Han van der Maas, Maartje Raijmakers, Pascal Hartelman and Peter Molenaar

# Reaction time as recovery time after perturbation


## Abstract

This paper introduces a preliminary model of reaction time that is derived from dynamic system theory. This alternative for the random walk model of reaction time applies another definition of information change and, as a consequence, another stop-criterion. Where the random walk in random walk models stops when a bound is reached, the walk in the dynamical model stops when it converges, or, in other words, when the system recovers stability. The mathematical formulation of the dynamical model is not yet complete. However, by computer simulation it can be shown that the model yields correct predictions about the reaction time distribution. First we will explain the background of this new model of reaction time, then the model is explained and compared with the traditional random walk model.


## Critical slowing down

A central concept in non-linear dynamic system theory (e.g. chaos theory, catastrophe theory, synergetics, bifurcation theory) is the concept of stability. These theories describe the time-course of behavioral variables of the system in terms of attractors and transitions between them. In this volume much attention is given to catastrophe theory. Catastrophe theory (Thom, 1975) can be used to study transitions between simple point attractors. These phase transitions are accompanied by so-called catastrophe flags. Hence, these flags indicate the presence of a non-linear phase transition. One of these flags is critical slowing down, a delayed recovery of stability after perturbation in the transition phase (Gilmore, 1981).

This flag can also be found in other theories of nonlinear dynamics. In earlier papers, to detect the flag critical slowing down, we suggested to look at reaction times. Our prediction, derived from catastrophe theory, is that reaction times are higher near a transition. This hypothesis is relatively easy to test. It is also consistent with common sense and many developmental models. For instance, Siegler and Jenkins (1989) have looked at higher reaction times of subjects in the transition between the so-called sum and min strategy for simple arithmetic problems. In a study of Hosenfeld, van der Maas, and van den Boom (1997) we tested this prediction for the development of analogical reasoning. In this study 80 children were tested 8 times three weeks apart. Evidence for the sudden jump in this study was not very convincing, but we did find statistical significant indications of anomalous variance and critical slowing down as measured by reaction time.

There is however a problem with this line of work. The prediction of longer reaction times is derived from an implicit and informal model of reaction time that seems to be inconsistent with the main models of reaction times in psychology.

## Reaction time models

Reaction time is a very general response measure in psychology. It is the time between stimulus presentation and response. The response itself could be any behavior, but is often simple and discrete. Typical examples are a choice reaction time test with two arrows as stimuli, a lexical decision task with correctly and incorrectly spelled words, and a multiple choice exam. In all these cases the standard model of a reaction time is the random walk model, shown in figure 1.

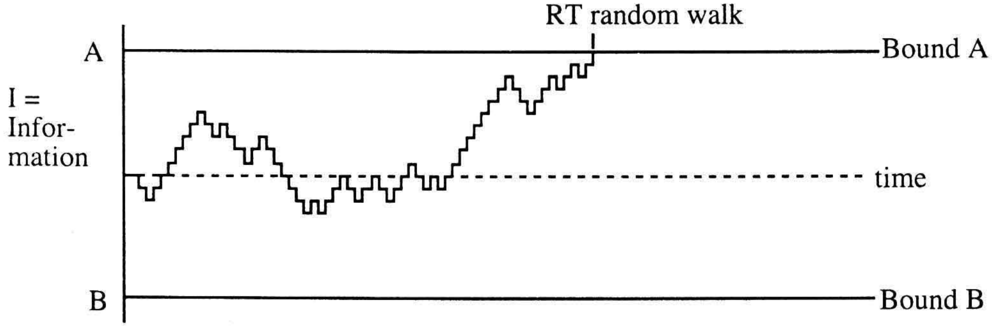



Fig. 1. The random walk model. Information I makes stationary steps over time in either the A or B direction. When a bound is reached the response is generated.

In this simplified model the two choices A and B (left or right, correct or incorrect, etc.) are depicted on the y-axis. The x-axis represents time. The system builds up information, each piece of information having a possible effect on the walk. Whenever the information hits the A or B bound the system makes a response with a reaction time RT. Bounds can be adjusted, possibly even during the run. There exist different mathematical formulations of this model and slightly different variants like the accumulator model and the run model (Luce, 1986; Ratcliff, 1988). These models lead to predictions about the form of the reaction time distribution, the relation between speed and accuracy, and the effects of various experimental manipulations.

