (i) that there is a multiplicity of mutually exclusive systems in many mathematical domains, and (ii) that the choice of a mathematical system for some empirical theory is an empirical matter (think about geometry and relativity theory). The latter point is the central one, of course. It entails that the multiplicity of mutually exclusive systems is not merely a technical matter;

only our continuing interaction with the world will enable us to decide which of the alternatives is appropriate to shape our theories about specific domains.

I know of one sensible argument for this exceptional position of logical constants: they would be fixed in our hardware. This sounds like a good argument, but is it true? There is no evidence to support the thesis that the logical terms from natural languages would be genetically determined. There is even no evidence for the thesis that those terms would be the same in all natural languages and in all subcultures. Quite to the contrary, we witness children mastering, from a certain age on, the way in which the (wide) variety of those terms is employed in their environments. Even as grown-ups we are able to master the logical terms from mutually exclusive logical systems (and to actually use them in proofs *about* logical systems). So, how could we even arrive at hypotheses that determine which meanings are genetically fixed and which are not? Apart from all this, it is much more plausible, by present evidence, that genetically fixed matters are quite remote from anything like logical terms.

4. Flexible meanings in formal systems

It takes only a small step from plurilogism to a position that may be summarized as follows. The terms occurring in formal logical systems are sharp, univocal and static, whereas those occurring in natural languages are vague, ambiguous and flexible. Formal logical systems might be fit for mathematics and science, especially handbook science; perhaps it might even be fit for other finished theories as well. But we need natural languages and argumentation in those domains and/or situations in which the exactness of formal systems cannot be reached or should be purposively avoided. Among the latter are the recently disclosed domains where creativity plays a role, both in and outside the sciences. Similar positions are advocated by Perelman and Olbrechts-Tyteca (1958) and Perelman (1968) and by many later argumentation theorists.

As small and obvious as this step may be, it is mistaken. There is no reason why we might not devise formal systems the terms of which have flexible meanings. Consider even programmed computers. There is no reason why these would not be able to use terms in a flexible way, and, where communicating with each other, to trace the meanings of the terms used by the other party. To refuse the label "formal system" for the underlying logical systems would be just a matter of fiat.

Formal results that allow for flexible meanings are quite meagre. This holds even for those parts of artificial intelligence that concern discovery and creativity. Simon's BACON, for example, merely adds new terms that are devisable from available ones by simple algorithmic means. Contrary to Simon's claims, there is hardly any relation with real historical creative processes, as was convincingly shown by Fernand Hallyn (1993). For this reason, I think that the results I report below are valuable to the present discussion.

Before I continue, there is a point that I need to stress again. A realistic approach based on plurilogism not only requires that the intended tool is capable of mapping some term, say from a natural language, to one of the logical terms of a given formal system. It would not even be sufficient that it did so with respect to a given set of mutually exclusive formal systems. The central issue is that new meanings for logical terms may originate, for example while one is trying to get a grasp on some domain, and that such terms may occur in communication (hence that the addressee should be able to detect them).

Work on the interpretation of texts is relevant in this respect, but quite remote from formal systems. I shall start at the other end of the continuum, from results that were arrived at for other reasons. The results are poor, but they are relevant to our problem and they allow for some generalization.

5. Adaptive logics: the problem

In the next section, I consider examples of two kinds of adaptive logics: inconsistency-adaptive and incompleteness-adaptive ones. Most results on (my preferred) inconsistency-adaptive logics have been published or are in print — see my (1989)[1], (1986) and (199+a), which contains the most comprehensive technical description of the (predicative) logics. Some results on decision methods have still to be written up, and so do all results on incompleteness-adaptive logics. But let us start with a general characterization of an adaptive logic.

Consider a theory $\langle \Gamma, L \rangle$, where Γ is the set of axioms and L the underlying logic. L will contain several presuppositions about the domain described. For example, classical logic presupposes that the domain (as approached by observational and operational, or other criteria) is consistent (that the criteria do not, for some A, lead to both A and $\sim$A). Sometimes Γ will violate some of these presuppositions, in which case we shall say that Γ has *abnormal* properties (with respect to the intended underlying logic). For example, where L is classical logic, the consequences of Γ may turn out to be inconsistent or to assert incompleteness (by way of non-logical theorems of the form $\sim(A \vee \sim A)$). If the abnormal properties cannot be readily removed, or if we have to reason about $\langle \Gamma, L \rangle$ in order to improve this theory, then neither L nor a monotonic weakening of L will do — see my (1989) for the inconsistency-adaptive case, other cases being analogous.

