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Decay of Monopolar Vortices in a Stratified Fluid

Abstract

This contribution describes some experimental observations of the decay of a monopolar vortex in a linearly stratified fluid. The vortex is generated by a rotating solid sphere, which is later removed. The observed decay of the planar flow in the disk-shaped vortical region is compared with three different theoretical models, and good agreement is obtained.

1. Introduction

Satellite observations have revealed the abundant occurrence of vortices in many parts of the world's oceans, see *e.g.* Robinson (1983). In addition to the surface vortices, which may be easily detected by satellite measurements of surface anomalies, oceanic vortex structures may also occur at some depth, and thus invisible for the eyes of satellites. Well-known examples of subsurface eddies are the 'Meddies', vortex structures originating from the gravitational collapse of Mediterranean Sea water that spilled over the sill in the Straits of Gibraltar. The existence of Meddies was first reported by McDowell and Rossby (1978) and later oceanographic measurements have yielded important information about the Meddies' dynamics (see *e.g.* Armi *et al.* 1989). In particular, it became clear that the Meddies occupy a relatively thin, pancake-shaped region with horizontal and vertical scales of roughly 100 *km* and 600 *m*, respectively. The relatively slow decay allows the Meddy to cross the Atlantic Ocean and reach the Bahamas after approximately one year.

This paper reports on a laboratory study of a monopolar vortex in a non-rotating, linearly stratified fluid. The flow evolution has been measured by application of digital image analysis techniques, and a comparison is made with a few simple decay models.

2. Generation and characteristics of monopolar vortices

The laboratory experiments were performed in a square perspex tank of horizontal dimensions 100 *cm* × 100 *cm* and a working depth of 30 *cm*, which was filled with a linearly salt-stratified fluid. Monopolar vortices were generated by a solid sphere (diameter 2.5 *cm*), placed in the fluid at mid-depth, rotating at constant speed Ω about a vertical axis, as also described in Flór *et al.* (1993). In the experiments discussed here, the rotation speed measured 344 *rev/min*, and the forcing was applied during typically 30 *s*.

During the forcing, fluid is swept away from the sphere in radial direction, and shadowgraph visualizations have revealed that the motion in the vicinity of the sphere is definitely turbulent at this stage. Then the rotation was stopped, and the sphere was removed by carefully lifting it. During the subsequent gravitational collapse, the vertical motions are substantially suppressed and the flow soon becomes laminar: at this stage the flow takes on the appearance of a monopolar vortex confined to a thin horizontal pancake-shaped region. In most cases, this vortex was observed to be stable, while gradually decaying owing to viscous effects.

The flow was visualized by seeding the fluid with small tracer particles. Special care was taken that their density exactly matched the fluid density at the mid-level of the vortex motion. The particle motions were recorded by video, and quantitative information about the horizontal flow field was obtained by applying a digital image analysis technique. After digitization the flow was characterized by a set of velocity vectors in the nodal points of a rectangular grid covering (part of) the flow domain. As a next step, the values of the vorticity $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, with $\mathbf{v} = (u, v)$ the velocity components in (x, y)-directions, and the stream function ψ (defined by $\mathbf{v} = \nabla \times \mathbf{k}\psi$, with $\mathbf{k}$ the unit vector in vertical direction) are calculated in each grid point. Some typical results of this procedure are shown in Figure 1 for a monopolar vortex 180 *s* after the forcing was stopped. The graphs show (a) contours of ω, (b) contours of ψ, (c) the ω, ψ scatter plot and (d) cross-sectional distributions of ω and the azimuthal velocity v_θ, respectively. Apparently, the core of the vortex is surrounded by a ring of very weak oppositely-signed vorticity. The scatter plot reveals a linear relationship between ω and ψ in the central part of the vortex, whereas ω is almost zero for larger radii. These characteristic features of the ω, ψ-relationship remain unchanged during the subsequent flow evolution, as can be seen in Figures 2a-c: although the maximum ω-value decreases during the decay, the scatter plots indicate preservation of the linearity at least in the vortex core. For this reason it is assumed that the monopolar vortex thus produced can in good approximation be characterized by a *linear* ω, ψ-relationship, *i.e.* by

