Shalom Lappin

# Logicality and Semantic Types for Natural Language*

## 1. Introduction

The problem of determining the set of semantic types which correspond to the syntactic categories of natural language is one of the central issues in semantic theory. The semantic type of an expression is the kind of entity which the expression denotes. It is often assumed, for example, that declarative sentences denote truth-values (true or false), and verb phrases denote sets of individuals (subsets of elements of the universe of discourse). The semantic types of a language partially determine the way in which the meanings of expressions are computed from the meanings of their constituents. The mapping from the syntactic categories of expressions to their semantic types specifies the syntax-semantics interface of the language. Specifically, this mapping defines the correspondence between syntactic structure (form) and semantic value (meaning) for the language.

In this paper I will explore the connection between the nature of semantic types and the property of logicality. I will consider the view that particular semantic types are logical in that all of their elements have this property. If this approach is correct, then logicality can be used as one of the criteria for deciding the semantic type of certain kinds of natural language expressions. I will argue that, in fact, this view is false. I will suggest that all types instantiated for natural language are heterogeneous with respect to logicality. This discussion will focus on the category-type correspondence for noun phrases.

Within the recent semantic literature it is possible to distinguish two alternative approaches to the type system of natural language. The first is broadly Davidsonian and seeks to project the types of first-order logic onto natural language. The second is Montague's view that the types of natural language are those of a higher-order formal system.

On the Davidsonian approach the core semantic types of natural language are those of

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a first-order language.[1] These include the types for the denotations of (i) individual terms (proper names and individual variables), (ii) k-place predicates, (iii) sentential connectives, and (iv) quantifiers. Applying Tarski's (1933) semantics for first-order logic to these categories, the Davidsonian approach yields the following category-type correspondence. Individual terms take elements of the domain E of a model as their values. K-place predicates denote k-tuples of E (elements of $E_1$ x...x $E_k$). Sentential connectives denote functions from ordered pairs of truth-values to truth-values. Quantifiers denote functions from open sentences (1-place predicate values) to truth-values.

By contrast, Montague (1974) treats the types of natural language as those of an independent higher-order formal system. On this view, the set of semantic types instantiated in natural languages is significantly different from those of a first-order language. It includes those corresponding to the denotations of (i) NP's, (ii) VP's, (iii) predicate modifiers (adjectives and VP adverbs), and (iv) connectives for a variety of categories (VP's and NP's, as well as sentences).

In Section 2 I will concentrate the comparison of these approaches on their respective semantic treatments of noun phrases. Specifically, I will look at the way in which each view uses generalized quantifiers (GQ's) to interpret NP's.

In Section 3 I will discuss the relation between logicality and types, with particular application to the semantic type of generalized quantifier (GQ). I will extend the notion of logical GQ to restricted quantifiers of the type which model noun phrases, and then use this notion to formulate the Logicality Thesis, which asserts that all quantified NP's denote logical GQ's. This thesis is compatible with the Davidsonian view of NP's, but not with the Montague account.

I argue in Section 4 that the Logicality Thesis does not hold, because there is an important class of quantified NP's which are not, in general, logical. Exception phrase NP's are heterogeneous with respect to logicality, but exhibit the major syntactic and semantic properties of other quantified NP's.

In Section 5 I suggest, as an alternative conjecture, the Non-Logicality Thesis, which maintains that there are no instantiated semantic types for natural language that are uniformly logical. I provide several arguments in support of the latter thesis and briefly consider its implications for the nature of semantic theory.

[1] See Davidson (1967a,b). For neo-Davidsonian accounts of the syntax-semantics interface, developed within the framework of Chomsky's (1981, 1986, 1995) Principles and Parameters models of grammar, see Higginbotham (1985, 1989) and May (1991). It is important to note that while Higginbotham and May accept Davidson's distinction between the respective types of proper names and quantified NP's, they are not committed to the claim that all quantified NP's can be modelled by first order generalized quantifiers.

## 2. Generalized quantifiers and the interpretation of NP'S

### 2.1 *Quantifiers in Logic*

Frege (1879) and (1891) established the foundations of modern logic in taking the existential and universal quantifiers of first-order logic ($\exists x$ and $\forall x$, respectively) as second-order functions on the sets denoted by open sentences. Equivalently, they correspond to sets of sets (properties) of individuals, where the existential quantifier is interpreted as the set of all sets containing at least one element of the universe of discourse E, and the universal quantifier is taken as the set of sets including all elements of E.

Mostowski (1957) generalizes Frege's characterization of the existential and universal quantifiers to the class of unary quantifiers. Generalized quantifiers of this type denote sets of subsets of the universe of discourse of a model M. The schema for interpreting this set of quantifiers is given in 1, with interpretations of the quantifiers *at least one*, *every*, *at least n*, and *cardinally many*, defined in (2a–d), respectively.