The random walk model does not refer to the concept of stability. The random walk model is not some kind of gradient system whereas this is required for an interpretation of critical slowing down in terms of reaction time. Such an interpretation refers to processes as depicted in figure 2.

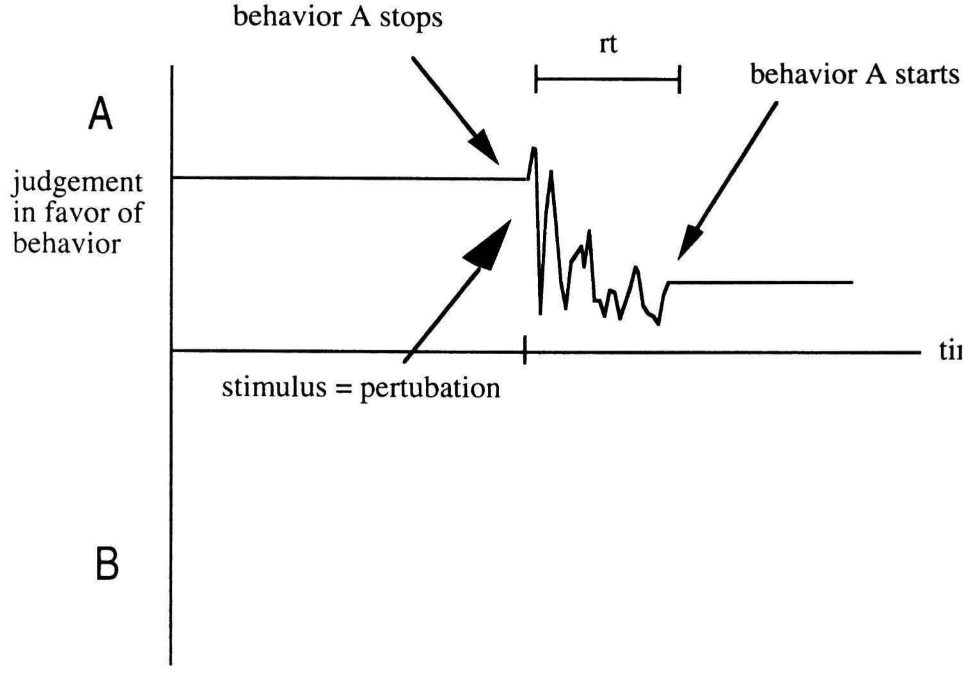



Fig. 2. Reaction time as recovery time after a perturbation. The perturbation is a stimulus, some new information, which, after some time, looses its effect, after which, in this example, behavior A continues.

This figure shows a dynamical system that is perturbed by a stimulus. Behavior A looses stability but is, after some time (the reaction time), recovered. In this case A is recovered but also B could have happened.

What is the big difference with the random walk model? The most striking difference is the stop criterion. We assume that a system will only give a response when it's judgment stops changing, e.g. when it has become stable (e.g. $dI/dt < \varepsilon$ for some time). Epsilon determines the accuracy of the process. Related to this difference in stop criterion is a more subtle differerence in the definition of information, especially in the defintion of information change. In figure 2 the change in information decreases after some time until information converges to a stable state. This means that information change is non-stationary.

In random walk models, however, information change is defined to be stationary ($dI/dt = c$). In the simplest random walk model at each time point I, information, makes one step to A or B. In more complex models the steps are sampled from a time invariant distribution (Ratcliff, 1988). In both cases the information change is stationary. If this change is stationary, the random walk always hits a bound (see Luce, 1986). If the information change over time goes to zero it is well possible that the bounds are never hit. All random walk models exclude this possibility because it can lead to infinite reaction times in these models.

So this assumption of stationary increments is very important. Is it also, as Luce (1986) says, by no means unreasonable?

We doubt this. Lets take a look at the definition of information. It could be neural activity, subjective certainty, or a measure of match, to give some examples. For each of these interpretations, information change varies over time. Luce (1986) already mentioned this for the neural interpretation: for constant input neural activity stabilizes quickly. If it were subjective certainty it seems reasonable to suppose that this also converges to a certain level. It will certainly not increase indefinitely. And also a match with some scheme will give non stationary increments in match value. Just after stimulus presentation dI/dt will be larger for most decision processes we can think of. For instance, a simple arrow to the right is identified as such within 300 ms and then the information growth probably ends. We are certain then and we don't increase certainty by looking longer at the stimulus. Note that dI/dt also decreases as function of T in computational techniques as optimizing, for instance, optimizing the likelihood of a statistical model. We think that for most interpretations or definitions of I, dI/dt decreases as function of t.