$$\omega = k^2\psi, \tag{1}$$

with k a proportionality constant. Because of the symmetry about the horizontal mid-depth level ($z = 0$), the flow can be considered as being 2D, so that (1) implies

$$\nabla^2\psi = -\omega = -k^2\psi. \tag{2}$$

The axisymmetric solution of this equation that is bounded in the origin $r=0$ is given by

$$\psi(r) = AJ_o(kr), \tag{3}$$

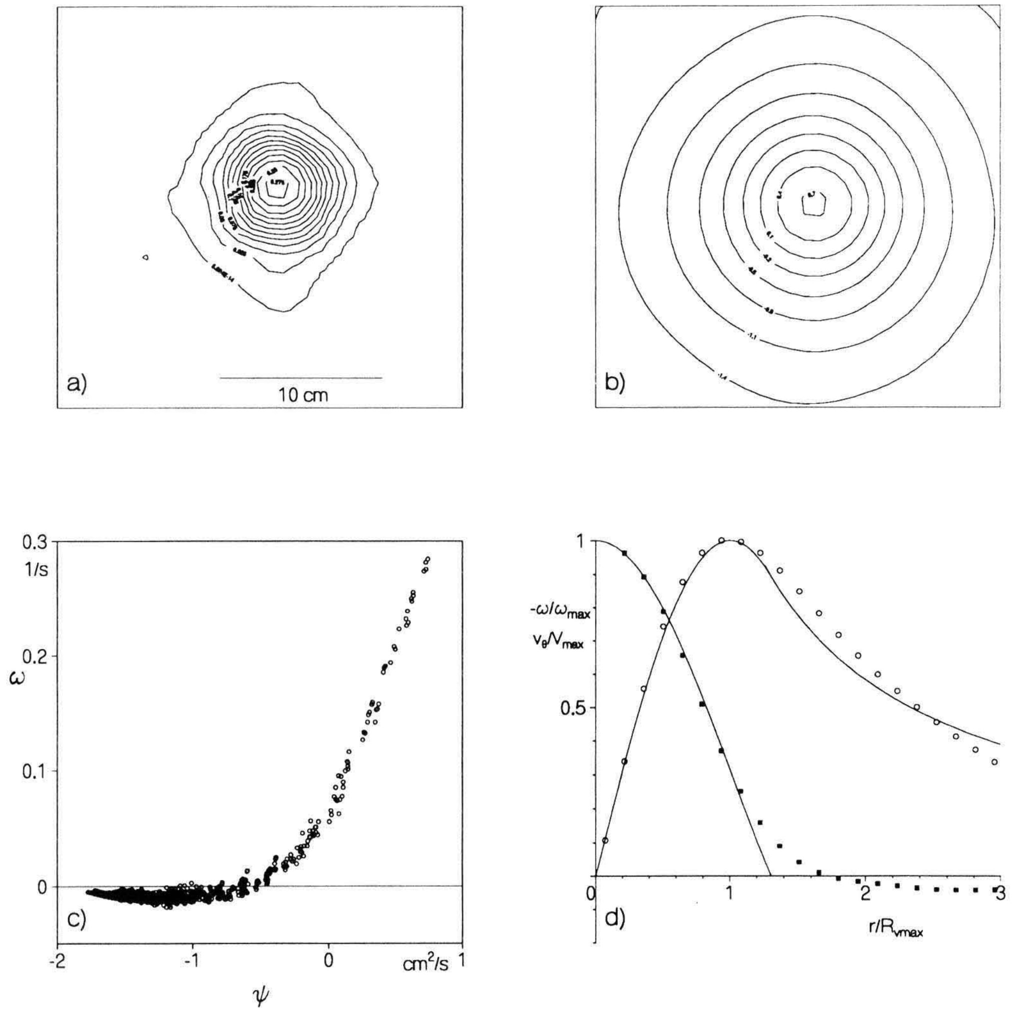

Fig. 1. Experimentally determined features of a characteristic monopolar vortex: (a) ω contours; (b) ψ contours; (c) ω, ψ scatter plot; (d) cross-sectional distributions of ω (symbol ■) and v_θ (symbol o). The profiles are scaled with their maximum values $\omega_{max} = 