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15. The development of attachment and attachment-related competences: a dynamic model

Abstract

This chapter introduces a method for building mathematical dynamic systems models of early mother-infant interaction, particularly those forms of interaction that are closely related with the attachment phenomenon. After an introduction to dynamic systems building in developmental psychology, especially dynamic growth models, an overview is given of developmental mechanisms and phenomena pertaining to the development of attachment. These mechanisms are then used to build a dynamic model which simulates the growth of behavioral variables such as proximity seeking or avoidance. These variables grow in interaction with changes in maternal behavior and sensitive responsiveness. The dynamic model produces various kinds of growth patterns, in addition to multi-modal distributions of end states. It also demonstrates a strongly non-linear relationship between the growth of the attachment variables at issue and sensitive responsiveness. The concluding section reviews some of the ways in which dynamic model building may contribute to research and theory building in a field like attachment development.

Introduction

Attachment is a complex, qualitative phenomenon. Those who study attachment 'in the wild' know that it shows a variety of forms and properties and that it consists of a broad range of aspects and variables. In this paper, however, I shall discuss an extremely impoverished notion of attachment which, in short, boils down to just a few quantitative variables. It is not my aim to argue that this particular view provides a deeper or more valid theory of attachment than the existing theories, which, in general, do more justice to the complexity of the phenomenon. I believe, however, that a considerable, if not overstretched reduction of a natural phenomenon is sometimes necessary to allow one to perform certain theoretical or empirical exercises that may help shed a new light on old issues. The main function of the simplification is to allow me to present a formal model of the growth of attachment and of its effect on the growth of competencis in the infant.

It is a model which is based on principles derived from *non-linear dynamics* (see Van Geert, 1991; 1994; Thelen & Smith, 1993; Smith and Thelen, 1994 for overviews). An important property of dynamic models is that they conceive of the mechanisms involved in the most general form possible. By doing so, these models can describe processes in the form of well-known mathematical formalisms. This is what distinguishes a dynamic model from a simulation in the traditional sense (although the distinction is gradual rather than categorical). A case in point is the application of mathematically derived indicators of discontinuous change ('catastrophes') to the domain of conservation development (Van der Maas & Molenaar, 1992) or to the domain of social phenomena such as conformity, aggression or love (Tesser & Achee, 1994). Many dynamic models deal with iterative processes, that is, processes that take their preceding product as the input for a new process (although it suffices for a model to take time into account in order to qualify as a dynamic model). Such iterative mechanisms are often responsible for the non-linear quality of dynamic models.

The model of attachment I shall discuss in this chapter is based on a simple, iterative concept of increase or decrease. This is the *growth model*, which was originally developed by the Belgian 19th century mathematician and demographer Verhulst.

A growth model of attachment

The generalized growth model

A growth model offers a mathematical description of a growth process, that is a process of increase or decrease of a numerical variable. It states that any next level of that variable is a function of three values. The first is the (or, more precisely, a) *previous level*. The second is a parameter of change, the *growth rate*. The third parameter forms the expression of all the available resources that altogether support the growth of that variable and help maintain its level. This is the so-called *carrying capacity* of an environment with regard to the variable at issue (for an extensive description, see Van Geert 1991, 1994). If growth is conceived as a process that occurs in discrete episodes (for instance, discrete reproductive seasons in animals, or discrete learning events in development), the model can be stated in the form of an iterative equation which displays interesting non-linear phenomena. Whereas the growth rate controls the speed of the process, the resource term controls its (eventual) equilibrium level. The delay between the cause (e.g., a reproductive season, a learning experience) and effect (the consequent increase or decrease of a population or of a skill or knowledge level) controls the nature of the attractor state, which can be either a point attractor (an equilibrium level), a cyclical attractor or a chaotic attractor. An attractor state is literally the state, level, or property to which a dynamic system is attracted to. A capital on an interest account at a bank, for instance, grows exponentially at a rate equal to the interest. Its attractor is infinity (if it is left on that account forever). The growth of a human body, on the other hand, depends on things like food intake or physical exercise. But no matter how much one eats (within certain viable limits, that is), a person's body will stabilize around a maximum weight. If that weight is fixed, it corresponds with a point on the weight scale (say, 175 kilograms). In that case, the person's body weight has evolved towards a point attractor. If the weight goes up and down, we can say there exists a cyclical attractor. If that cycle is irregular but nevertheless has some underlying regularity (which may be very hard to find) we call it a chaotic attractor.

It is easy to use the basic growth equation as a building block for more complicated models dealing with connected variables. For instance, it is highly likely that a tutor adapts his or her level of instruction to the level of knowledge or skill of the pupil. On the other hand, we may expect that the level of knowledge or skill is a level of the demands made and the instruction given. Thus it is possible to express the growth in both the level of instruction and the level of skill as a system of coupled growers. The skill level, for instance, is represented in the form of an equation that takes as its input a previous level and the level of instruction presented.

It is important to note that, as far as the relationship with empirical or measurable variables is concerned, most of the parameters involved in the growth model are concatenated dimensions. By a concatenated dimension I understand a one-dimensional representation of a multi-dimensional process. The resources needed for the growth of cognitive competence, for instance, are multi-dimensional in that they are both internal and subject-dependent, as well as external (environmental). Moreover, they may take a variety of forms. Nevertheless, they all contribute to a one-dimensional value, namely the equilibrium level of the growing variable. The same is true for the growth rate. For instance, a variety of factors contribute to the rate with which new knowledge is appropriated (all support factors assumed to be equal): speed of information processing, size of working memory, available knowledge, and so forth. All these variables contribute to the rate with which knowledge grows, and which can be expressed in the form of a single parameter.

Growth models operate with numerical variables that represent psychological properties. For instance, we can model part of the attachment process by describing the increase of a quantitative property 'experienced security'. The growth levels are mathematical abstractions. Their association with empirical data should be established separately. For instance, some variables act only as hidden variables, necessary for the model to work but impossible to measure. Their function is to explain the changes in observable or measurable variables. It is also likely that numerical variables can only be measured in the form of qualitative nominal distinctions. For instance, if variables such as proximity seeking or avoidance cluster into multi-modal distributions, we can probably determine whether a child falls in the avoidant or in the non-avoidant class, but we have no means to measure the exact degrees or levels of avoidance. The task of the growth model is to finally lead to testable predictions that critically distinguish between competing models.

What shall we model?

Attachment is a complicated and maybe also rather diffuse qualitative phenomenon. It cannot be reduced to a few simple quantitative variables. On the other hand, the study of attachment, for instance in the Strange Situation, requires that, among others, a small number of quantifiable variables are observed and scored. For instance, children differ in the degree in which they avoid contact, resist interference from their mothers, and so forth. Empirically these variables are probably far from the ideal of a neat numerical scale, but at least for the sake of model building we can pretend our measurements are crude approximations of numerically scaled variables. That is, a significant part of the study and description of attachment boils down to measuring and describing quantitative variables. They are the variables we will address in this model building exercise.

A core finding of attachment research is that attachment falls apart in various types. The classic A, B, and C types represent differences on a variety of constituent dimensions. For instance, babies differ on avoidance, maintaining contact, resistance after separation, fear for strangers, seeking contact and proximity, exploratory behavior, and so forth. If these differences were gradual, we wouldn't have found any types or clusters, but rather a normal unimodal distribution over the variables. If the types are real, we should expect to find variables that naturally cluster into bimodal or multi-modal distributions.