(1) $\|Qx\Phi\|^{M,g} = t$ iff $\|\Phi\|^{M,g} \in \|Q\|^{M}$

(2) a. $\|\exists\|^{M} = \{X \subseteq M: X\}$
b. $\|\forall\|^{M} = \{M\}$
c. $\|\exists \geq n\|^{M} = \{X \subseteq M: |X| \geq n\}$
d. $\|CM\|^{M} = \{X \subseteq M: |X| \leq \aleph_0\}$

Lindström (1966) further generalizes the set of GQ's by defining a *k*-ary GQ as a relation which holds for an ordered k-tuple of subsets of E. (3) gives the interpretation of the binary GQ *most* of *Most A are B*, where $\|\Phi(x)\|^{M}$ ($\|\Psi(y)\|^{M}$) $= \{a: \|\Phi(x)\|^{M,g(x)/a} = t\}$ ($\{a: \|\Psi(y)\|^{M,g(y)/a} = t\}$).

(3) a. $\|most\|_{M} = \{X \subseteq M: |X \cap Y| > |X - Y|\}$
b. $\|most\ x,y(\Phi(x),\Psi(y))\|^{M,g} = t$ iff
$|\ \|\Phi(x)\|^{M} \cap \|\Psi(y)\|^{M}\ | > |\ \|\Phi(x)\|^{M} - \|\Psi(y)\|^{M}\ |$

It is important to note that, on the definition given in (3), *most* is not a first-order quantifier. There is no first-order formula which can be substituted on the right side of (3a) or (3b) which has the same truth conditions as the set theoretic statement that appears there.[2]

### 2.2 *Quantified NP's in Natural Language: the Davidsonian Approach*

The Davidsonian view partitions the class of NP's into two distinct syntactic categories at the level of syntactic representation which provides the syntax-semantics interface. Proper names appear *in situ* as arguments of predicates. Quantified NP's, by contrast, are restricted quantifiers consisting of a determiner denoting a quantifier and an N' predicate that restricts

[2] See Barwise and Cooper (1981) and Keenan (1996a) for discussion of this point.

the domain of the quantifier. A rule of quantifier raising (QR) adjoins quantified NP's to VP or IP. This rule partially defines an abstract (non-overt) level of syntactic structure LF, in which a quantified NP is an operator binding a syntactic variable (an A'-bound trace) in its original argument position. These structures provide the input to rules of semantic interpretation that take quantified NP's to be restricted quantifiers and the traces which they bind to be bound variables. Names are not within the domain of QR, and so they remain *in situ* at LF. Proper names correspond to the semantic type of individual constants (referring expressions), while quantified NP's are interpreted by GQ's.[3] The structures in (4) represent the distinct LF roles of proper names and quantified NP's, respectively.

(4) a. $[_{IP}\ [_{NP}$ John$][_{VP}$ sings$]]$
b. $[_{IP'}\ [_{NP}$ every student$]_1[_{IP}\ t_1$ sings$]]$

(5b) and (5c) give the partially disambiguated scope readings of (5a), where *most students* has wide scope relative to *a paper* in (5b) and narrow scope in (5c).

(5) a. Most students completed a paper.
b. $[_{IP'}\ [_{NP}$ most students$]_1[t_1\ [_{VP'}\ [_{NP}$ a paper$]_2[_{VP}$ completed $t_2]]]$
c. $[_{NP'}\ [_{NP}$ a paper$]_2[_{NP}$ most students$]_1][_{IP}\ t_1$ completed $t_2]$

Higginbotham (1980) and May (1985) cite several empirical arguments for the syntactic distinction between names (in fact, referring expressions in general) and quantified NP's. They observe that quantified NP's exhibit a range of syntactic and semantic properties which names and other referring expressions do not. Three central properties of this kind are as follows.

(i) Inverse scope readings and scope ambiguity are possible for quantified NP's within the scope of other quantified NP's, but not for proper names. On the preferred reading of (6a) *every city* has wide scope relative to *a representative*. However, there is no scope interaction between *a representative* and *London* in (6b).

(6) a. A representative of every city attended the meeting.
b. A representative of London attended the meeting.

Similarly, *every student* can be understood as taking wide or narrow scope relative to the object NP *a logic course* in (7a), but no such scope ambiguity exists in the interpretation of (7b).

(7) a. Every student attended a logic course.
b. Every student attended Logic 101.

(ii) Quantified NP's impose a bound variable reading on the pronouns which they bind. Pronouns interpreted as coreferential with a name do not receive a bound variable reading. Therefore, *his* is taken as a variable bound by *no student* in (8a), while *her* is understood as *Mary's* in (8b).

[3] Higginbotham and May (1981), Higginbotham (1985), and May (1985, 1989, 1991) develop this view of NP's.

(8) a. [no student]$_1$ submitted his$_1$ paper
b. Mary$_1$ submitted her$_1$ paper

(iii) A quantified NP cannot bind a pronoun if it does not c-command it, as in a weak cross over structure, while a proper name can be interpreted as coreferential with a pronoun which it does not c-command. (9a,b) illustrate this contrast.