0.28\ s^{-1}$, $V_{max} = 0.30\ ms^{-1}$ and $R = 3.2\ cm$. The drawn lines represent the vortex model (4). These quantities were determined at mid-depth of a monopolar vortex created by a rotating sphere in a linearly stratified fluid with $N = 1.98\ rad\ s^{-1}$, at $t = 180\ s$ after the forcing was stopped.

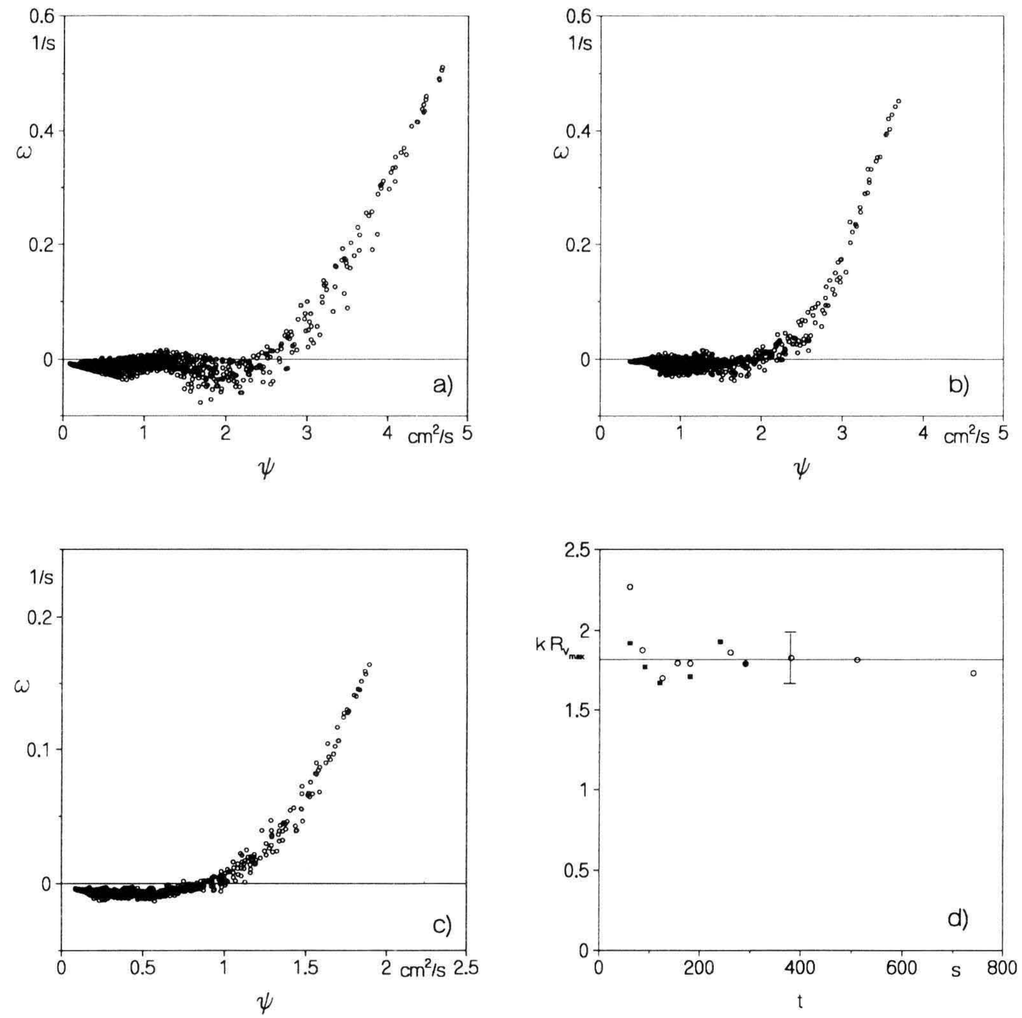

Fig. 2. Sequence of ω, ψ-plots measured at $t = 90\ s$ (a), $120\ s$ (b) and $240\ s$ (c) after the forcing was stopped (same experiment as in Figure 1). The experimental kR-values are shown graphically for two different experimental runs as a function of time in (d), with the horizontal line representing the value $kR = 1.8$.