Which assumptions do we make?

Types of assumptions

I shall make a distinction between two kinds of assumptions. The first are the general model assumptions, which, in our case, will be adapted from dynamic growth theory. I shall discuss these assumptions in the next section, where I deal with the model itself. The second set of assumptions comes from the domain and phenomenon that we intend to model, which is attachment development. These assumptions can be divided, first, into a subset which is common to a large variety of comparable phenomena. We may think about general assumptions concerning learning, instruction, help, information processing and so forth. A second subset contains assumptions specific to the domain in question: attachment models are about actions, behaviors and properties of a dyad, more particularly the mother-infant pair.

There exists a considerable literature on what may be called the antecedents of attachment, in particular secure attachment. These antecedents are traditionally sought in characteristics of the mother. For instance, if the mother is affectionate, positive and avoids intrusive vocalizations she is likely to make her infant grow into a securely attached person (Isabella, 1993; Roggman et al. 1987). A very important characteristic of maternal behavior is sensitivity, a concept which is discussed and defined elsewhere in this book (see for instance Van IJzendoorn et al., 1992). Sensitivity pertains to reading the infant's behavioral signals and responding to them promptly and adequately and it is affected by the infant's behavioral properties (Ainsworth, 1983; Bretherton, 1987).

Assumptions regarding mother-infant interaction

In his study of mother-infant interaction with preterms, Wijnroks (1995) has presented a review of the studies carried out on this topic and has formulated a theoretical model. Wijnroks describes maternal contribution in terms of the quality and quantity of the help, support and information the mother gives to her child. The help, support or information given can be analyzed in terms of intensity and frequency. For both aspects we can claim that the relationship with an eventual developmental effect is probably curvilinear. The quality of the information and help given is expressed in aspects such as contingency and appropriateness. Both quality and quantity of the help and information given depend on the following two variables. The first concern is the exact timing, i.e., when is help and information given, relative to the infant's activity. The second is the adaptation of help and information to the infant's state of arousal. The infant's contribution to the developmental process is supported by various aspects. For instance, infants differ as to their natural sociability and exploratory tendency. They also differ with regard to speed and quality of information processing. Infants have different optimality ranges as far as arousal, timing, length and content of the mother's intervention is concerned.

In this model there still exists an asymmetry between the mother and the infant. The mother acts basically as the 'sender', the infant as the 'receiver'. The notion of sensitivity, however, requires that the infant on his turn acts as a 'sender' and the mother as a 'receiver'. The sender-receiver model involves an inadequate metaphor, in that it presupposes a form of turn-taking that is not or at best only moderately present in mother-infant interaction (Fogel). What we see is an ongoing, continuous adaptation of one participant to the other. However, the model I am going to present does not describe attachment growth by modeling the processes taking place during single social 'frames' (e.g., an actual interaction between a mother and her infant focusing on feeding or playing). The model proceeds by single steps. Each step is conceived as the result or summary of an interaction 'frame' (although one should realize that the boundaries between consecutive frames are not always very sharply distinguished). The continuous nature of the mother-infant adaptation is simulated by letting the infant- and the mother-part change simultaneously, in answer to the preceding conditions.

The simultaneity of the process does not imply that a form of asymmetry between mother and infant is nonexistent. The major asymmetry between mother and infant is that the mother's range of control reaches considerably farther than that of the infant. This is especially true for processes in which a form of teaching takes place, aiming, for instance, towards learning cognitive skills.

The structure of a growth model of attachment

The growth of attachment dimensions is conceived of as an ongoing, iterative process of adaptation. It is a form of co-adaptation, in that the infant adapts to qualities of the environment, primarily mediated by the mother, whereas the mother adapts to the infant. Adaptation is not always a positive property. Mother and infant may co-adapt to a form of interaction which is highly intrusive or even aggressive and with a high amount of avoidance and withdrawal from the infant.

The model I shall present here is an adaptation of a simple model aimed at describing basic features of educational interactions (Van Geert, 1991, 1994a). A more complicated version has been discussed in the framework of the Vygotskyan notion of the *zone of proximal development* (Van Geert, 1994b). The choice of this particular model does not imply that I see the emergence of attachment as the outcome of an explicit process of teaching or education. In my view, the basic property of an educational interaction, which makes it suitable as a model for attachment development, is the fact that a more mature person interacts with a less mature one in a process of mutual adaptation, with a basically benevolent intention from the side of the more mature person. In general, educational interactions are goal-driven, but they don't need to be. Many aspects of such an interaction are not covered, and need not be covered, by explicit and conscious aims or educational goals. When I use the term 'educational' in the framework of attachment theory, I basically refer to the mutually adaptive and benevolent nature of a continuous interaction process.

A first major assumption is that if interactions are seen in the framework of a growth process, educational interactions are basically about *resource management*. A significant part of the resources that contribute to the growth of a psychological variable are under the control of the educator. The mother's problem is to adapt those resources to the growth level of the child in such a way that the child profits optimally from those resources and shows the expected psychological growth process. Help given by the mother, for instance, is an important resource function. The amount of help given, and the form of that help should be adapted to the child's growing level, for instance of the mastery of some particular skill. The basic idea is that there exists an *optimal distance* between the child's actual developmental level and the mother-dependent resources that contribute to further developmental change.

Infants differ as to the *optimality ranges* they display (Wijnroks, 1995). For instance, some infants may have a narrow arousal range, which implies that, if one goes beyond the boundaries of that range, the support may no longer have any effect (or even turn into a negative effect). This means that they are more vulnerable to an overstimulation effect than infants with a more robust arousal system. Infants differ also with regard to the speed and intensity of information processing. That is, they differ with regard to the parameter that governs the speed of learning, namely the growth rate. Smart infants may also have a bigger optimality range, in that they are capable of bridging a bigger gap between what they already can and the level of help or information given and therefore progress with bigger 'steps', so to speak.

A second major assumption is that mother-infant interactions are not just determined by the properties and needs of the child alone. They are also determined by the mother's 'habits', which is defined as the mother's usual or preferred way to interact with a child. This assumption implies, among others, that the current help, support or information given depends on the preceding level of help, support or information, thus accounting for a certain, mother-dependent continuity.

The psychological meaning of the model parameters

In this chapter I shall refrain from a technical presentation of the model and confine myself to a qualitative description, particularly of the psychological meaning or correlates of the parameters employed (interested readers can find more details in the Appendix). In its simplest form the model consists of two coupled equations (or more precisely, two groups of coupled equations). One refers to the infant's role and properties, the other to the mother's.

Infant parameters

The first equation specifies the change in an infant variable. For instance, we may assume that the level of security in the attachment relationship is a variable that increases from an initial state level to some (eventually temporary) equilibrium level. Other examples are the probability that an infant will actively seek the company of its mother, or the infant's tendency to avoid contacts with the caretaker. I repeat that it is not necessary that we can measure those variables with the same level of accuracy that the model needs in order to perform its computations. What matters is that the results obtained with the model can be mapped, in one way or another, onto empirical, observable variables. For instance, a continuous model variable such as the level of security can eventually be mapped onto a dichotomous classification into securely and insecurely attached infants.