In the literature on random walk models of reaction time the possibility of nonstationary information change is sometimes recognized. Ratcliff (1980) formulates a model that allows discrete change in the drift parameter of the diffusion process. Heath (1981) proposes a tandem random walk model for non-stationary random walks. In both proposals, however, the stop criterion is still based on bounds and not on convergence. If we use convergence as stop criterion we get the dynamical model of reaction time.

## The dynamical model of reaction time

Figure 3 illustrates the dynamical model of reaction time in which dI/dt decreases over time. Stimulus evaluation (A or B) is performed by some kind of neural system that minimizes a measure of mismatch between stimulus and internal representation. The stimulus can be considered as a perturbation of this system that restores equilibrium as soon as possible. As such, the system can be understood as a kind of gradient system. In its most elementary form the potential function is quadratic.

The dynamical model and the random walk model agree in most aspects. In both models information is build up for making a choice or decision. The dependent variable is either the amount of information (somehow quantified) or the subjective certainty of the choice. In both cases a continuously changing behavior is forced to become discrete. The system should make this simplification in some way. Here the two models differ. The dynamical model assumes that this occurs when the state of the system, in terms of the amount of information or certainty, is not changing anymore, whereas in the random walk model it occurs whenever the amount of information or certainty reaches a higher or lower bound. In simple words, according to the random walk model we take decisions when we are sure enough, according to the dynamical model we take decisions when our jugdement stops changing. Figure 4 shows the time course of both models.