(9) a. *his$_1$ mother loves [every boy]$_1$
b. his$_1$ mother loves John$_1$

## 2.3 *The Montague Approach*

On Montague's treatment of NP's, this set of expressions constitutes a unified syntactic category which corresponds to a single semantic type. Names and quantified NP's are all interpreted by GQ's, and so every NP denotes a set of sets (or a set of properties). In a quantified NP generated by applying a determiner to an N', the determiner denotes a function from a set (the denotation of the N') to a set of sets (the GQ which the NP denotes). Alternatively, the determiner can be taken to denote a relation between the N' set and the VP (predicate) set. A proper name does not denote an element of E but the set of sets which contain a specified element of E.[4]

(10a) defines the GQ denoted by the proper name *John*, and (11b,c) give the GQ interpretations of *every student* and *most student.*

(10) a. $\|\text{John}\| = \{X \subseteq E: j \in X\}$
b. $\|\text{every student}\| = \{X \subseteq E: \text{Students} \subseteq X\}$
c. $\|\text{most students}\| = \{X \subseteq E: |\text{Students} \cap X| > |\text{Students} - X|\}$

When the NP's that denote the GQ's specified in (10) are combined as subjects with the VP *sings*, the resulting sentences receive the interpretations given in (11).

(11) a. $\|\text{John sings}\| = t$ iff $\text{Sings} \in \{X \subseteq E: j \in X\}$ iff $j \in \text{Sings}$
b. $\|\text{every student sings}\| = t$ iff $\text{Students} \subseteq \text{Sings}$
c. $\|\text{most students sing}\| = t$ iff $|\text{Students} \cap \text{Sings}| > |\text{Students} - \text{Sings}|$

## 2.4 *Constraints on Natural Language Determiner Functions*

Barwise and Cooper (1981) (B&C), and Keenan and Stavi (1986) (K&S) suggest that all natural language determiner functions are conservative, where the set of conservative binary determiner functions is defined in (12).

(12) A binary determiner function det is conservative iff, for every $A,B \subseteq E$,
$B \in \text{det}(A) \Leftrightarrow (A \cap B) \in \text{det}(A)$.

[4] See Montague (1974), Barwise and Cooper (1981), Cooper (1983), Keenan (1996a), Keenan and Moss (1985), Keenan and Stavi (1986), Keenan and Westerståhl (1997), van Benthem (1986, 1989), and Westerståhl (1989) for versions of the unified GQ view of NP's.

The truth-value of a sentence whose subject is a quantified NP with a conservative determiner (a determiner that denotes a conservative function) depends only on the N' set of the subject NP and the intersection of this set with the predicate (VP) set. The conservativity of $\|all\|$, $\|no\|$, $\|five\|$, and $\|most\|$ sustains the validity of the implications in (13).

(13) All/No/Five/Most students sing. $\Leftrightarrow$
All/No/Five/Most students are students who sing.

Conservativity excludes from the set of determiner functions those functions which specify relations between an N' set A and a predicate set B that depend on objects which are in B but not in A. If det is conservative, then elements of B which are not also contained in A need not be considered when determining whether $B \in det(A)$.

Van Benthem (1984) and Westerståhl (1989) claim that all natural language determiner functions satisfy the condition of Extension (EXT), defined in (14).

(14) A binary determiner det satisfies EXT iff, for any two models M and M', and any $A \subseteq E$, if $A \subseteq E_M \subseteq E_{M'}$, then $det_M(A) = det_{M'}(A)$.

EXT rules out determiner functions which specify relations between A and B that depend on objects outside of both A and B. If det satisfies both conservativity and EXT, then to determine if $B \in det(A)$ it is only necessary to consider the entities in $A - B$ and $A \cap B$.

Conservativity is a condition on determiner functions, but, as Moltmann (1995) shows, it can be straightforwardly extended to NP denotations (GQ's). (15) defines a conservative GQ relative to a set A.

(15) $\|NP\|$ is conservative for a set A iff, for every $X \subseteq E$, $X \in \|NP\| \Leftrightarrow (A \cap X) \in \|NP\|$.

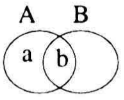
## 3. Logicality

The notion of a logical term can be intuitively understood as one whose meaning depends only upon formal properties, and so is insensitive to the actual properties of the individuals in the domain of a model. Mostowski (1957) characterizes a unary quantifier as a logical constant iff its interpretation remains constant under all permutations of the elements of the domain E, where a permutation is an automorphism of E (a mapping of E onto itself) which respects the cardinality of the subsets of E. Lindström (1966), van Benthem (1986, 1989), and Sher (1991, 1996) progressively generalize the notion of logicality across syntactic categories to define a logical constant as a term whose interpretation is invariant under isomorphic structures defined on E.

The set of logical determiners is the set which includes all and only those determiners denoting relations that depend solely upon the cardinality of the sets among which they hold and the cardinality of the intersections of these sets. These relations are insensitive to the identity of the elements of the sets among which they hold. Westerståhl (1989) points out that, in addition to permutation invariance for isomorphic structures defined on E (Westerståhl's condition of Quantity), logical determiner functions must also satisfy the conditions of Conservativity and Extension. Assuming that all natural language det's satisfy Conservativity and EXT, the distinction between logical and non-logical natural language determiner functions depends upon the property of invariance under isomorphic structures defined on E.