An essential parameter is the *growth rate*, that is the amount of progress made, given a certain amount of help and support. The growth rate is an abstract, concatenated variable which covers psychological aspects such as the child's speed of information processing, the ease with which he or she adapts to a particular situation (involving cognitive style properties such field dependence), but also the infant's sensitivity towards particular kinds of signals or information (for instance, some infants may be more sensitive to the social than to the object-related dimensions of a problem situation). Growth rate is moderated by two other model parameters which relate to the optimal range of effective experience.

When given help, support or information, children may differ as to the most effective distance between that help and their actual level of development. Some children require help that is very close to what they already know and master, whereas others thrive only if the help (and the information or challenges presented) are relatively far ahead from their present level. Determinants of the *optimum gap* are psychological factors such as intelligence, but also more functional aspects such as the particular knowledge, schemes and representations a child already has and which help him or her to structure and retrieve information from experiences. Thus, in addition to growth rate *per se* we should reckon with an optimal distance aspect, which differs among children. But optimal distance alone is not enough.

When presented with challenges that are too high for their capacities, some children get easily fatigued or overaroused or loose their interest, whereas others are less sensitive and go on trying to cope with the problem (which, given the fact that it is far beyond their current scope will seldom lead to success, or to any progress, for that matter). Similarly, when confronted with help that is too close to what they already manage (for instance, when the mother does things for the child he can do himself), some children will get bored very easily, even become angry, whereas other children patiently follow the well-meant but ineffective interventions of the adult. Wijnroks (1994) summarizes some of the literature on the relationship between intensity of stimulation given by the mother and arousal level in the infant. If children are too aroused, they will insufficiently profit from the stimulation given. Although Wijnroks concludes that regulation of arousal is better explained by the quality of the mother's stimulation than by the infants arousal regulation ability, he also points to the fact that preterm infants have a more problematic arousal regulation pattern than full terms which makes it more difficult for the mothers to provide high quality stimulation. We may conclude, therefore, that there exist individual differences in the *optimal distance range*, that is, in how far the stimulation, help or information given may diverge from the optimal level and still have a positive effect on the infant's development.

Maternal parameters

The second equation describes the evolution of the resources that contribute to the growth (or decline) of the infant variable at issue. Examples of such a variable are: security, the tendency to seek proximity, the likelihood that an infant will explore an environment on his own and without support, and so forth. Since a resource level technically corresponds with a (potential) equilibrium level of the variable it affects, we can represent the resources along the same dimension as the variable itself, notwithstanding the fact that the resource level is the result of a great many, eventually independent influences and factors. I stated earlier that an interaction between a mother and an infant basically deals with resource management, more precisely with attempting to adapt the resources to the infants continuing development. We can therefore conceive of the resource level as a representation of the mother's interactional activities (which are 'educational' in the broad sense of the word). Of course, the resource level also depends on internal resources present in the infant, but for the sake of model building we can pretend those factors are constants. Thus in the equation describing the resource adaptation I confine myself to parameters that refer to psychological properties of the mother. Those properties are of two kinds, first, perceptual, second, focusing on actual activity.

The *perceptual aspect* is related to the mother's ability to make a reliable estimation of the infant's needs, that is, the infant's current developmental level, arousal state, interest, and so forth. Infants may of course differ in how clear and consistent they are in signaling these properties, but for the sake of simplicity I refrained from adding this infant-governed variation into the equations. Mothers may differ in how sensitive they are to the infant's developmental signals. Moreover, they may show biases, for instance by overestimating either signs of success or signs of failure.

In addition to the perceptual aspect, there are parameters that affect the action component. Notwithstanding the fact that a mother has made a reliable estimation of her infant's needs, she may not react adequately. That is, mothers may differ with respect to their talent to act in accordance with the situational demands. For instance, some mothers may be relatively impatient, and give the infant little opportunity to try his own things. Others may be too placid and cling to a laissez-faire attitude that leaves the initiative entirely to the infant. Mothers may give help that is either simply too far from what the in-

fant can assimilate, or that is too close to what the infant can already accomplish without help. We may expect that mothers who can reliably estimate their infant's needs are in general also capable of adequate action. Whatever this association, it is probably weak enough to allow for a considerable variation between the perceptual and actional parameters.
It should be noted that the probability of an adequate reaction also depends on the mother's investment of time and effort into the educational process. Mothers may be very busy with other things than just this particular infant. They simply don't have the time to respond to whatever signal the infant emits. Some mothers may have time, but are simply not motivated to pay much attention to the educational task. These variations in investment exist between mothers, but also 'within' subjects. That is, we may expect rather considerable variation across time (just consider a mother being ill for some time, or busy finishing her dissertation). I do not pretend that a mother should always be there for her child (and I even assume that too much interference is probably not good at all). However, whatever the optimal investment in terms of time or effort spent in the educational process, we should reckon with considerable within- and between-subject variations (again I should add that children differ in what *they* experience as an optimal investment, but I have kept this factor constant in the model).

The reader may be struck by the fact that nowhere in this list of properties the concept of sensitivity or sensitive responsiveness can be found. The answer to this omission is that sensitive responsiveness is a variable which expresses a relation between the educational activity of the mother and the activity of the infant. I fully agree with those authors who view sensitive responsiveness as an interactive, not an individual property (Hoeksma & Koomen, 1991). In the model, sensitive responsiveness can be defined as the match between the optimal help level of the infant and the actual help given, which is represented by the second main variable in the model (see also the Appendix for technical details). This matching level corresponds with a psychological property of the mother, namely her ability to correctly estimate the child's actual needs and her ability to act accordingly.

Common parameters

The model simulates the growth process in the form of a series of steps. Each step corresponds with an 'encounter' between mother and infant (or 'frame', in Fogel's terminology, see Fogel, this volume, and Fogel, 1993). Not all encounters (whatever their nature) lead to an effective interaction. For instance, an infant may express its need for proximity or support, but the mother may lack the time to actually respond. Or, alternatively, a mother may try to help an infant who is not particularly willing to accept the help, or even withdraws from the mother. Put differently, there is a certain probability that an interaction goes no further than just its initiation, or that it has no effect on either mother or infant, because (at least) one of the participants does not respond or withdraws. This probability is based on two parameters. One is the probability that a mother actively engages in the interaction, which depends on factors such as available time, effort, and so forth, but also her belief in how independent her infant should be and whether or not she should respond to even the slightest signal from the infant. A second parameter is the infant's tendency to engage in the interaction, which is probably mostly affected by the infant's tendency to avoid contact, or withdraw.

The parameters described in the preceding sections are constants, i.e., numbers with a fixed value. It is unlikely, however, that the infant's effective growth rate, for instance, is similar over all interaction situations. Information processing qualities, which I associated with the abstract growth rate notion, are likely to vary, for instance as a consequence of varying levels of effort, fatigue, interest and so on. These variations, although eventually regular in their own right (cyclical fluctuations in interest and effort, for instance), can best be conceived of as the cause of random deviations from the fixed parameter values. Thus, instead of writing equations with fixed values we shall employ randomized values. The constant value can be seen as the average of a series of randomly varying values.