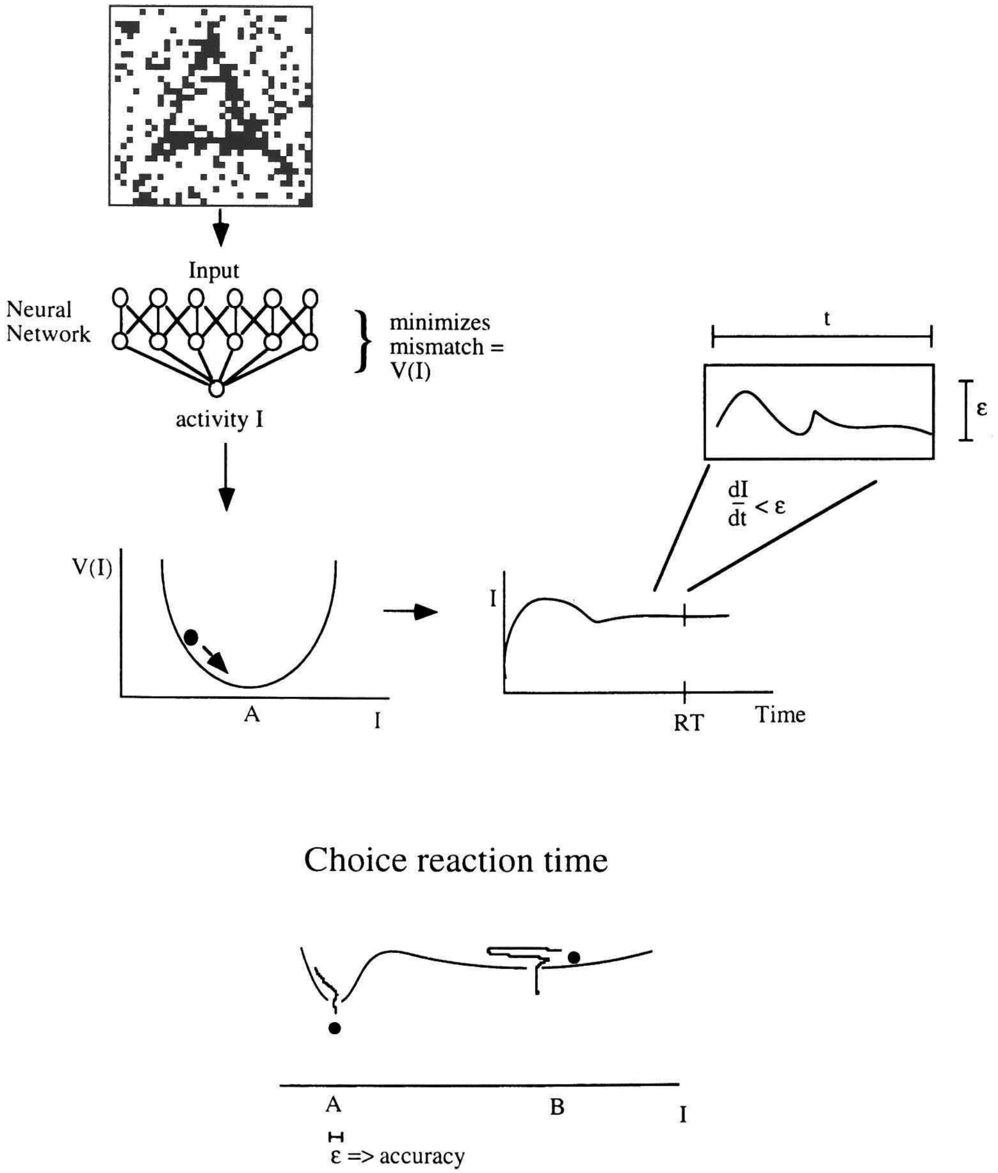



Fig. 3. A dynamical model of reaction time. The information processing system is modelled as a simple recursive neural network in which a measure of mismatch with pre-learned patterns is minimized. As soon as the mismatch value converges a reaction is produced. At the bottom of this figure a possible model is shown for a two choice task. The location of the little ball represents the present state of certainty or information, either in the A or B region. The discrete response is only given when the ball has lost its speed and falls through the small opening in one of the minima[1].

[1] The figure shows a ball in each minimum to illustrate that reaction time depends on the curvature of the minimum. If we use one ball the model is more similar to the random walk model, if we use two balls the model is more similar to the accumulator model.

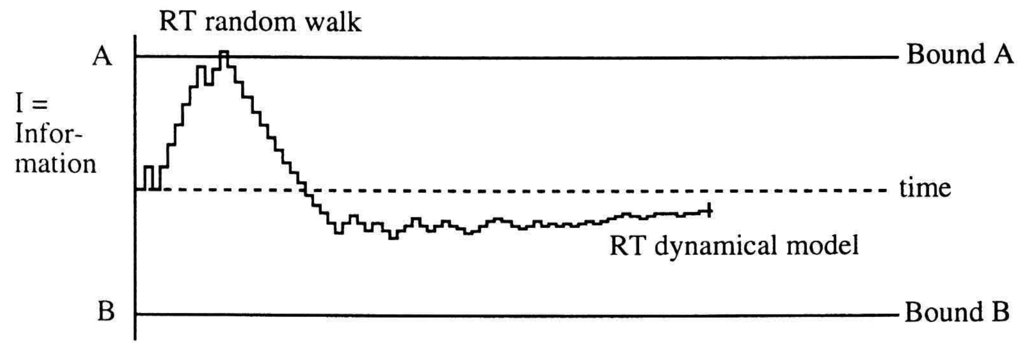


Fig. 4. The random walk and the dynamical model compared.

Our prediction of higher reaction times in the transition phase is derived from this dynamical model of reaction times. Catastrophe theory predicts that the curvature of the existing minima decreases near the transition point. This flattening of the minima is essential for critical slowing down. It leads, among other things, to a delayed recovery of equilibrium. Since we took equilibrium as stop criterion in the dynamical model of reaction time, we predict reaction times to be longer.

Note that both models explain the speed-accuracy trade-off by use of adjustments of stop criterion parameters. In a random walk model the bounds are adjusted to change this trade-off, in the dynamical model $\varepsilon$ is changed. Furthermore, the random walk model generates the same predictions as the dynamical model for elementary reaction time tasks. In these cases the dynamical stop rule can be replaced with the classical stop rule without too much risk. It is then safe to assume that a random walk towards A will never return to B. But for more complex tasks, like a multiple choice exam, we only respond when our judgment stops changing.