It is possible to characterize a logical det as a function from an ordered pair of cardinality values to a truth-value. Let A be any N' set and B any VP set. If det is logical, then it is a function from $<|A-B|, |A \cap B|>$ to $\{t, f\}$. For the sets in (16) let $a = |A - B|$ and $b = |A \cap B|$. Examples of cardinality definitions for logical det's are given in (17).

(16)




(17) a. $every(<a,b>) = t$ iff $a = 0$ and $b = n$ $(0 \leq n)$.
b. $no(<a,b>) = t$ iff $a = n$ and $b = 0$.
c. $some(<a,b>) = t$ iff $a = n$ and $b \geq 1$.
d. at least $five(<a,b>) = t$ iff $a = n$ and $b \geq 5$.
e. $most(<a,b>) = t$ iff $b > a$.

It is possible to extend the definition of logicality from determiner functions to GQ's. Let A be the smallest set for which $\|NP\|$ is conservative.

(18) $\|NP\|$ is a logical GQ iff there is a function $f$ from pairs of cardinality values to $\{t,f\}$ such that for every $B \subseteq E$, $B \in \|NP\|$ iff $f(<|A-B|, |A \cap B|>) = t$.

To obtain the function for a particular logical det, it is necessary to define the set of possible cardinality pairs for which the function gives the value t. Each definition places constraints on the cardinal values which can appear as elements of these pairs and the relations which hold between them. The logical $\|NP\|$'s corresponding to the logical det's in (17) are specified in 19, where the smallest set for which each $\|NP\|$ is conservative is the set of students.

(19) a. $B \in \|$every student$\|$ iff $|Students - B| = 0$.
b. $B \in \|$no student$\|$ iff $|Students \cap B| = 0$.
c. $B \in \|$some student$\|$ iff $|Students \cap B|$ 0.
d. $B \in \|$at least five students$\|$ iff $|Students \cap B| \geq 5$.
e. $B \in \|$most students$\|$ iff $|Students \cap B| > |Students - B|$.

The function specified in (19a), for example, which corresponds to $\|$every student$\|$, assigns t to the pair of cardinality values $<0, n>$, where 0 is the value of $|Students - B|$ and n is any positive integer representing the cardinality of $|Students \cap B|$. (19d) characterizes the function for $\|$five students$\|$ as assigning t to the pair $<j, k>$, where j is any positive integer for the value of $|Students - B|$ and k (the value of $|Students \cap B|$) is greater than 5 . The pair of cardinalities for which the function for $\|$most students$\|$ defined in (19e) yields t is any $<j, k>$ such that $k > j$.

Proper names and NP's formed by conjoining quantified NP's with proper names do not denote logical GQ's. We can see this by considering the definitions of ‖John‖ and ‖Mary and every student‖ given in (20).

(20) a. $B \in \|John\|$ iff $john \in B$ iff $\{john\} \cap B \neq \emptyset$
(where {john} is the smallest set for which ‖John‖ is conservative).
b. $B \in \|Mary\ and\ every\ student\|$ iff $(\{mary\} \cup Students) \subseteq B$ iff $mary \in B$ and $Students \subseteq B$
(where $\{mary\} \cup Students$ is the smallest set for which ‖Mary and every student‖ is conservative)

If the individual bill is substituted for john under a permutation of E, then (20a) will be false for many values of B. The identity of john as well as the cardinality of $\{john\} \cap B$ is significant in determining the truth-value of (20a) for any given value of B. Similarly, the identity of mary, as well as the cardinality of $(\{mary\} \cup Students) - B$ plays a role in determining the truth-value of (20b) for any given value of B. Therefore, the GQ's denoted by these NP's cannot be specified by a cardinality function of the kind indicated in (18).

May (1991) proposes what I will refer to as The Logicality Thesis. He claims that the property of logicality is the criterion for distinguishing between quantified and non-quantified NP's. Specifically, he suggests that those NP's which correspond to restricted quantifiers are constructed by the application of a logical determiner (determiners which denote logical det functions) to an N'. He takes such NP's to denote logical GQ's. Non-quantified NP's, on the other hand, are treated as non-logical expressions. The assertion that the distinction between logical and non-logical NP's corresponds to a difference in syntactic category and semantic type is an empirical claim concerning the organization of categories and types in the grammar of natural language. The Logicality Thesis claims that for natural languages, the semantic type GQ includes only logical functions.

It is important to recognize that this claim is independent of the question of whether a GQ is first-order definable. A function is first-order definable iff it can be defined by a set of sentences in a first-order language. NP's formed by applying a proportional determiner like *most* or *exactly half the* to an N' denote GQ's which are not first-order definable. However, while GQ's of the form ‖most (A)‖ and ‖exactly half the (A)‖ are not first-order definable, they are logical, as (17e/19e) (and analogous definitions for other proportional dets) show.