Here *adaptive* logics come in. They localize the abnormal properties of the theory, safeguard the theory for triviality by preventing specific rules of L from being applied to abnormal consequences of Γ, but behave exactly like L in all other cases.

The easiest way to understand how all this proceeds, is to realize that an adaptive logic 'oscillates' between the original logic L and a fragment Lf of L that differs from

[1] The semantics presented in 1989 (written around 1981) should be forgotten as soon as possible in view of the extremely clarifying 1986 semantics.

L in not sanctioning the abnormal properties involved. If the abnormal property displayed by Γ is inconsistency, *Lf* will allow for inconsistencies (will not lead from inconsistency to triviality); if the abnormal property is (negation-)incompleteness *Lf* will allow for incompleteness (by not having such theorems as $A \vee \sim A$ or such rules as $A \supset B, \sim A \supset B$ / B). That the adaptive logic *La* oscillates between *L* and *Lf* may now be characterized intuitively, but somewhat inaccurately, by saying that *La* allows for the application of the (i.e. all) rules of *L*, except for applications to sets of consequences of Γ for which it is derivable from Γ that they display abnormal properties. This formulation is inaccurate because the "derivable" is not specified. The correct specification is somewhat complicated but, on closer inspection, turns out extremely intuitive.

I hope that the previous paragraph clarifies that an adaptive logic *localizes* the abnormal properties. At the syntactic level, a rule operates on finite sets of consequences of Γ (as in the case of any other logic); if a rule presupposes that a (specific) abnormal property is not involved, then it will be applicable or not applicable according as it is or is not derivable from Γ that the formulas included in the set have the abnormal property. To phrase it differently, the adaptive logic prevents that abnormal properties of specific consequences of Γ result in a trivial consequence set, but it does not restrict the rules of *L* in as far as they are applied to consequences of Γ that do not display abnormal properties. If applied to a normal theory, nothing has to be restricted and the adaptive logic *La* leads to exactly the same set of consequences as *L* itself.

Another way to look upon adaptive logics is to say that they interpret the premises as maximally normal. *L* presupposes normality. *Lf* gives up this presupposition (for some form of normality), thus heavily restricting on the set of consequences of Γ. *La* takes into account that Γ is abnormal at specific points, but goes on presupposing normality elsewhere, thus leading to a set of consequences that is a real subset of the *L*-consequence set iff the latter is trivial[2], but is in general a real superset of the *Lf*-consequence set.

It should be stressed that the adaptive character of the logics does not rely on any inventiveness (or even any intervention) on the part of whoever applies them: applying the adaptive logic leads to correct, although not necessarily interesting, results. Also, adaptive logics, at least, those I report upon below, have a nice and intuitive semantics that is directed precisely at maximizing normality.

Adaptive logics are non-monotonic (if $\Gamma \cup \{A\}$ is more abnormal than Γ, some B derivable from the latter need not be derivable from the former). Some adaptive logics, e.g., the examples I shall discuss, are decidable at the propositional level and exactly as undecidable as classical logic at the predicative level.

To end this section, I record some facts. Adaptive logics differ from the kind of logics usually labelled "non-monotonic logics" because of two (related) properties: (i)

[2] In the present paper, the trivial set of sentences is the set of all formulas. This convention is handier here than the usual convention that calls a set trivial iff all formulas are derivable from it.

they do not involve any non-logical preferences and (ii) they do not rule out the abnormal properties. In a sense, they form the purely logical basis for some (the usual) non-monotonic logics: they localize the problems but do not resolve them. This result is established in Batens (199+b) in that a circumscription logic is reconstructed by (i) an inconsistency-adaptive logic (that 'minimizes' the inconsistent consequences), (ii) a purely logical mechanism, defined in terms of transformation rules, that connects a set of consistent models to the set of inconsistent models, and (iii) a (non-logical) preferential mechanism that selects the preferred set of models from the set of consistent models.