Special attention should be given to the randomization of the help-and-support variable, the growth of which is primarily based on maternal parameters (basically because I have kept the eventual influence by infant parameters constant, such as the interpretability of the infant's behavior). We may assume that help and support given are not just functions of the mother's influence. Occasionally her educational and didactic interventions are supported and intensified because she obtains the help of a mate. On a different occasion her interaction with the infant is counteracted because somebody else (her husband for instance) calls for her attention. Sometimes she uses tools (such as picture books or toys) that enhance the infant's attention, but it is equally likely that these objects sometimes diminish the effectiveness of the interaction, for instance because the infant is not particularly interested in them or dislikes them. Put differently, on top of the random variation of the parameters I add a certain amount of random variation to the help and support variable itself. All the randomization parameters can be manipulated, thus accounting for individual or family differences in fluctuations of the quality of the educational interactions.

Sensitive responsiveness

I have stated earlier that sensitive responsiveness is a property that *emerges* in the interaction between mother and infant. Since this interaction is the product of the parameters and the rules of interaction, we may conceive of sensitivity, or, more precisely, sensitive responsiveness, as a 'dependent' variable. An optimally sensitive mother can be defined as a mother who manages to keep the help, support or information given to the child at the child's optimum level, that is, at a level the child will maximally profit from. Since sensitive responsiveness is a product of the interaction process, it may vary, and eventually vary considerably, over the course of that process (see the Appendix for a more technical description of how sensitive responsiveness has been defined and calculated in the model). It is possible, therefore, that simulations produce 'mothers' that show a high level of sensitive responsiveness for a certain period of time, and then altogether lose their sensitivity and subsequently pay only little attention to the child's particular needs.

What is a sensitive mother sensitive to? A mother can be sensitive to the child's progress in competencies such as language, social understanding or problem solving, and stimulate her child in acquiring more words, higher syntactic proficiency or social skills. But the mother can also be sensitive to things such as the child's tendency to withdraw from an interaction, or its unwillingness to seek proximity. For instance, the mother may see the child's withdrawal as an expression of its ingratitude or whatever other negative personality trait of her infant she thinks responsible for its behavior. She may instantly respond to the child's withdrawal, for instance by punishing it, or forcing it to come back and join the activity. This, no doubt, will increase the child's unwillingness to engage in interesting activities with his mother. That is, the mother is positively responsive to the child's withdrawal tendency in that she's sensitive to the occurrence of withdrawal and is actively engaged in stimulating and enhancing this tendency (although her conscious mind probably tells her she's doing the opposite).

Summary and conclusion

In a nutshell, the model contains two growers, that is, two variables that grow towards an equilibrium level. One is a behavioral variable in the child, for instance, the child's ability to cope with social situations involving unfamiliar people, his tendency to seek proximity, or the level of some cognitive skill. This variable grows towards an attractor state which is a function both of the environment and of the child him- or herself. In the present model the child-dependent aspect is kept constant. This fact allows us to experiment with the environmental aspects alone. The latter are modeled in the form of psychological and action-properties of the mother. They consist of aspects such as the quality of the mother's estimation of her child's needs and abilities, the rate with which she adapts her help, intervention and information to changes in the child's behavior, and so forth. Concepts such as sensitivity are covered by several model parameters and are essentially dependent on the conditions that the child brings in into the interaction (and those conditions are, in their turn, affected by the ongoing interaction).

Results of simulation experiments

Basic properties of the simulations

The time scale of a simulation

A single simulation of the model is based on 2000 consecutive steps. Since each step represents a particular interaction 'frame' or learning encounter the real time scale of a simulation (or 'run' of the model) depends on our estimation of how many of such frames occur on average during a day. Let us assume that an interaction frame amounts to such interactive activities as a common lunch, putting the child to bed or in bath, playing, the mother's trying to keep her infant quiet while's she's having a chat with her neighbor, mother and infant looking in a picture book, and so forth. Assume that we have estimated the average number of such interactions at about 20 a day. This implies that a single run of the model (2000 steps) spans about a hundred days. It is not unlikely that our 20-a-day estimation is much too optimistic. If it is, we must adapt the length of the models and the parameters used to whatever we consider a realistic estimation of the ongoing mother-infant interaction. Our model shouldn't cover a time stretch which exceeds the period during which the mechanisms held responsible for the emergence of attachment are operational.

Forms of random variation across simulations

An important aspect of the present dynamic growth model is that it implies a certain degree of randomness. This randomness accounts for a variety of interesting phenomena, which I will discuss in later sections. Let me first explain the nature of the randomness applied in the simulations.

Basically, there are three sources of random variation in the simulations. Two of them refer to *intra-subject randomness*, that is, random variations that occur within a single run.

The first depends on the successful-encounter probability parameter. Recall that the model makes a distinction between successful and unsuccessful encounters or interaction frames. An unsuccessful one does not go any farther than just an unsuccessful initiative, for instance. The encounter probability has the form of a randomly distributed dichotomous variable (there either is an effect or there isn't). It is determined by either the mother's possibilities to respond to the infant or the infant's tendency to withdraw from an interaction initiated by the mother.

A second source of intra-subject randomness lies in the random variations of the parameter values across steps (or encounters). A particular chosen parameter value (such as the infant's growth rate r) is nothing but an average value. It's real value depends on things like effort, attention, fatigue, and so on, whose variation may eventually be systematic (e.g., cyclical). Since I cannot control for these dynamic variables in the present model, I treat them as random variability.

The third source of randomness deals with the distribution of parameter values in (simulated) populations of mother-infant dyads. In order to simulate a population, I start with fixed parameter values (e.g., I choose an infant growth rate parameter $r=0.1$). A population consists of a certain number of cases (separate runs of the model) with parameter values that are randomly picked from a (quasi-)normal distribution around the chosen default values. By increasing the random variation parameter I can manipulate the variation in the simulated population. By setting this randomization parameter to zero I can simulate the effect of 'repeated lives', that is, I can compare the effect of intra-subject randomness upon variables such as the outcome of the growth process or the correlation between sensitive responsiveness and peaks in the growth variable.

Simulations of attachment growth

Base-line simulations

The first check on a dynamic model is to see whether or not it works. The minimal requirements are, first, that the model produces systematic growth patterns of either increase or decrease. Second, it should generate deficient trajectories under parameter conditions that we consider deviant or deficient and normal trajectories under normal parameter conditions. By 'normal' trajectories we understand trajectories that lead to a successful growth of the behavioral variable we simulate. What we call 'successful' depends on our expectations of what the normal pattern should be. For instance, we may either expect a smooth S-shaped increase towards a stable equilibrium, or a relatively sudden peak in the behavioral variable at issue (for instance withdrawal tendency or fear for strangers), followed by a gradual move towards a (lower) equilibrium state.

The model does not simulate qualitatively specified patterns. It shows how a variable, whatever its nature, changes in relation to other variables. In the case of an attachment model, the simulated variable could be the child's inclination towards seeking proximity, the probability it avoids contact with his mother, and so forth. In order to justify the names given to the variables (e.g., variable A is proximity seeking), those variables should have all the relevant dynamic properties of the empirical phenomena to which they refer (e.g., A should react to its model-theoretic environment in the same way as proximity seeking should react to its empirical environment).

