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Dynamical modeling of cool supergiant atmospheres

Abstract

This paper reviews the results of dynamical atmosphere modeling for relatively low-mass long-period pulsating stars, then describes some results obtained by applying the same methods to more massive stars. The behavior of massive stars is generally similar but somewhat more complex. The period most clearly seen in their light curves may be associated with their atmospheric acoustic cutoff period, not with interior pulsation.

The atmospheres of pulsating stars

Atmospheres of strongly pulsating cool stars are very different in structure and behavior from those of otherwise similar stars which are not pulsating. Waves generated by the pulsation always grow into shocks, which dissipate energy, transport momentum, and cause enormous extension of the atmosphere. The velocity, density, temperature, and composition of a given parcel of gas are all functions of time. The gas temperature is determined jointly by effects of mechanical work during compression/expansion; energy changes from ionization/recombination and molecule formation/dissociation; non-LTE radiative transfer in a medium where the composition and velocity fields both vary; and collisional energy exchange between gas and grains. The resulting temperature and pressure gradients play a significant part in the dynamics. Radiation pressure on grains and molecules affects the structure of the atmosphere and helps to drive an outflowing wind, which plays a key role in stellar evolution.

These dynamic atmospheres are thus complex systems with nonlinear interactions among numerous time-dependent processes, some of which may be far from equilibrium for much of the time. Their analysis is not simple! Separate study of one isolated process at a time is at best a crude first approximation; it can be quite misleading. It is not surprising that progress toward a good understanding of the complete system has been slow. A few noteworthy steps along the way are briefly described below.

Wood (1979) made hydrodynamic modeling calculations for the atmosphere of a 1-$M_\odot$, 373-d overtone-mode pulsator driven by pressure variations at the model's inner boundary. Shocks resulted and the model atmosphere became greatly extended. When the shocks were assumed to be adiabatic everywhere, there was unrealistically rapid mass loss. Isothermal shocks alone gave little mass loss, but addition of radiation pressure on an assumed dust distribution gave a reasonable mass loss rate.

Willson and Hill (1979) and Hill and Willson (1979) reported numerical hydrodynamic calculations for isothermal models driven by radial oscillation of their inner boundary. Radiation pressure on dust was not included. Little mass loss was found with isothermal shocks, but inclusion of a transition from isothermal to adiabatic behavior in low-density outer regions, where cooling is slow, gave suggestive results. They also described analytic results which give insight into the models' behavior.

Gail, Sedlmayr, and collaborators have published an important series of papers (e.g. Gauger, Gail, and Sedlmayr 1990, and references therein) on grain formation in stellar winds. Almost all of their work so far has dealt with carbon grains and with time-independent flows, but applications to an increasingly wide range of problems should be possible and fruitful.

The work of Cuntz (see contribution to this volume and references therein) with stochastic shock waves appears specially promising for understanding systems that do not execute coherent large-scale periodic pulsation. Cuntz and Stencel (this volume) have also presented an overview emphasizing the multiplicity of interrelated processes and phenomena that can be involved in stellar structure, mass loss, and evolution.

Bowen (1988a, 1988b, 1990) has pursued numerical hydrodynamic and thermodynamic modeling of the atmospheres of stars with periodic pulsation. Approximate treatments are included for time-dependent thermal relaxation by radiative transfer, for the radiation pressure cross section of grains, and for the collisional exchange of energy as well as momentum between gas and grains. The behavior of the models has been studied for a wide range of parameters. This work will be used as the basis for most of the following discussion.

The purpose of this paper is not to essay any sort of definitive treatment, but to call attention to the complex time-dependent interactions that abound in these dynamical systems, to point out a few effects that may be of special interest in connection with massive stars, and to attempt to stimulate further thought and investigation by others working in this field.

Low-mass stars

I think it is best to start by considering results for stars of lower mass, which have been more widely studied. We can then use the methods and insights developed there as the basis for extending our study to more massive stars. We shall note what things become different, and try to understand why.