where J_o is the zeroth-order Bessel function of the first kind, and A is a constant. The corresponding solutions of ω and v_θ are

$$\omega(r) = Ak^2 J_o(kr), \qquad v_\theta(r) = AkJ_1(kr), \tag{4}$$

where J_1 is the first-order Bessel function of the first kind.

In view of the fact that the azimuthal motion is unidirectional, the v_θ-solution should be truncated at a radius a_1 for which $ka_1 \leqslant 3.83171$, this being the first zero of J_1. However, a single-signed vorticity distribution (as is approximately the case in the experiments, compare with Figure 1d), is obtained by truncating the solutions at the first zero of J_o, *i.e.* at $ka_2 = 2.40483$. By defining a potential (outer) flow $v_\theta(r) = \frac{A}{r} kJ_1(ka_2)$ for $r > a_2$ only an unphysical discontinuity occurs in ω at the radius $r = a_2$. It is shown in Figure 1d that this model gives at least a reasonable description of the flow in the central part of the vortex. The experimental kR values, k being determined from the slope of the scatter plot and R being the radius at which $v_\theta = V_{max}$, are presented for a typical vortex as a function of time in Figure 2d. According to the model, $kR = 1.84118$. Although some scatter is present in the data, the measured kR-values agree well with the value 1.8, at least within the experimental error.

During the decay, the vorticity magnitude shows a considerable decrease (see Figures 2a-c), while k and R only show a marginal change: after $t = 120s$ k decreases slowly and R increases somewhat, but in such a way that the combination kR is approximately constant (Figure 2d).

3. Decay models

In an attempt to describe some characteristic features of the decaying monopolar pancake-shaped vortex in a linearly stratified fluid, we will now consider three approximative theoretical models, in order of increasing sophistication.

(i) *purely 2D decay*

In the approximation that the decay of the planar vortex flow can be considered as purely 2D, the flow evolution is governed by the vorticity equation

$$\frac{\partial \omega}{\partial t} + J(\omega, \psi) = \nu \nabla_h^2 \omega \tag{5}$$

where ν is the kinematic fluid viscosity, ∇_h^2 the horizontal Laplacian, and J the Jacobian. Any axisymmetric vortex satisfies $J(\omega, \psi) = 0$, so that for the case of a Bessel vortex, as discussed in section 2, equation (5) takes the following form (see also Batchelor, 1967):

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega = -\nu k^2 \omega \tag{6}$$

Apparently, the decaying vortex solution is

$$\omega(r, t) = \omega_o(r) \exp(-t/\tau_{2D}) \tag{7}$$

with $\omega_o(r)$ the vorticity according to the Bessel vortex, (4), and

$$\tau_{2D} = (\nu k^2)^{-1} \tag{8}$$

the 2D decay time.

(ii) *the 'constant-thickness' model*

The purely 2D decay model can be somewhat refined by taking into account the vertical structure of the planar vortex. Experiments have revealed that the vertical distribution of the horizontal velocity field is closely approximated by a Gaussian profile of the form $\exp(-z^2/2\sigma^2)$, with σ a vertical scale. It is now assumed that the thickness 2σ of the vertical region is constant during (at least) the first stages of the decay, so that in the region $z \ll \sigma$ the vorticity distribution can be approximated by

$$\omega(r, z, t) = \omega_o(r) \exp(-z^2/2\sigma^2)\, h(t), \tag{9}$$

with $h(t)$ a time-dependent amplitude function. Near the mid-level of the vortex region, the vorticity is close to vertical, and its evolution is described by

$$\frac{\partial \omega}{\partial t} + J(\omega, \psi) = \nu \nabla_h^2 \omega + \nu \frac{\partial^2 \omega}{\partial z^2} \tag{10}$$