Let me first give an overview of the qualitative growth patterns that emerge from the model simulations. One particular parameter represents the rate with which the (simulated) mother adapts to changes in the behavior of the child. I took a value which presents a significant risk for deficient trajectories. The question I tried to answer was: which qualitative patterns will emerge for parameter constellations that vary randomly around this particular, potentially deficient value? The results were as follows. About half of the cases (51% out of 484 runs) showed a *zero pattern*. That is, the growth level of the behavioral variable L remained about the same as its initial level. This means that with deficient parameter sets growth doesn't get off the ground. The initial state level is a stable attractor (more precisely, there's a certain range around that level that serves as a small regional attractor). The remaining cases (49%) that led to an effective growth process showed the following variation in patterns. Sixty-eight percent (68 %) of the successful cases showed a peak pattern. That is, the behavioral level grows towards a peak and then falls back to a lower level. There are two patterns. About half of the cases evolve towards a stable end state (Figure 1, top left). These end states are approximately normally distributed with a mean which is about .75 (recall that the inbuilt final equilibrium is 1). These end states are 'frozen' states in that the growth rate has dropped to zero (on average). This happens because the support (the attractor state basically caused by the mother's activity) has fallen away completely. This is an interesting finding in that it was in no way built into the model: in a considerable number of cases the maternal support for a certain kind of behavior disappears. It is interesting to see that this downswing can have two different effects. I already mentioned one: the child's behavioral level with regard to some habit, skill or competence, remains stable. There exists a second possibility which occurs in about half of the cases that show a peak trajectory: the regression in the supporting maternal behaviors is adapted to the rate of change

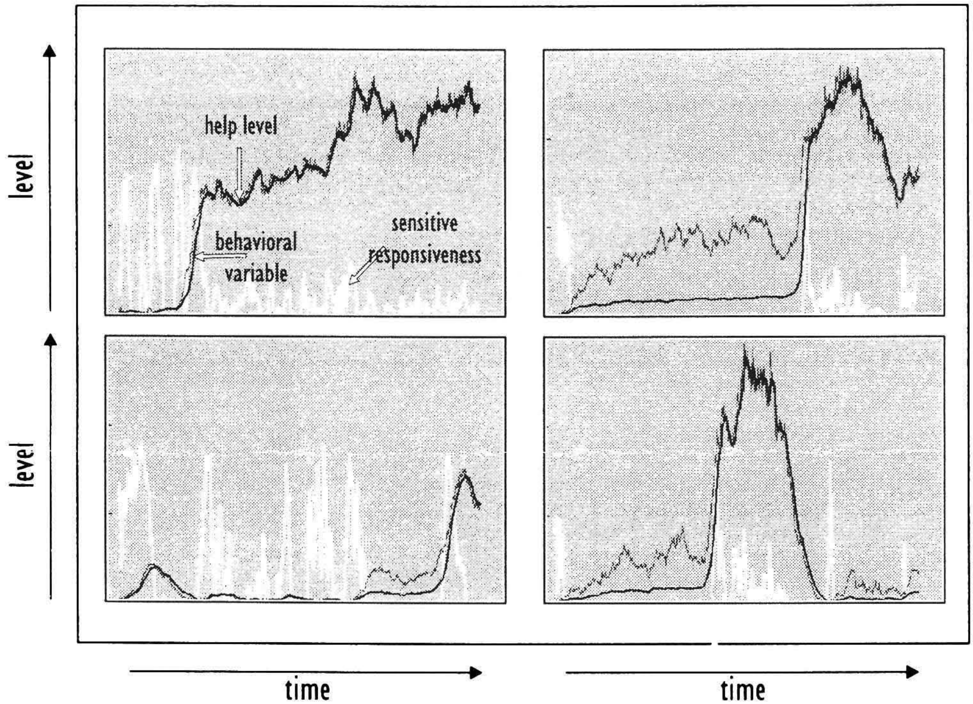


Fig. 1. Four growth patterns of a behavioral variable, with associated help-and-support and sensitivity levels; although sensitive responsiveness is an important factor for determining the course and rate of growth, it is certainly not the only factor: growth is determined by a dynamic interaction between the variables, leading to a variety of growth patterns.

in the child's behavior. This leads to a mutually supported downswing of both the behavior and the support (see for instance Figure 1, bottom right). and the support (see for instance Figure 1, bottom right).

The remaining 32% of the successful trajectories falls into two patterns. One is an ordinary logistic pattern: the behavioral level L is attracted by the attractor state. This either locks the attractor state into some oscillating equilibrium, or the attractor state disappears after a while and leaves a 'frozen' equilibrium level of L. The logistic pattern (S-shaped increase) occurs in 22% of the cases. The remaining 10% consists of sudden jumps: the L-level stays approximately the same for quite some time, while the attractor state steadily increases. Then a sudden jump follows, which brings the L-level in the proximity of the attractor (Figure 1, top right).

The basic hypothesis, that the model produces a rich set of growth trajectories, which can be expected in real psychological growth, can be confirmed. One should note, however, that the richness of the patterns depends very much on whether or not the parameter values are at a point which marks the boundary between various qualitative 'regimes'. For the present simulation I have chosen a set of values around such a boundary, thus allowing for a maximal qualitative variability. Small random fluctuations will push the trajectories in one or another pattern, thus exploring a rich variety of forms. Many of the combinations of parameter values produce smaller varieties of growth patterns. I shall discuss a few of those patterns in the coming sections.

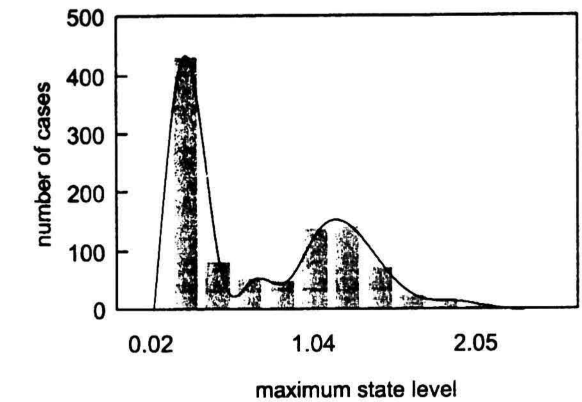
(Multi-)modality of the distributions

Any empirical typology that describes development of *types* (inter-subject typology) or *stages* (intra-subject typology) should be based on one or several multimodal distributions of distinctive properties. For instance, if children develop in stages, we expect significantly more children in a stage than at a level between two stages (see Van der Maas & Molenaar, 1992). Attachment occurs in various types (the famous A-B-C typology) based on a few distinctive properties, such as proximity seeking, fear of strangers and so on. We may expect, therefore, that those properties show a multi-modal distribution in a population, once the attachment pattern has emerged. Eventually, the patterns result from bimodal distributions of distinctive features (the behavioral dimensions on which the patterns are scored).

Our simulations clearly show bi- and eventually multi-modal distributions, notwithstanding the fact that all of the parameters are either normally or evenly distributed. The present model has a second source of multi-modality, which lies in the randomized nature of how the parameters affect the growth patterns. Recall that the parameter values with which the model operates are just averages. Their actual value varies randomly from step to step (or from learning encounter to learning encounter, or from interaction to interaction). The random variation is caused by the intervention of any influencing factor that is not covered by the model or by any of its parameters. By means of a randomization parameter, one can control the amount of variation, for instance, ranging from very small to very substantial. In one simulation of 1000 cases I applied a fully randomized distribution of all the parameters involved (within certain general boundaries, e.g. keeping the initial state levels confined to a - broad - low range). The distributions of the following variables were plotted: average level (averaged over the 2000 steps in each simulated case), the maximal level of each case (which gives an idea of the eventual peaks) and, finally, the end state (the average value of the last 50 steps in each simulated case). The resulting bimodal distribution is most explicitly present in the maximum state (or peak state) distributions (see Figure 2). Average and end state show bimodal distributions where the second modality is smeared out over a much larger range than the first one.