Consider in particular the Mira variables, which are large-amplitude fundamental-mode radial pulsators, with periods of 200-500 d. They typically have atmospheric shocks, considerable circumstellar silicate dust, and slow, cool winds giving mass loss rates up to a few x 10^{-6} $M_{\odot}$ yr^{-1}. They are cool (type M), but very large (a few x 10^2 $R_{\odot}$) and hence very luminous (several x 10^3 $L_{\odot}$) stars on the asymptotic giant branch. The most common Mira masses are thought to be in the range 1-2 $M_{\odot}$. This is a normal stage in the late evolution of all ordinary stars of relatively low mass; the wind removes their envelope, and their degenerate core becomes a white dwarf.

The methods I have used for modeling Mira atmospheres are described by Bowen (1988a, 1988b). These can be summarized as follows. Spherical symmetry and negligible magnetic effects are assumed. Basic hydrodynamic and thermodynamic equations are written for thin concentric Lagrangian zones, with artificial viscosity included to handle shocks. Time-dependent thermal relaxation toward a local radiative equilibrium temperature calculated with an Eddington approximation is included, using an approximate radiative cooling function which depends on the local density and temperature. The cross section of dust for radiation pressure is assumed to be a simple function of the local temperature, and grains are assumed to be fully momentum coupled to the gas. The inner boundary of the model, placed inside the photosphere and a little outside the pulsation driving zone, is constrained to move radially as a sinusoidal function of the time. The equations are solved by explicit numerical integration to learn the resulting structure and behavior of the atmosphere.

A variety of graphs showing results for low-mass Mira models have been presented by Bowen (1988a, 1988b, 1989, 1990) and will not be repeated here. The reader can refer to those papers or to the similar figures for 5 and 10 $M_{\odot}$ models printed below; the latter are qualitatively much the same, except as noted.

When the inner boundary of an initially hydrostatic model (often referred to as the "piston") is made to oscillate radially, the resulting acoustic waves move outward in the steep density gradient, growing rapidly in amplitude; they quickly develop into shocks. If the piston amplitude increases smoothly from a very small initial value to a much larger final value, the mass distribution quickly adjusts to give a stable dynamical model. It is that kind of steady-state model, with a large driving amplitude, which we now describe.

A plot of radial velocity vs. radius in such a model (e.g. Figure 1) shows formation of shocks near the photosphere. The shocks reach maximum velocity amplitude almost at once, then slowly decrease in amplitude as they propagate outward. Shock luminosity is very large in the inner region, where the density is relatively high, but it decreases rapidly. At radii where the temperature is low enough to permit dust formation, there is a large acceleration, and the outflow soon approaches an almost constant speed which exceeds the escape velocity; this is the circumstellar wind region.

A plot of density vs. radius (Figure 2) shows that inside the first shock there is an approximately exponential decrease of density with radius which is almost the same as $\rho(r)$ for the initial hydrostatic model. In the wind region the density, ρ, decreases very nearly as r^{-2}, as it must, since the velocity, v, is almost independent of r, and the mass loss rate $\dot{m} = 4\pi r^2 \rho v$ is constant. This very slow decrease of ρ in the wind results in an enormous circumstellar region in which there is low but still significant density. Note that the density in the (inner) wind region is determined primarily by how far the density drops in the region inside the first shock, hence by the scale height there.

A plot of the gas kinetic temperature vs. radius (Figure 3) shows a temperature spike at each shock in the inner region, where the shocks are strong but postshock cooling is rapid because the density is fairly high. Radiative cooling depends on collisional excitation, however, so at greater radii, where the density is much lower, the thermal relaxation time becomes comparable to the period between shocks, then even larger, and processes become effectively adiabatic. There is a region not far outside the strongest shocks (say ~1.5-2.0 R_*) where rapid and essentially adiabatic expansion between shocks drops the temperature to ~1000-1100 K for much of a cycle in almost all models. This may be where and how the nucleation of grains commonly takes place. If so, this dynamical effect gives a nice illustration of the interactions that can occur between quite different processes. At large radii the temperature is determined primarily by radiation from molecules and grains.