Finally, note how the dynamical model explains how the system responds when there is simply not enough information to make a confident response. As long as I (information) converges the system responds. Consequently, the system will not give a response when the system is captured in a non-converging trajectory (an oscillation for instance). In the random walk model the picture is different. Although random drift will always lead to a hit of the bound (Luce, 1986), lack of information leads per definition to long reaction times. In contrast, in the dynamical model responses can be uncertain but fast. This is one example of a differential prediction of the dynamical model.

## Mathematical formulation

A further evaluation of this dynamical interpretation of reaction times requires a mathematical analysis of the consequences of this stop criterion. To what kind of reaction time distributions does the dynamical model lead? That is, what is the distribution of stop times of the ball when it is randomly placed in the neighborhood of a minimum. As in random walk models the answer depends on several assumptions concerning the form of the minimum, the form of the distribution of start positions of the ball, the kind of dynamics (gradient or Newtonian gradient), and the $\varepsilon$ parameter for stopping. If we assume that the minimum is quadratic and the initial distribution is normal, the distribution of stop times is skewed to the left, where it should be skewed to the right. This is derived as follows:

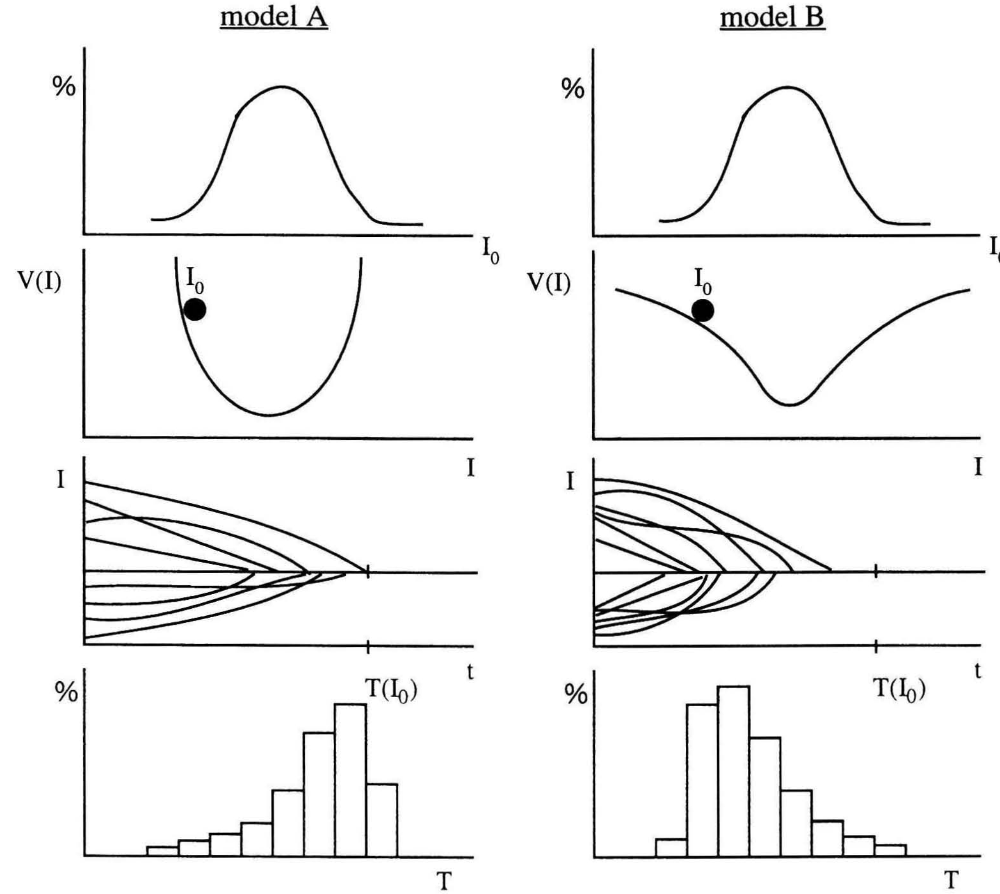
We take a gradient system $di/dt = - \lambda\, i$, i.e. potential $.5\, \lambda\, i^2$.
then $i(t) = i_0 \exp(-\lambda t)$ $\lambda>0$, $i_0$ is the random initial value
Suppose $i_0 > \varepsilon > 0$ and stop when $|di/dt| < \varepsilon$, then $T=(1/\lambda) \ln(i_0\, \lambda/\varepsilon)$
Suppose distribution of $i_0$ is f, cumulative F.
then $p(T>t) = P((1/\lambda) \ln(i_0\, \lambda/\varepsilon)>t) = P(i_0 > (\varepsilon/\lambda) \exp(\lambda t)) = 1 - F((\varepsilon/\lambda) \exp(\lambda t))$
and distribution of T: $f((\varepsilon/\lambda) \exp(\lambda t))\, \varepsilon \exp(\lambda t)$. This distribution T, assuming f is a normal distrubition, is skewed to the left instead of the right.