Theoretically, there are three equilibrium or attractor levels in the model. One is a level close to the initial level (or some other very low level). Another is the maximum level, based on the equilibrium level built into the help and support function. The third and most interesting is an equilibrium level somewhere between the minimum and maximum levels. It depends on the ratio between the child's growth parameter and the adult's adaptation parameter (the parameter that governs the rate of adaptation of help, support and information to the perceived

Fig. 2. An example of a bimodal end state distribution. The sharp frequency peak at the low end refers to a large number of cases where the behavioral variable didn't get off the ground. The second mode takes the form of a normal distribution with a peak at about 1.1



amount of progress or change in the infant). In dynamic terms, the model has three *attractors* and two *separatrices*, that is, lines that mark the boundary between trajectories that will evolve towards a particular attractor. If the model is entirely deterministic, the parameter values determine at which side of the separatrix a particular trajectory will fall, that is, towards which attractor it will evolve. If the model is randomized, the random fluctuation will have only minor effects as long as they stay sufficiently far from the separatrices. However, in the vicinity of a separatrix, a small random fluctuation may have a dramatic effect, in that it may push the process 'over the rim', so to speak, and into a new attractor basin.

Do we always find bi- or multi-modal distributions? The answer is, not really, or more precisely, the bi- or multi-modal distributions are not always equally clear. For instance, if the successful interaction probability is set to its default value (50%) clear bimodal distributions are found when the additional intra-subject randomness is fairly low (low in comparison to an arbitrarily chosen *default value*, which has no intrinsic psychological meaning and is just a technical calibration point). If the successful encounter probability is high (say 75 % or more), bimodal distributions can be found with the 'default' randomness parameters. This example serves just to illustrate the fact that although bimodality is a general and frequently occurring phenomenon, its distribution over the parameter space depends on a variety of factors.

Where does bimodality come from in the trajectories? It may result from one part of the trajectories zooming in on a low end state level and another part progressing towards a much higher level. But it may also result from an oscillation in all the trajectories between a low level and a high level (Hartelmann, 1995). From a general inspection of the patterns resulting from many different parameter values it seems that, although the first scenario (two end states) occurs particularly if intra-subject randomization is low, the major scenario is one in which the variable at issue shows a single major peak and then falls down to some low or intermediate end state. This pattern corresponds to what one expects from attachment research: the typical attachment pattern occurs quite suddenly, lasts for some time, and then either partially or completely disappears. It goes without saying that the underlying feeling of security or perceived competence does not disappear together with this particular behavioral expression. It is taken over by alternative forms or expressions of security, social anxiety or whatever. If the infant growth and maternal adaptation rates are high and if the model has high intra-subject randomization parameters, the trajectories tend towards oscillatory patterns, showing repeated peaks of different lengths and heights.

The association between sensitive responsiveness and the growth of behavioral variables

In the model, sensitive responsiveness is a product variable which is computed from the match between, on the one hand, the mother's accuracy to 'read' her child's progress or developmental level, the nature of her reaction (the magnitude of the adaptation to the child's growth or change), and, on the other hand, the child's optimal level of help, support or assistance. What is the relationship between the growth of a behavioral variable (such as the probability of avoidance or proximity seeking) and the level of sensitive responsiveness?
As a preliminary answer to this question, I present the analysis of some combinations of parameter values. The analysis can be compared with the work of an archeologist who, instead of excavating the whole site, confines himself to digging a few ditches and from there on tries to anticipate the structures he shall find if the whole field is opened. Each simulation has been based on 300 runs. In general, this number more than suffices to let the computed correlation values stabilize around a narrow range. Each run is based on a new set of parameter values, randomly drawn from a sample that roughly approached a normal distribution around the parameter values that were chosen as the anchoring points for a particular simulation. For each parameter, the range of variation was -.3 to +.3 of its assigned value. From the simulations it can be concluded that the majority of parameter combinations possible in the dynamic model yields correlations between either average level, end level or maximum growth level on the one hand and sensitivity on the other that are within the significant range (which I arbitrarily define as correlations bigger than .3).

Now that we have seen that there exists a correlation between sensitivity and growth of a behavioral variable, the second main question is, is there any *general* line specifying the relationship between sensitivity and the outcome of simulations?

A first interesting conclusion is that simulations with no intra-subject randomization of the parameters, *except for the successful encounter probability*, show fairly low correlations. This is so because the outcomes are determined by the distribution of successful encounters over the entire range of steps. If, accidentally, successful encounters are infrequent at the very beginning of the growth process, the trajectory aims towards a low end state. Once it enters this particular attractor basin it is very difficult to get out of it, basically because the adaptation is almost entirely determined by fixed adaptation rules and less by randomized factors that might push the process out of this basin. One may conclude, therefore, that under the present probability conditions, the first stage is highly decisive as to which growth pattern will result.

A second conclusion pertains to the relationship between the level of intra-subject randomization and the resulting correlations. We have seen that if intra-subject randomization is zero, the correlation is low (r is around .2). Figure 3 shows the complex relationship between randomization level and outcome/sensitivity correlations (for the set of parameters I have arbitrarily chosen as my default set).

Let me now reinstall the default randomization parameter (which, as I explained earlier, is a more or less arbitrary anchoring point) and experiment with the relationship between sensitivity and growth level under varying values of the growth rate variable.

The simulation results in a linear relationship between sensitivity and level, beginning with a high positive correlation (.54) for the lowest growth rate chosen (half the default) and ending with a negative correlation (-.3) for a growth rate twice the default.

If I vary the encounter parameter (that is, the parameter determining the probability that an 'encounter' will evolve into a successful interaction), I find a non-linear relationship between level and sensitivity when I hold the randomization at the default level, but for a low randomization rate, the relationship is clearly linear and proportional to the encounter probability. By varying the optimality parameter we find that the relationship between level and sensitivity is U-shaped for simulations with a default randomization value. With a low randomization, however, the relationship is linear (with the line somewhat bent) and runs opposite to the increase in the optimality value.

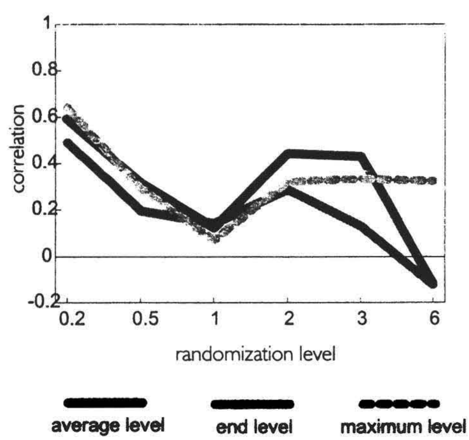


Fig. 3. Plots of the correlation between growth and sensitive responsiveness with a varying randomization level (left) and a varying growth rate (right). Whereas increasing growth rate results in a linear decline of the correlational level, an increasing randomization leads to a non-linear pattern with two local minima (one at level 1, the other at level 6). The number 1 on the x-axis refers to the default parameter value (e.g., the value 0.5 of the growth rate axis represents a growth rate which is half the default value used in the simulations).