Some aspects of Nicholas Rescher's famous mechanism — Rescher (1968) and several later publications — are somewhat similar to adaptive logics. The main difference is that, at the syntactic level, Rescher's mechanism operates in terms of sets of premises, whereas adaptive logics operate in terms of deductive proofs. As a result, Rescher's mechanism is extremely dependent on the *formulation* of the set of premises; for example, {p, ~p, q, ...} and {p, ~p&q, ...} determine different sets of 'weak consequences'. For some applications this dependency is suitable, for others adaptive logics are preferable.

In his (1991) Graham Priest invokes adaptive logics to an end that is completely different from the one I originally intended, but proves very interesting from his philosophical stand. Priest is a (monologistic) dialetheist for whom the true logic is a paraconsistent (and relevant) one. He agrees, however, that in many situations we are justified in presupposing consistency. He goes on to show that, if his preferred paraconsistent logic *LP* (from Priest (1987)) is turned into an adaptive logic LP^m by assuming consistency "until and unless proven otherwise,"[3] then LP^m recaptures all classical reasoning where it is sensible (according to his so qualified dialetheist view).

6. Two adaptive logics: semantics

Although the syntax of adaptive logics is both more impressive and more realistic (with respect to actual revisionist thinking) than the semantics, I start with the latter because it is simple and intuitive. But first to the two forms of abnormality.

A simple paraconsistent logic is obtained by giving up the consistency requirement from classical logic *CL*. We keep binary connectives, quantifiers and identity unchanged but weaken negation to the completeness requirement (if v(A) = 0, then v(~A) = 1), dropping its converse, which is the consistency requirement. Let us call this logic *PIL*. Actually, *PIL* has an infinite number of paraconsistent extensions, obtained by adding such requirements as v(~~A) = v(A), and some of these are maximally paraconsistent (have *CL* as their only non-trivial extension). Each of these

3 This phrase is appealing but only accurate if it is not derivable from the premises that some formulas are connected with respect to their inconsistent behaviour. For example, p and q are so connected if (p&~p) ∨ (q&~q) is derivable from the premises, but neither disjunct is. In this case exactly one of the contradictions is true in each model of the premises.

results in different inconsistency-adaptive logics, but astonishing as it might seem, *adaptive* logics based in *PIL* seems preferable for most applications. A simple paracomplete logic *POL* is obtained by giving up the completeness-requirement instead of the consistency requirement. *POL* has an infinite number of paracomplete extensions, etc.

Consider the set of *PIL*-models.[4] All *CL*-models are *PIL*-models (and all *PIL*-models that do not contain any inconsistency are *CL*-models). Consider some set Γ of formulas and some formula A. A is a *CL*-consequence of Γ iff A is true in all classical models of Γ. A is a *PIL*-consequence of Γ iff A is true in all paraconsistent models of Γ. Clearly, as any set of premises has more paraconsistent models than classical models, its set of paraconsistent consequences will in general be a subset of its set of classical consequences.[5]

Where M is a *PIL*-model, let K(M) be the set of contradictions (formulas of the form A&~A) occurring in M.[6] There are at least two strategies to select maximally normal models from the *PIL*-models of some set of premises Γ. The first selects models on the basis of *reliability*. The idea is that, if (p&~p)∨(q&~q) is a (*PIL*-)consequence of the premises and neither p&~p nor q&~q is, then both p and q are considered unreliable. This leads to the inconsistency-adaptive logic *APIL1*. The second strategy, which is less cautious, proceeds by *minimizing abnormality*. A *PIL*-model M of Γ is selected if and only if there is no *PIL*-model M′ of Γ such that K(M′) ⊂ K(M). In other words, there are no models of Γ that are strictly less inconsistent than M. This strategy leads to the inconsistency-adaptive logic *APIL2*. The choice of a logic will obviously depend on the appropriateness of the strategy in a specific situation.