## 4. Exception phrase NP's and logicaltiy

### 4.1 *Exception Phrase NP's as GQ's*

The Logicality Thesis is compatible with the Davidsonian view of NP's, but not with Montague's unified GQ treatment of NP's. Given that proper names are not logical terms, if they are taken as a subset of a unified syntactic category and corresponding semantic type, then the latter cannot be logical. The debate between these two approaches has, in part, focused on the question of how to accommodate the interpretation of proper names and referring expressions within the semantic type system of natural language. However, there is a class of quantified NP's whose semantic properties provide important evidence against the Logicality Thesis. Exception phrase NP's offer an interesting challenge to the Logicality Thesis from within the set of quantified NP's. The subjects of (21a) and (21b) are examples of exception phrase NP's.

(21) a. Every student except five (students)/five law students/John arrived.
b. No student except five (students)/five law students/John arrived.

Following Hoeksema (1991), Moltmann (1995), and Lappin (1996a), I will take exception phrases to be syntactic functions from NP's to NP's (NP modifiers), and so they denote functions from GQ's to GQ's. In order to specify the interpretation of exception phrase GQ's it is necessary to introduce two preliminary notions.

Moltmann (1995) (modifying B&C) defines a witness set as in (22).

(22) If A is the smallest set for which $\|NP\|$ is conservative, then W is a witness set for $\|NP\|$ iff $W \subseteq A$ and $W \in \|NP\|$.

Any set of five students is a witness set for $\|$five students$\|$, {john} is the only witness set for $\|$John$\|$, and any set whose elements are John and three physics students is a witness set for $\|$John and three physics students$\|$. For any generalized quantifier $\|NP\|$, let $w(\|NP\|)$ = the set of witness sets for $\|NP\|$. Lappin (1996a) defines the set of total relations as in (23).

(23) R is total iff (i) $R = \subseteq$, or (ii) for any two sets A, B, R(A,B) iff $A \cap B = \varnothing$.

R is total iff it imposes a condition of inclusion or exclusion between two sets, and nothing more.

Let $NP_2$ be the NP to which the exception phrase *except*($NP_1$) applies, and assume that $\|NP_2\| = \{X \subseteq E: R(A,X)\}$, where A is the smallest set for which $\|NP_2\|$ is conservative. The domain of the function which an exception phrase denotes is restricted to NP arguments for which R is total in every model M such that the value of $\|NP\|$ is defined in M. For any set X, let X' be the complement of X. Lappin (1996a) proposes (24) as the interpretation of exception phrase NP's.

(24) $(\|except\|(\|NP_1\|))(\|NP_2\|) = \{X \subseteq E: R(A^{rem},X)$, where $\|NP_2\| = \{X \subseteq E: R(A,X)\}$, and $\exists S(S \in w(\|NP_1\|)\ \&\ S \subseteq A\ \&\ A^{rem} = A - S\ \&\ R(S,X'))\}$, if R is total and A.
= undefined otherwise.

According to (24) an exception phrase denotes the set of sets X such that X stands in the appropriate total relation R to the remnant set $A^{rem}$. This remnant set is computed by subtracting a witness set W of the GQ denoted by the NP argument of $\|except\|$ from A, the restriction set of the $\|NP\|$ to which the entire exception phrase modifier applies (A is the smallest set for which $\|NP_2\|$ is conservative), where W bears the total relation to the complement of each set X in the denotation of the exception phrase NP.

Restricting the domain of exception phrase modifiers to GQ's that impose total relations between their restriction sets and the VP sets captures the fact that these modifiers only apply to universal NP's.

(25) *Five/most/not many/neither/both students except John arrived.

Applying the GQ modifier function ||except five law students|| to ||every student|| yields the GQ given in (26).

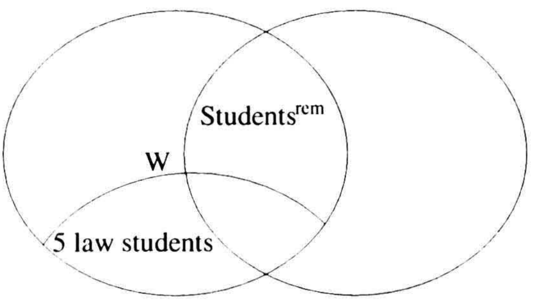
(26) $(\|\text{except}\|(\{X \subseteq E: |\text{Law\_Students} \cap X| \geq 5\}))(\{X \subseteq E: \text{Students} \subseteq X\}) = \{X \subseteq E:$
$\text{Students}^{rem} \subseteq X$, where
$\exists S(S \in w(\{X \subseteq E: |\text{Law\_Students} \cap X| \geq 5\})\ \&\ S \subseteq \text{Students}\ \&$
$\text{Students}^{rem} = \text{Students} - S\ \&\ S \subseteq X')\}$.