Finally, experimenting with the arousal range parameter results in an inverted U-shape relationship between level and sensitivity.

What can we conclude from this short, somewhat arbitrary but probably highly confusing list of relationships between a 'dependent' variable (the growth level of a behavioral tendency, skill, ability or whatever) and sensitivity? Some relationships are linear, others are curvilinear, still others are non-linear in a more complex way. However, all these simulations have been based on a single model of mother-infant interaction. They differ only in what part of the parameter landscape they explore, not in the underlying mechanisms and principles. Thus taken as a whole, they provide strong evidence for the non-linearity of the relationships between growth level and sensitivity that the present model predicts (and deductively generates in the form of simulations).

Of course, one can easily get away with this conclusion by stating that the simulations show that the relationship between growth and sensitivity is context-dependent, which is not very surprising anyway. One should make a distinction, however, between actual contexts featuring in a developmental process (real or simulated) and context in the sense of particular combinations of model parameters. In the first sense, the context-relatedness of development amounts to an almost trivial observation. In the second sense, however, (context as the set of parameters that govern a particular process) context-dependency is far less trivial. We have seen that the nature and form of the association between sensitivity and growth is a non-linear property of the model, and, more importantly, that the relationship between the parameters and that association is a systematic property of the model (and not just a random connection). The present simulations provide only a first approximation of the hidden structure of the model, just like the few trenches dug by an archeologist show little more than a crude estimation of what the excavation site may reveal.

Dynamic models of interaction: some conclusions and discussion

Summary

In this chapter I have presented the outlines and backgrounds of a dynamic systems model of early mother-infant interaction applied to (some highly abstracted aspects of) attachment development. The model is based on a combination of an existing non-linear growth model with a set of theoretical statements about expected relationships between empirical variables. The statements are derived from the literature on attachment and the learning of early competencis. Our simulations showed that the simulated variables (a particular behavioral variable or tendency, an ability, and so forth) can grow in a variety of ways. They can increase more or less linearly (with random fluctuations added), grow in an S-shaped form, or show one or several sudden growth spurts. After that they can stay at an equilibrium level, but also remain at their peak level for only a little while and then glide downward either to an intermediate stable level or to a level close to their original starting point. The latter may occur only once, but also repeatedly. Which pattern will be followed is, to a certain extent, a coincidence, but one whose chances are dependent on which parameter values are actually used.

In the simulation studies I concentrated on two issues. One was the expected bi- or multi-modality of the distributions, an expectation which is justified by the fact that attachment patterns occur in types. Bi- or even multi-modality proved to be the rule rather than the exception. I should add, however, that in many cases where such multi-modality was found it was more explicitly based on temporary peaks than on a sharp division of end states across a simulated population. The second issue pertained to the nature of the relationships between sensitivity and growth (for instance the growth of the tendency to withdraw from an interaction situation, or the growth of security or of the ability to do something oneself, without help). I found that 'locally' specified relationships (i.e., for one particular parameter varying) where often of the standard linear, curvilinear or (inverted) U-shaped form. As a whole, however, that is, with all the local contexts combined, the model specifies a strongly non-linear relationship between sensitivity and observed growth.

The function of non-linear dynamic model building

In this chapter, I haven't built any models of concrete empirical data from experiments, and neither did I present new discoveries of how attachment and related infant competencis develop. What is this all good for? The answer to this question pertains not only to mother-infant interaction models, but to the relationship between dynamic mathematical model building on the one hand and developmental theorizing and research on the other.

First of all, non-linear dynamic model building provides a *computational justification* for the theoretical claims of a model. For instance, if I claim that a certain relationship between two variables suffices to explain the sudden emergence of a third, I need a computational model to formally or deductively demonstrate that my claim is right. This demonstration has of course no bearings on the empirical truth of the claim. The computational, deductive test is only a meager substitute for a real-life experiment. But since such experiments are so difficult to carry out (and also provided that they can be performed, which is not always the case in developmental psychology or the social sciences in general, for that matter) it is much more convenient to test the claims of the theory in the form of a dynamic systems model that can be tested in the computer. It is basically a test for the theoretical legitimacy of the if-then statements that any model implicitly contains: if such-and-such mechanisms and conditions apply, then such-and-such effects will result.

The second function is *heuristic*. Very often, dynamic models of existing theories produce not only the expected patterns (linear increase, for instance) but also unexpected ones, such as sudden growth spurts, temporary regressions, peaks, cycles, and so on. These patterns often make theoretical and empirical sense, albeit in a vague way perhaps, but they were not explicitly expected simply because the theoretical models used did not allow for a deductively based and exhaustive search of the possibilities implied in those models. The fact that dynamic models can show, for instance, that a threshold crossing in one or other parameter produces a different growth pattern (for instance, a single S-shape turns into a step-wise pattern or eventually a cycle) makes them very suitable for explorative theorizing. The function of such theorizing is to guide and inspire our empirical search for new or alternative developmental patterns.

The heuristic value not only extends to the domain of hypothesizing, that is, the formation of hypotheses about which type of developmental trajectories will eventually be found. It also applies to the *methodological* issues. For instance, it supports and justifies the demand for much more detailed longitudinal studies that collect time series data from individual children in a quest for the underlying patterns of change.

In our simulations we have seen that correlations between sensitive responsiveness and the growth and equilibrium of a particular developing variable is not unequivocal. In some domains of the parameter space correlations are linearly related to some control variable, in other parts the relationship is U-shaped. Moreover, also the direction of these relationships (increasing, decreasing, inverted U or straight U, monotonic or complex...) differs for different regions in the parameter space. From this one may conclude that the present dynamic model justifies all possible, eventually contradicting empirical findings. Of course it does not: what it tells is that empirical relationships make sense only given certain parameter values and sets. As long as those conditions are not explicitly taken into account, the results may eventually remain seemingly contradictory. One way out of this methodological impasse is to no longer concentrate on comparative group-based research designs and instead focus almost exclusively on single-case studies. This is what happened with language development research in the seventies, when researchers turned massively to studies of single children. At present, the results of all these individual studies have been collected in an electronic data base (*Childes*) and now allow for generalizations and group comparisons.

The third function of non-linear dynamics is to provide *empirically testable models* of quantitative and qualitative phenomena of developmental change. For instance, it is possible to describe lexical and syntactic growth in the form of a simple logistic growth model which fits very well with the available empirical data (Van Geert, 1991; 1994a; Ruhland & Van Geert, 1995). It should be noted that this is a form of 'high level' model building, 'high level' in the sense that abstraction is made from the actual neurological, cognitive or sensory-motor processes that constitute the acts of learning or development underlying the empirical phenomena at issue. Growth modeling claims that an abstract conceptualization in terms of overarching growth processes provides a good generalization of what happens in a wide variety of developmental domains.