Given all this, we define: A is an *APIL1*-consequence of Γ iff it is true in all *APIL1*-models of Γ; in other words, in all *PIL*-models in which only unreliable formulas behave inconsistently. Similarly, A is an *APIL2*-consequence of Γ iff it is true in all *PIL*-models of Γ that are not more inconsistent than is required by Γ. If Γ is consistent, both logics select exactly the classical models of Γ. If it is not, they select no classical model, but in general (i.e., unless Γ is trivial) they select a subset of the *PIL*-models of Γ — and the *APIL2* models form a subset of the *APIL1* models. In the former case, the inconsistency-adaptive consequence set will be identical to the classical consequence set; in the latter case, the inconsistency-adaptive consequence set will be in general (i.e., whenever Γ is not trivial) a subset of the *CL*-consequence set

[4] To keep things simple, let us consider a model as an ω-complete set of formulas throughout this paper.

[5] There is only one exception, viz. when the set of premises is trivial itself.

[6] If ω-incomplete models are included, the treatment becomes quite a bit more difficult — see Batens 199+a.

(the trivial set), but will be a superset of the *PIL*-consequence set (that takes a *larger* set of models into account).[7]

Although results were readily within reach, I stopped working, some ten years ago, on adaptive logics based on *POL* because I did not find sensible applications. Recently, a discussion with Diederik Aerts, who is doing advanced research in quantum physics, suddenly made me see the light: an incompleteness-adaptive logic (the philosophical rationale of which I evidently cannot discuss here) *might* spare us the awkward properties of quantum logics. The *APOL*-systems are nice counterparts of the *APIL*-systems, the two strategies now being defined with respect to formulas of the form $\sim(A \vee \sim A)$. Again, if Γ is complete, *APOL*-systems define the same consequence set as *CL*, whereas they define a poorer consequence set, but one richer than the *POL*-consequence set, in the opposite case.

7. Two adaptive logics: a glimpse on the syntax

The proof theory of *APIL*-systems and *APOL*-systems is most interesting and (in a specific sense) realistic. The proof procedure is dynamic (or revisionist) in that we start from the supposition that all formulas derivable from Γ are consistent (respectively negation-complete) unless and until proven otherwise — but compare note 3. It turns out indeed that the articulation of a proof procedure leaves us no other way than to rely on the formulas *that actually occur in the proof.* This is realistic in the sense of conforming to what happens in our 'natural' thinking: *to revise our view according as our understanding improves.* However, the proof procedure is still deterministic in the sense that, if we proceed sensibly, we shall eventually arrive at a result that may be defined statically. In other words, even if different people set off in different directions from the same set of premises Γ, they will all end up at the same fixed point.

The technical details are complicated but intuitive. A central feature is that formulas may be *connected* with respect to their inconsistent — respectively incomplete — behaviour. This will be expressed by formulas of the form $(A_1 \& \sim A_1) \vee \ldots \vee (A_n \& \sim A_n)$ — respectively $\sim(A_1 \vee \sim A_1) \vee \ldots \vee \sim(A_n \vee \sim A_n))$ — occurring in the proof in the absence of sub-disjunctions of them. Also, the aforementioned "proceeding sensibly" is a bit tricky, but strictly definable. And then, all this is simple at the propositional level, where everything is effectively decidable, but at the predicative level the usual lack of an algorithm for derivability interferes, and forbids even a general algorithm for "proceeding sensibly".

There is more tricky stuff, like the notion of a theorem. Both *APIL*-systems and *APOL*-systems have exactly the same set of theorems as *CL* if a theorem is defined by derivability from the empty set. However, their theorems reduce to those of *PIL* and

[7] In view of the dialectical properties displayed by *APIL1* at the syntactic level, and also because I did not at that time recognize the importance of other adaptive logics, I originally called it *DDL* (dynamic dialectical logic).

POL respectively, if theorems are defined as formulas derivable from any set of formulas.

Although all this is too technical to be continued here, I hope I made it clear that the notion of a formal system may have some very unexpected properties. All this is at the level of 'logic', even formal logic in the strict sense; no external preferences are involved, no non-logical terms, not even relevance requirements on either connectives or derivability.