(27) specifies the truth conditions for *Every student except five law students arrived.*

(27) $\|\text{every student except five law students arrived}\| = t$ iff
$\text{Students}^{rem} \subseteq \{a: a \text{ arrived}\}$, where
$\exists S(S \in w(\{X \subseteq E: |\text{Law\_Students} \cap X| \geq 5\})\ \&\ S \subseteq \text{Students}\ \&$
$\text{Students}^{rem} = \text{Students} - S\ \&\ S \subseteq \{a: a \text{ arrived}\}')$.

To see how the definition in (27) works, consider (28).

(28)




(27) states that the sentence is true iff (i) there is a witness set W of five students which is a subset of the set of Students, (ii) the remnant set $\text{Students}^{rem}$ is computed by subtracting W from the set of Students, (iii) $\text{Students}^{rem}$ is a subset of the set Arrived (objects which arrived), and (iv) the intersection of W and Arrived is empty.

There is a problem with (24).[5] It assumes the existence of a single remnant set $A^{rem}$ derived by the substraction of a witness set for the GQ argument of ||except||. But (29) suggests that this assumption is unwarranted.

(29) Every student except five (students) passed the math exam, and every student except five (students) passed the physics exam.

(29) is true in a situation where different witness sets of five students did not pass the math and the physics exam, respectively. But in this case, a distinct remnant set $\text{Students}^{rem}$ is required for the denotation of *every student except five* in each conjunct of (29).

[5] I am grateful to Hans Kamp for pointing out this difficulty to me.

We can solve this problem by existentially quantifiying over the set of remnant sets, as in (30).

(30) $,(\|except\|(\|NP_1\|))(\|NP_2\|) = \{X \subseteq E: \exists A^{rem}(R(A^{rem},X)$, where $\|NP_2\| = \{X \subseteq E: R(A,X)\}$, and $\exists S(S \in w(\|NP_1\|)$ & $S \subseteq A$ & $A^{rem} = A - S$ & $R(S,X')))\}$, if R is total and A.
= undefined otherwise.

According to this revised definition, exception GQ's denote the set of sets which stand in the specified total relation R to some remnant set $A^{rem}$ obtained by subtracting a witness set of the GQ argument of $\|except\|$ from A. The value of the remnant set variable varies with the selection of distinct witness sets S, and so the denotation of $\|$every student except five$\|$ remains constant across both conjuncts of (29) while still yielding the correct interpretation for each sentence.

(30) generates (31) and (32) as the revised versions of (26) and (27), respectively.

(31) $(\|except\|(\{X \subseteq E: |Law\_Students \cap X| \geq 5\}))(\{X \subseteq E: Students \subseteq X\}) = \{X \subseteq E:$
$\exists A^{rem}(A^{rem} \subseteq X$, where
$\exists S(S \in w(\{X \subseteq E: |Law\_Students \cap X| \geq 5\})$ & $S \subseteq Students$ &
$A^{rem} = Students - S$ & $S \subseteq X'))\}$.

(32) $\|$every student except five law students arrived$\|$ = t iff
$\exists A^{rem}(A^{rem} \subseteq \{a: a \text{ arrived}\}$, where
$\exists S(S \in w(\{X \subseteq E: |Law\_Students \cap X| \geq 5\})$ & $S \subseteq Students$ &
$A^{rem} = Students - S$ & $S \subseteq \{a: a \text{ arrived}\}'))$.

(28) can be used to understand (32) in the same way as it was for 27, with the additional condition that there is a set $Students^{rem}$ which is the result of subtracting a witness set W from Students.

### 4.2 *The Logically Heterogenous Nature of Exception Phrase NP's*

Exception phrase NP's are heterogeneous with respect to logicality. An exception phrase NP is a logical GQ iff it is of the form *every/no A except det A*, and det is a logical determiner. To show that this is the case, it is necessary to establish that the smallest set for which the GQ denoted by an exception phrase NP is conservative is the restriction set A of $\|NP_2\|$ (the $\|NP\|$ argument to which $\|except\|(\|NP_1\|)$ applies), rather than a remnant set $A^{rem}$ for the GQ which the exception phrase denotes.

An exception phrase NP is conservative for A iff, for any $B \subseteq E$, $B \in (\|except\|(\|NP_1\|))(\|NP_2\|)$ iff $(A \cap B) \in (\|except\|(\|NP_1\|))(\|NP_2\|)$. The proof that exception phrase NP's satisfy this condition for the restriction set A of $\|NP_2\|$ is straightforward.

(33) $B \in (\|except\|(\|NP_1\|))(\|NP_2\|)$ iff
$\exists A^{rem}(R(A^{rem},B)$, where $\exists S(S \in w(\|NP_1\|)$ & $S \subseteq A$ & $A^{rem} = A - S$ & $R(S,B'))) \Leftrightarrow$
$\exists A^{rem}(R(A^{rem},(A \cap B))$, where $\exists S(S \in w(\|NP_1\|)$ & $S \subseteq A$ & $A^{rem} = A - S$ &
$R(S,(A \cap B)')))$ iff
$A \cap B \in (\|except\|(\|NP_1\|))(\|NP_2\|)$

While a GQ of the form $(\|\text{except}\|(\|NP_1\|))(\|NP_2\|)$ satisfies the *if* clause of the conservativity condition for any of its remnant sets, the converse does not hold.