The fitting of theoretical curves to individual growth trajectories is but one of the possible empirical applications of non- linear growth theory. A second form of application attempts to explain group-based phenomena of longitudinal change. Examples are correlations found among groups or variables, but also of different kinds of relationships among variables (linear, but also curvilinear, U-shaped, inverted U-shaped and various non-monotonic and non-linear relationships). I have presented several examples of this variability by simulating collections of individual trajectories under specific conditions of randomization.

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Appendix

The first equation describes the growth of a behavioral variable in the infant. Its eventual equilibrium state is given by the help and resources invested by the adult:

$$L_{t+1} = L_t + L_t \cdot \langle (1 + D\_R \cdot rand_{-1}^{+1}) \cdot R_t - L_t \cdot (1 + D\_R \cdot rand_{-1}^{+1}) \cdot R_t / K_t \rangle \quad \text{(eq. 1)}$$

- L_t and L_{t+1} are the levels of a behavioral variable in the infant at times t and t+1
- $(1 + D\_R \cdot rand_{-1}^{+1}) \cdot R_t$ is a variable, randomized growth rate
- $rand_{-1}^{+1}$ is a normally distributed random number between -1 and +1 with standard deviation .4 and mean 0
- D_R is a parameter that damps the effect of the random number
- R_t is the optimized growth rate (equation 2) of the variable L
- K_t is the local attractor state of L, which is a function of the behavior of the mother, K_t equals M_t unless $M_t < 0$, in which case K_t is equal to a minimal equilibrium level

$$R_t = \langle R\_L - | K_{t-1} / L_{t-1} - L\_OPT) | \cdot R\_DAMP \cdot (1 - K_t) \rangle \cdot Av_t \quad \text{(eq. 2)}$$

- R_L is a fixed growth rate parameter (e.g. R_L=0.1)
- L_OPT is an optimality value; for instance if L_OPT=1.5, the value of R_t is maximal if K_{t-1} is 1.5 times bigger than the value of L_{t-1}
- R_DAMP is a parameter which dampens the effect of the optimality variable; the bigger R_DAMP, the smaller the range within which R_t still has a significant value
- AV_t is a binary parameter which has value 1 if a randomly drawn number is smaller than the value of an avoidance parameter, and 0 in all other cases

(for a more detailed discussion of comparable equations I refer to Van Geert, 1994b).

These two equations describe the infant's part of the model. The mother's part of the model is represented by an attractor state equation. The attractor state should in no way be seen as a property of the mother herself. It is a property of the situation, more precisely of the infant's potential for development (more precisely, for growth in the infant variable at issue). The attractor state variable is directly dependent on the mother's educational activity and therefore represents maternal properties, such as the mother's ability to adapt her actions to the needs of her infant.

Basically, the attractor state equation says that the level of the attractor state grows towards a final equilibrium as a function of, first, the preceding increase (or decrease) in the behavioral variable, and, second, the quality and magnitude of maternal actions aiming at changing this behavioral variable (note however, that the mother need not have a conscious goal representation, the goal is a property of the system).

$$M_{t+1} = \langle M_t + M_t \cdot [(1 + D\_RAND\_M \cdot rand_{-1}^{+1}) \cdot R\_EFFECT_{t+1} \cdot \Delta L_t \cdot (1 + M_t)] \rangle \cdot \langle 1 + K\_R \cdot rand_{-1}^{+1} \rangle \quad \text{(eq. 3)}$$

- M_{t+1}, the attractor state level as defined by maternal influences; the actual attractor K_{t+1} is a function of M_{t+1} (see equation 1)
- D_RAND_M is a randomizing parameter, specifying the degree in which the rate of change is randomized; it can be used to specify specific maternal biases (see further)
- R_EFFECT_{t+1} is a composite growth rate parameter
  $R\_EFFECT_{t+1} = B_{t+1} \cdot D \cdot (1 + D\_R \cdot rand_{-1}^{+1})$
  For B_{t+1} a number from a Bernouilli sequence, i.e., a sequence where 1's and 0's occur with a fixed random probability B; and D a parameter describing the average rate of increase, given a certain increase in the behavioral variable L
- ΔL_t is the increase or decrease in the behavioral variable L over the preceding encounter; in the model I usually use the relative increase, that is, the increase divided by the previous level of L
- K_R is a parameter determining the amount of contextual randomization upon the attractor level; contextual randomization is that part of the random variation that does not depend on random variation in either maternal or infant parameters

As a starting point of simulations with these equations one can take any initial level of L_0 and M_0. For instance, as initial level of L take a small number which is a fraction of 1, for instance 0.01. As initial level of M_0 I take a number which is a random fraction bigger than the value chosen for L_0.

The sensitive responsiveness score S for the mother in any particular simulation can be computed with the following equation. Let sensitive responsiveness S be defined as the mother providing just that kind of help, support or information that is optimally suited to the child's needs and possibilities. This optimality point is defined by the OPT parameter. If the maximum score is arbitrarily set to 1, it follows that

$$\text{If } (L_i \cdot OPT) / M_i = 1 \text{ then } S(1) = (L_i \cdot OPT) / M_i = 1 \text{ (maximum)} \quad \text{(eq. 4)}$$

(By S(1) I mean the sensitivity score computed by this particular, i.e., first, method)
When is sensitive responsiveness minimal? If help, support or information given by the mother is at the same level as the child's current developmental level, or even below that level, we can say that the mother is completely insensitive to what the child would need in order to make any progress (this reasoning is only partially true; it may be necessary for a mother to just follow or mirror the child every now and then, to enable the infant to relax or present him with a model of what he's doing; but this should be what it is, namely an exception).
Thus

$$\text{if } L_i \geq M_i \text{ then } S(1) = 0 \text{ (minimal)} \quad \text{(eq. 5)}$$

Since the fraction from equation 4 may also be bigger than 1, and since 1 is the maximum value of S(1), we need to transform all values bigger than 1 into values smaller than 1. This can be done by subtracting the part bigger than 1 from the maximum value, which is 1:

if $(L_i \cdot OPT) / M_i > 1$ then $S(2) = 1 - ((L_i \cdot OPT) / M_i - 1) = 2 - (L_i \cdot OPT) / M_i$

else $S(2) = (L_i \cdot OPT) / M_i$ (eq. 6)

Given equation 5, we know that S(2) approaches its minimum value if L approaches M (or vice versa), which implies that

$S(2) = 0$ if $L_i = M_i$

which implies that

$(L_i \cdot OPT) / M_i = OPT$ (eq. 7)

Since OPT is, by definition, bigger than 1, the main condition from equation 6 applies, and therefore S must be minimal if

$S(2) = 2 - OPT$ (eq. 8)

This implies that the range between the maximum value of S and its minimal value must equal the maximum value (1) minus the minimum value:

$Range_{S(2)} = 1 - (2 - OPT) = OPT - 1$ (eq. 9)

In order to compute a sensitive responsiveness score which ranges from 0 to 1, we have to rescale the currently computed S(2) values such that the value of (OPT -1) corresponds with the value 1. Given that if L>M S(2) is by definition 0, we may simplify the rescaling as follows:

if $L_i > M_i$ then $S(3) = 0$

else $S(3) = 1 - (1 - S(2)) / (OPT -1)$ (eq. 10)

(Recall that S(2) refers to sensitivity as computed with the if-statement from equation 6).

I have used equation 10 to compute the sensitive responsiveness score for each step of a simulation.