8. Adaptive logics and argumentation

The main point I wanted to make is that it is quite possible for a formal logic (defined with respect to a formal language) to deal with *flexible* meanings of *logical* terms. Of course, some aim need to be determined in some way or other. In adaptive logics, it is determined by the specific points at which a theory displays abnormal properties of the given kind. The flexibility displayed by this procedure has some generality already. This shows at least a certain similarity with such problems as discovering new meanings from the interaction with some domain, or grasping unknown meanings hidden in a text. In the latter case, there clearly is not an intended logic (as the L in our couple $\langle \Gamma, L \rangle$). Yet, a preferred logic is given by assumptions deriving from the pragmatic context.

Of course, it would be nicer if these pragmatic assumptions themselves might be incorporated within the formal machinery. But then I set myself a modest task, which was to report on a machinery — an exact and formal one for that matter — that, given those pragmatic assumptions, proves able to deal with flexible meanings of logical terms.

The force of a logical machinery manifests itself in its applications. Fortunately, some applications of inconsistency-adaptive logics have been tried out, and with success. The most impressive one concerns the reconstruction of a creative discovery process from the history of thermodynamics. In her 199+, Joke Meheus considers the case of Clausius who forged a consistent theory from the inconsistent set comprising Carnot's thermodynamics as well as Joule's principle on the conversion of work to heat (and back) and a set of experimental results (mainly obtained by Joule). By relying on concrete passages from Clausius's text, she convincingly shows that the process may be reconstructed in terms of adaptive logics; and not in terms of classical logic, Rescher's aforementioned mechanism, or (the usual) non-monotonic logics (that are directed at handling rules with exceptions). The reconstruction is especially interesting because, in the presence of inconsistent premises, Clausius nevertheless applies a Reductio ad Absurdum (concludes to the falsehood of a supposition by showing that it leads to an inconsistency), and this application is indeed justified in view of the inconsistency-adaptive logic.

During the process by which Clausius transformed the inconsistent set of theories and data into his consistent theory, the meaning of "heat" (and many other terms) changed drastically. A rather impressive aspect of the Meheus' reconstruction is that this change does not require any special treatment. The following hypothesis seems plausible in view of the reconstruction. Relying on non-logical preferences[8], Clausius arrives at his theory by stepwise eliminating (halves of) inconsistencies from the consequence set of the premises. But as some of the eliminated statements pertain directly to the meanings of the (non-logical) terms, the latter are modified at once. The elimination of other statements indirectly modifies those meanings. Roughly, this happens because of the connection between the meaning of terms and the accepted statements in which they occur (needless to say, this should be further specified[9]). This aspect has not been sufficiently studied. But if the analysis is roughly correct, *some* changes in the meanings of non-logical terms may be understood as consequences of the elimination of inconsistencies. In other words, the explicit flexibility of the meaning of negation, as it occurs in inconsistency-adaptive logics, is sufficient, *in such cases*, to understand the (implicit) flexibility of non-logical terms.

All I said up to here was related to a given logical frame, viz. a specific adaptive logic determined by a given maximal logic (*CL*) and a hypothesized minimal logic (*PIL*, respectively *POL*). Let me now turn to the question whether it is within the reach of algorithmic means to devise a minimal logic and next an adaptive logic in view of a specific abnormality problem. If we succeed in establishing this, we take another, rather remarkable, step from the stability of logical systems to the flexibility of argumentative procedures.

9. Devising adaptive logics

We shall learn more about flexible meanings by turning to the question as to how we arrive at an adaptive logic. (I shall try to stay as close as possible to the facts: the available systems, how I arrived at them, and how, by trying to generalize, other problems may be handled.)

We start from a problem: a theory showing some abnormal properties. This first of all presupposes a notion of normality (a mapping of the (logical) terms of some text to the terms of some given formal system). We may safely consider this to be determined by pragmatic considerations external to the formal task under discussion. Next we need a *criterion* for abnormality. Triviality clearly is a good indicator, but at first sight it seems too narrow. That a theory turns out non-trivial under some interpretation, does not seem to warrant the correctness of the interpretation. Yet, some reflection leads to a rather startling conclusion: triviality is sufficient as a criterion. Suppose that our interpretation of a text entails that its author X subscribes to a view

8 Mainly: favouring data over theoretical statements and favouring some principles over others — the preferred ones derive from his world-view, which, however, is modified itself as an effect of his analysis.