(34) $B \in (\|\text{except}\|(\|NP_1\|))(\|NP_2\|)$ iff
(i) $\exists A^{rem}(R(A^{rem},B)$, where $\exists S(S \in w(\|NP_1\|)\ \&\ S \subseteq A\ \&\ A^{rem} = A - S\ \&\ R(S,B')))$
$\Rightarrow$
(ii) $\exists A^{rem}(R(A^{rem},(A^{rem} \cap B))$, where $\exists S(S \in w(\|NP_1\|)\ \&\ S \subseteq A\ \&\ A^{rem} = A - S\ \&\ R(S,(A^{rem} \cap B)')))$ iff
(iii) $(A^{rem} \cap B) \in (\|\text{except}\|(\|NP_1\|))(\|NP_2\|)$, for any value of $A^{rem}$ which satisfies the open sentence in the scope of the existential quantifier binding $A^{rem}$ in (i).

(35) $(A^{rem} \cap B) \in (\|\text{except}\|(\|NP_1\|))(\|NP_2\|)$ iff
(i) $\exists A^{rem}(R(A^{rem},(A^{rem} \cap B))$, where $\exists S(S \in w(\|NP_1\|)\ \&\ S \subseteq A\ \&\ A^{rem} = A - S\ \&\ R(S,(A^{rem} \cap B)')))> \not\Rightarrow$
(ii) $\exists A^{rem}(R(A^{rem},B)$, where $\exists S(S \in w(\|NP_1\|)\ \&\ S \subseteq A\ \&\ A^{rem} = A - S\ \&\ R(S,B')))$

(i) does not imply (ii) in (35) by virtue of the fact that it is possible for a total relation R to hold between a witness set W of $\|NP_2\|$ and the complement of $A^{rem} \cap B$ (for a given value of $A^{rem}$), but not between W and the complement of B. So, for example, if $R = \subseteq$, then some of the elements of W could be contained in a subset of $A \cap B$ which is outside of $A^{rem}$. In this case, $W \subseteq (A^{rem} \cap B)'$, but $W \not\subseteq B'$.

We can characterize the denotations of exception phrase NP's of the form *every/no A except det A* as logical GQ's on analogy with the definitions given in (19).

(36) $B \in \|\text{every student except five (students)}\|$ iff
$|Students - B| = 5$

(37) $B \in \|\text{no student except five (students)}\|$ iff
$|Students \cap B| = 5$

$\|\text{every student except five (students)}\|$ denotes a function $f$ from pairs of cardinality values such that $f(<|A - B|,|A \cap B|>) = t$ iff $|A - B| = 5$ and $|A \cap B| \geq 0$. The function that $\|\text{no student except five (students)}\|$ denotes assigns t only to the pair $<n,5>$ $(0 \leq n)$.

By contrast, the exception phrase GQ's defined in 38–41 are not logical.

(38) $B \in \|\text{every student except five law students}\|$ iff
$|Students - B| = 5$ &
$(Students - B) \subseteq Law\_Students$

(39) $B \in \|\text{no student except five law students}\|$ iff
$|Students \cap B| = 5$ &
$(Students \cap B) \subseteq Law\_Students$

(40) $B \in \|\text{every student except John}\|$ iff
$(Students - B) = \{john\}$

(41) $B \in \|\text{no student except John}\|$ iff
$(Students \cap B) = \{john\}$

Each of these definitions contains a condition that makes essential reference to the elements of a witness set for the argument of ||except||. Therefore, they cannot be encoded in cardinality functions of the sort which satisfy (18).

### 4.3 *Exception Phrase NP's as Quantified Noun Phrases*

Exception phrase NP's (both logical and non-logical) exhibit the same syntactic and semantic properties as other quantified NP's. Specifically, they generate inverse scope readings and scope ambiguity.

(42) a. A representative of every city except Haifa attended the meeting.
 b. Every student except the law students attended a logic course.

They force bound variable readings of pronouns.

(43) [no student except Mary]$_1$ submitted her$_1$ paper

They produce a weak cross over effect with non-c-commanding pronouns which they do not c-command.

(44) *his$_1$ mother loves [every boy except Bill]$_1$

Therefore, the same considerations which lead Higginbotham and May to map quantified NP's to the semantic type GQ apply to exception phrase NP's. As exception phrase NP's are heterogeneous with respect to logicality, the Logicality Thesis does not hold. It is important to recognize that this argument against the Logicality Thesis is independent of the type assignment for proper names and other referring expressions. The argument shows that the Logicality Thesis cannot be sustained for the class of quantified NP's.