However, we could also take another form of the minimum, for instance potential $\ln(i^2+1)$. In this case the landscape is flat with one minimum. Then $di/dt = 2i / (i^2+1)$. By computer simulation it can be shown that, for normally distributed initial positions and for a large set of values of the parameters, the distribution of stop times is skewed to the right (see figure 5). It may be possible to give a complete model by replacing the diffusion model of Ratcliff (1980) by some variant of the Ornstein-Uhlenbeck process (Gardiner, 1994).

## Discussion

According to catastrophe theory phase transitions can be recognized by a number of qualitative properties. Some of these properties like a sudden jump, bimodality and anomalous variance are already known and used in developmental research. Other catastrophe flags, like hysteresis and critical slowing down, open up new possibilities to test for transitions and to interpret developmental processes. To operationalize critical slowing down we looked into the consequences of instability in transition periods on reaction times. Based on a dynamical model of reaction time we expect reaction times to be importantly longer when the system is in transition. This dynamical model differs from the standard random walk model with respect the nature of information increments (stationary or not), and consequently with respect to to the stop criterion. We explored the consequences of these differences in some detail. It is clear that stability considerations are not part of the standard model and lead to a different view of reaction time. From a dynamical point of view, the assumption of stationary increments of information does not hold, and a choice is made when this information converges, not when it hits a bound.

The research of dr. Han L. J. van der Maas has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences. Correspondence should be addressed to the first author Han van der Maas, Developmental Psychology, Department of Psychology, University of Amsterdam, Roetersstraat 15, 1018 WB Amsterdam, The Netherlands. E-mail: op_maas@macmail.psy.uva.nl




$V(I) = I^2$

$dI/dt = - c\ I + \text{(diffusion)}$
$I(t) = I_0 \exp(-c\ t)$

$f(T) = f(\varepsilon \exp(ct))\ \varepsilon\ c \exp(ct)$

where f is distribution of $I_0$

$\varepsilon$ is stop criterion :

$dI/dt < \varepsilon$ then stop

$V(I) = \ln(I^2+1)$

$dI/dt = -2I/(I^2+1)$

$I(t) = ?$
$f(T) = ?$

Fig. 5. Derivation of reaction time distribution for two possible dynamical models. Model A, in contrast to model B, can be analyzed easily but leads to wrong predictions. The difference is found in the choice of potential function. Model B can be simulated on the computer and very robustly leads to correctly skewed distributions.

## References


Gardiner, C.W. (1994). *Handbook of stochastic methods for physics, chemistry and the natural sciences*, second edition. Berlin: Springer-Verlag.

Gilmore, R. (1981). *Catastrophe Theory for Scientists and Engineers*. New York: John Wiley and Sons.

Heath, R.A. (1981). A tandom random walk model for psychological discrimination. *British Journal of Mathematical and Statistical Pschychology*, **34**, 76-92.

Hosenfeld, B, H.L.J. van der Maas and D.C. van den Boom (1995). Indicators of discontinuous change in the development of analogical reasoning. *Journal of Experimental Child Psychology*, **64**, 367-395.

Luce, R.D. (1986). *Response Times: Their role in inferring elementary mental organization*. New York: Oxford University press.

Ratcliff, R. (1980). A note on modeling accumulation of information when the rate of accumulation changes over time. *Journal of Mathematical Psychology*, **21**, 178-184.

Ratcliff, R. (1988). Continuous versus discrete information processing: Modeling accumulation of partial information. *Psychological Review*, **95**, 238-255.

Siegler, R.S. and E. Jenkins (1989). *How children discover new strategies*. New Jersey: Erlbaum.

Thom, R. (1975). *Structural Stability and Morphogenesis*. Reading, Mass.: Benjamin.



University of Amsterdam