9 The main missing aspects are the role of *interpretations* and the (empirically supported) fact that only 'pieces' of the meanings of terms play a role in specific thought episodes.

A, whereas we assumed him or her to reject A. If we take other interpretations to be possible, we shall not consider our interpretation a sufficient reason to reject the assumption that X rejects A. Taking "X accepts A" and "X rejects A" to be strong negations of each other (presumably the only sensibly assumption) we arrive at a strong contradiction and hence at triviality. Or consider a case in which our interpretation leads to the conclusion that X utters a truism (like in interpreting "If it rains, it rains" as a statement of the form "$p \rightarrow p$"), whereas we assumed that X was not uttering a truism; here again we end up with triviality. In other words, if we take our assumptions serious, triviality is a sufficient criterion for localizing problems.[10]

As a second step, we might identify the inference rules that lead to the problem. This offers important but confusing information. Indeed, the result will highly depend on the actual inference rules, whereas we know these to be exchangeable. In other words, the gathered information will be highly contingent on arbitrary choices.

Identifying combinations of derivable inference rules that lead to the problem sounds better, but again there is a difficulty: whenever an arbitrary statement A is derivable from some (obviously finite) set of premises Γ, there is (in general) an infinite number of sets of derivable rules of inference (even if we rule out supersets of other sets) by which A may be derived from Γ.

In order to arrive at a diagnosis of the problem, we need a theory about the *elements* of the meanings of the logical terms ("meaning-elements" for short). For example, with respect to classical logic, we standardly consider the meaning of implication to consist of three elements: if $v(A) = 0$, then $v(A \supset B) = 1$; if $v(B) = 1$, then $v(A \supset B) = 1$; if $v(A) = 1$ and $v(B) = 0$, then $v(A \supset B) = 0$. Please remark that these are not the standard semantic clauses, but rather a way of summarizing them (that mainly derives from standard metatheoretical proofs about classical logic).

Given a theory about the meaning-elements of the logical terms, the information about derivable rules of inference becomes relevant. We may now study which rules depend on which meaning-elements. Once this is accomplished, we know which of the meaning-elements leads to the problem. More correctly, we will have arrived at a set of sets of meaning-elements and we know that restrictions should be imposed on one of these sets.

If some set of meaning-elements is a subset of another, we leave out the superset. There are two reasons to do so. The first lies with the notion of normality: for a start we try to keep as close as possible to the standard interpretation defining normality. The second reason is a bit disappointing: if we do not introduce this restriction, then each meaning-element will occur in some of the aforementioned sets.[11] This does not entail that we might not have a good reason, later on, to get further away from the

10 I am not arguing that a theory about the assumptions involved in the interpretation of texts is irrelevant, but rather that it need not be incorporated in the formal machinery for devising adaptive logics.

11 Consider any meaning-element of some logical term. If A is derivable from Γ, then there is a derivation of A from Γ in which is applied a rule relying on the considered meaning-element.

standard interpretation defining normality (for example, doing so might lead to greater systematicity in the interpretation of the logical terms). But this again will be a consideration that is external to the formal machinery under discussion (compare note 10).

The third step consists in choosing the set of meaning-elements that we shall impose restrictions upon. If the abnormal property occurs systematically and frequently, there is only one such set (and nothing to choose). Whenever there is a choice to be made, we shall again have to rely upon assumptions external to our logical machinery. We might pick the set containing the least elements, or a set containing meaning-elements of one term only. Even these, however, are to be considered as extra-logical assumptions.