## 5. The non-logicality thesis

I propose the following conjecture concerning the semantic types of natural language. There are no semantic types for natural language all of whose elements satisfy the condition of logicality. This conjecture, which I will refer to as the Non-Logicality Thesis, asserts that all semantic types for NL are heterogeneous with respect to logicality.

Let us briefly consider the major semantic types of natural language in turn. Predicates (VP and N') denotations are clearly non-logical, given that they are sets of individuals, which are not invariant under permutations of their elements (and similarly for the k-ary relations denoted by k-place verbs). If we take VP adverbs to denote functions from VP denotations to VP denotations (ie. from sets of individuals to sets of individuals), then they are also non-logical. The value of such a function does not, in general, depend solely upon the cardinalities of the pairs of sets which correspond to its argument and its value, respectively (and similarly for N modifiers).[6] A sentential adverb is interpreted

[6] If one takes the Davidsonian view that VP modifiers denote properties of events (and N modifiers denote properties of individuals) then they are non-logical for the same reason that

by a function from propositions to truth-values, and so it may be regarded as denoting a set of propositions. The interpretations of at least some sentential adverbs are sensitive to the identity of the propositions in the sets they denote. So, for example ‖*surprisingly*‖ depends, in part, on individual properties, particularly the cognitive status of the proposition to which it applies. There may be people for whom (45a) is true, but it unlikely if this is the case for (45b).

(45) a. Surprisingly the square root of 9 is 3.
  b. Surprisingly 3 is 3.

It would seem, then that the only semantic types which are plausible candidates for logicality are (i) determiner functions, (ii) GQ's, and (iii) sentential connectives. As van Benthem (1986, Ch. 1) points out, possessive determiners headed by proper names, like *Mary's*, are non-logical.[7] Assume that the intersection of the set of books and the set of things about linguistics has the same cardinality as the intersection of the set of books and the set of things about physics. It does not follow that ‖*Mary's books are about linguistics*‖ = ‖*Mary's books are about physics*‖. The relation between sets which a possessive determiner denotes depends, in part, on the identity of the possessor. We have already observed that the fact that exception phrase GQ's do not, in general, satisfy logicality indicates that the second type is not logical.

How can we characterize logicality for connectives?[8] Keenan (1996b) points out that the set of truth-values {t,f} is structured by the partial ordering relation ≤, such that f ≤ f, f ≤ t, and t ≤ t. He observes that the interpretations of truth-functional connectives are logical in that they are invariant under the set of permutations which preserve the structure of {t,f}. Therefore, they are invariant under substitution of {1,0} for {t,f}, for example. However, the interpretations of sentential connectives like *but*, and *although* depend, at least partially, upon pragmatic/discourse properties (such as factors related to speakers' expectations and assumptions) of the sequences of sentences to which they apply. If we take a connective as denoting a function from pairs (or ordered k-tuples) of propositions to truth-values, then such a discourse sensitive connective is not logical. The truth-value which it assigns to a pair of propositions $<p_i, p_j>$ depends not only on the formal properties of $p_i$ and $p_j$, specified as functions from possible worlds or situations to truth-values, but also on pragmatic properties, which are not part of their propositional content. These considerations provide at least initial motivation for the Non-Logicality Thesis.

It would not be surprising if the Non-Logicality Thesis does, in fact, turn out to be true. Logical types like GQ's and sentential connectives in first-order logic are designed to facilitate the identification of the set of valid sentences and valid inferences in the language. Validity depends upon the most abstract formal features of models, specifically, those properties which hold across the set of possible models. Natural languages are

predicates in general are.

[7] I am following van Benthem, Keenan and Stavi (1986), and Keenan (1996a) in treating possessives as determiners. I am grateful to Yoad Winter for useful discussion on this point.

[8] I am grateful to Hans Kamp for useful discussion on this point.

formal syntactic and semantic systems, but they are biologically evolved rather than designed systems. There is no reason to believe that the property of validity is formally encoded in the semantics of natural language through the existence of distinguished semantic types satisfying the condition of logicality.

Inferences which are sustained by the semantic properties of classes of lexical items (like the semantic entailments of different verb classes) are not less significant for speakers of natural languages than valid inferences. These lexically driven inferences depend upon properties of objects and events which are not invariant under permutations of the entities in the universe of discourse. Hence, it is reasonable to expect that the set of semantic types instantiated for natural language will be independent of logicality.

## 6. Conclusion

The Logicality Thesis is compatible with (and, in a sense, implied by) the Davidsonian approach to determining the semantic types of natural language, but not with the Montague approach. The fact that the GQ's denoted by exception phrase NP's are not, in general, logical indicates that the Logicality Thesis does not hold, and so provides support for the Non-Logicality Thesis. More generally, the properties of exception phrase NP's offer motivation for the Montague view of the relation between syntactic categories and semantic types in natural language in that they sustain the idea that GQ's are non-uniform with respect to logicality.

Natural languages are evolved, and so they are not designed to facilitate the specification of the set of valid sentences and the set of valid inferences. Therefore, it is not surprising that logicality is not a factor which distinguishes among semantic types for natural language.

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