Substition of (9) into (10) yields

$$\frac{\partial \omega}{\partial t} = \nu \left(-\lambda^2 + \frac{\varepsilon^2}{\sigma^2} \right) \omega \tag{11}$$

with $\varepsilon = z/\sigma$ and $\lambda^2 = k^2 + 1/\sigma^2$. Under the restriction $|\varepsilon| \ll (1 + \sigma^2 k^2)^{1/2}$ one obtains

$$\omega(r, z, t) = \omega_o(r) \exp(-t/\tau_{ct}) \exp - (z^2/2\sigma^2) - O(\nu\varepsilon^2 t/\sigma^2) \tag{12}$$

with

$$\tau_{ct} = (\nu \lambda^2)^{-1}. \tag{13}$$

The last term in (12) is negligible as $t \ll \sigma^2/\nu\varepsilon^2$, which is easily met in most experiments as long as ε is small. Apparently, the decay is again exponential.

(iii) *the vertical diffusion model*

A more accurate model is obtained when the vertical diffusion of the (vertical) vorticity is allowed to result in an increasing thickness of the vertical region. Again limiting the validity of the model to a thin region around the midplane level $z=0$, we put

$$\omega(r, z, t,) = \omega_o(r)\,\gamma(z, t), \tag{14}$$

with $\omega_o(r)$ again the Bessel-vortex solution. By substitution into (10), one obtains for the amplitude function γ:

$$\frac{\partial \gamma}{\partial t} = -\nu k^2 \gamma + \nu \frac{\partial^2 \gamma}{\partial z^2}. \tag{15}$$

By the transformation $\gamma = \Phi(z, t)\exp(-\nu k^2 t)$ one obtains a diffusion equation for Φ:

$$\frac{\partial \Phi}{\partial t} = \nu \frac{\partial^2 \Phi}{\partial z^2}. \tag{16}$$

Under the assumption that initially the vorticity is confined to a thin region at midplane depth, according to $\Phi(t=0) = \Phi_o \delta(z)$, the solution of (16) is

$$\Phi(z, t) = \frac{\Phi_o}{\sqrt{t}} \exp(-z^2/4\nu t). \tag{17}$$

The solution for the vorticity is then

$$\omega(r, z, t) = \omega_o(r) \frac{1}{\sqrt{t}} \exp(-t/\tau_{dif}) \exp(-z^2/4\nu t), \tag{18}$$

with the timescale

$$\tau_{dif} = (\nu k^2)^{-1}, \tag{19}$$

and the constant Φ_o being incorporated in the amplitude of $\omega_o(r)$. Note that this timescale τ_{dif} is identical to τ_{2D}, see (8), as derived for the purely 2D decay.

4. Comparison between experimental observations and decay models

(*i*) *the constant-thickness model*

A useful quantity to characterize the decay of the vortex is the maximum velocity V_{max}, which is proportional to the amplitude A, see (4). Figure 3a shows the behaviour of V_{max} as a function of time for two different experiments.

was stopped was taken as $t = 0$. The graph clearly shows that the model describes a similar decay trend as the experiments. The obtained decay timescales are

	$\tau_{dif}(V_{max})$	$\tau_{dif}(\omega_{max}a)$
I	$440 \pm 26\ s$	$759 \pm 185\ s$
II	$357 \pm 17\ s$	$450 \pm 56\ s$

while the decay value $\tau_{2D} = 1/\nu k^2$ obtained from scatter plots for experiments I and II are $322 \pm 60\ s$, and $280 \pm 100\ s$, respectively. The deviation from the experimental decay values is probably due to the assumption $\omega = \omega_0 \delta(z)$ at $t = 0$ and the fact that $t = 0$ is chosen after the forcing was stopped; in reality a vortex with a certain thickness has already been formed at that time. Besides, the vortex slightly expands horizontally, an effect which is not incorporated in the model. Nevertheless, the decay timescales are of the same order of magnitude. Also, the model describes a trend in the decay that is very similar to that in the measured quantities.

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