The fourth and final step consists in devising the adaptive logic. The extra-logical element here is that we have to choose a strategy (minimizing abnormality, or reliability — but more might be discovered later). Once this is done, devising the adaptive logic is completely straightforward. We first define a (monotonic) subsystem *Lf* of the original logic *L* by dropping the meaning-elements of the chosen set. Then we define abnormal properties of models; these are typically properties of *Lf*-models that do not occur in the *L*-models.[12] Here are some examples: the presence of a formula of the form A&~A, the absence of a formula of the form A∨~A, the absence of both A and B in the presence of A∨B, etc. The adaptive logic *La* is arrived at by defining the maximally normal models in view of the chosen strategy. This procedure is clearly algorithmic.[13]

All this may seem somewhat theoretical. However, some nice and ready problems may function as a test for the procedure sketched. Simple one's, like the lottery paradox, and complex ones like the paradox of Curry and Moh Shaw Kwei. Neither of these has anything to do with negation.

Before leaving the matter, I want to stress the important role played by the theory about the meaning-elements of the logical terms. If this theory is based on the standard semantics of classical logic, the meaning of each connective reduces to two or three elements only. If it is based on, say, the Routley-Meyer semantics for relevant logics, the meaning of each connective (of classical logic) consists of a host of (independent) elements — see, e.g., Routley (1982). In the latter case, deviations from normality will be much smaller, which is not necessarily preferable. The importance of philosophical theories should be stressed in this connection. Each of these semantic systems, as most others, leads to theories that *isolate* the meanings of logical terms. Again, there is a philosophical position behind this. The position may be justified, but it is neither unimportant nor straightforward (as many logicians seem to presume). Frege did some

[12] By the definition of *Lf*, all *L*-models are *Lf*-models, but not conversely.

[13] It is still an open problem whether there is an algorithm for devising proof methods that are dynamic in the sense explained in section 7.

excellent thinking for classical logic; after him, we got too much technique and bad metaphysics.

References

Apostel, L. (1971). 'Assertion logic and argumentation.' *Philosophy and Rhetoric* 4, pp. 92-110.

Batens, D. (1985). 'Meaning, acceptance, and dialectics.' In: J. C. Pitt (ed.), *Change and Progress in Modern Science.* Dordrecht: Reidel, pp. 333-360.

Batens, D. (1986). 'Dialectical dynamics within formal logics.' *Logique et Analyse* 114, pp. 161-173.

Batens, D. (1989). 'Dynamic dialectical logics.' In: G. Priest, R. Routley & J. Norman (eds.), *Paraconsistent Logic.* München: Philosophia Verlag, pp. 187-217.

Batens, D. (1990). 'Against global paraconsistency.' *Studies in Soviet Thought* 37, pp. 27-47.

Batens, D. (1992a). 'Do we need a hierarchical model of science?' In: J. Earman (ed.), *Inference, Explanation, and Other Frustrations. Essays in the Philosophy of Science.* Berkeley-Los Angeles-Oxford: University of California Press, pp. 199-215.

Batens, D. (1992b). *Menselijke Kennis. Pleidooi voor een bruikbare rationaliteit.* Garant: Leuven-Apeldoorn.

Batens, D. (199+a). 'Inconsistency-Adaptive Logics.' In: E. Orłowska (ed.) *Logic at Work. Essays Dedicated to the Memory of Helena Rasiowa.* Dordrecht: Kluwer.

Batens, D. (199+b). 'Inconsistency-Adaptive Logics and the Foundation of Non-Monotonic Logic.' *Logique et Analyse.*

Hallyn, F. (1993). 'La troisième loi de Kepler et la 'psychologie de la découverte'.' *Archives internationales d'histoire des sciences.* vol. 43, pp. 247-257.

Meheus, J. (199+). 'Adaptive logic in scientific discovery: the case of Clausius.' *Logique et Analyse.*

Perelman, Ch. (1968). *Logique et argumentation.* Bruxelles: Presses Universitaires de Bruxelles.

Perelman, Ch. & L. Olbrechts-Tyteca (1958). *Traité de l'argumentation.* Paris: PUF (many later editions).

Priest, G. (1987). *In Contradiction. A Study of the Transconsistent.* Dordrecht: Nijhoff.

Priest, G. (1991). 'Minimally inconsistent LP.' *Studia Logica* 50, pp. 321-331.

Rescher, N. (1968). *Hypothetical Reasoning.* Amsterdam: North-Holland.

Routley, R. (1982). *Relevant Logics and their Rivals.* vol. 1, Atascadero, Ca., Ridgeview.