Dust absorbs momentum from the radiation field of the star and transfers it to the gas by way of collisions, thus helping to drive mass loss. In the models this effect is clearly present: any change in parameters that increases the radiation pressure on dust also increases the wind momentum and the mass loss rate by corresponding amounts. Some mass loss can take place in the models in the absence of dust, however. What then drives the wind? The wind speed and hence the expansion rate in the outer atmosphere are much slower, and there is little cooling by effectively adiabatic expansion; the cooling by radiation from grains is also lost. An extended region of elevated temperature thus develops, and the resulting pressure gradient drives a very slow wind. The mass loss rate without dust is typically several percent of the value with dust: it is small, but not negligible. It does not occur, however, unless density-dependent cooling rates are included in the model.

Mass loss rates for these models appear to be similar to the values observed for stars. Moreover their dependence on stellar parameters predicts evolutionary effects of great potential interest. Bowen and Willson (1991) describe results for a large grid of models with masses from 0.7 to 2.4 $M_\odot$ and fundamental-mode periods from 150 to 800 d. Their calculations indicate an approximately exponential growth of m with time because of natural evolutionary changes in the stellar parameters. They used model parameters which were consistent with the Iben (1984) mass-luminosity-radius relationship for evolving AGB stars and the Ostlie and Cox (1986) period-mass-radius relationship for Mira variables. The wind develops in an entirely natural, indeed inevitable way and exhibits the properties expected of the oft-discussed "superwind" (Renzini 1981).

Plots showing the radius as a function of time for a number of model zones (Figure 4) help one to visual the actual motion in the atmosphere. In simplest terms, this shows the oscillating inner boundary, a standing wave between that and the photosphere, the formation of shocks just outside the photosphere, and the roughly ballistic motion of material between encounters with shocks; in a more extended region one would also see weakening of the shocks and the development of a steady outflow (the "wind") at large r. The picture is complicated, however, by two additional shocks within each pulsation period -- an initially puzzling clue whose solution led to recognition of some other interesting effects (Bowen 1990).

It is well known that sinusoidal acoustic waves cannot propagate radially outward in a gravitationally produced density gradient if their period is greater than $P_{AC} = 4\pi H/c_s$, where $H = R_{gas}T/\mu g$ is the density scale height and c_s is the sound speed (Stein and Leibacher 1974). There is a minimum in P_{AC} just inside the photosphere of typical models, which leads to reflection back into the interior of waves with periods longer than that value, and transmission into the atmosphere of waves with shorter periods. This period-dependent behavior has effects on the structure of the star, on the shape of the light curve, and on the power needed to drive pulsation -- hence on the damping of different oscillation modes and the determination of the limiting pulsation amplitude.

Bowen (1990) explored these effects for a 1-$M_\odot$ model which had a fundamental period of 320 d. Its overtone period was 150 d, and its acoustic cutoff period was 135-140 d. When driven at 150 d there was very little reflection, and a plot like that of Figure 4 looked like traveling waves everywhere. For overtone pulsation the atmosphere became, in effect, a giant sink which received and dissipated acoustic power. The average mechanical power delivered by the piston, even at modest amplitude, was necessarily very large.

When the same model was driven at its fundamental period, 320 d, strong reflection near the photosphere gave rise to a standing wave in the interior. The waves reaching the reflecting region contained components of shorter period, of course, and some power was transmitted, predominantly at about the acoustic cutoff period. This gave rise to an extra, or "intraperiod" shock for which the luminosity was briefly very high. The average piston power was almost two orders of magnitude smaller than for overtone oscillation of the same amplitude, however. Beach (1990) has shown that this intraperiod shock can give rise to the "bumps" often observed in Mira light curves.

It appears to be true in general that for low-mass Miras there will be severe damping of overtone-mode pulsation through acoustic power dissipation in the atmosphere, but damping of the fundamental mode will be weak until the pulsation amplitude becomes very large. In fact, even though linear interior modeling has shown large growth rates for both the overtone and fundamental modes (Ostlie and Cox 1986), this damping by the atmosphere may be sufficient to prevent overtone pulsation entirely. Miras would then perforce be fundamental-mode pulsators with large amplitudes -- as indeed they seem to be.

We now consider the results of similar modeling calculations for more massive stars.

More massive stars

The methods summarized above were used to model several more massive stars. In particular, results will be described for two models characterized by the following sets of parameters:

M	=	5 $M_\odot$	10 $M_\odot$
P_o	=	800 d	800 d
R	=	736 $R_\odot$	967 $R_\odot$
T_{eff}	=	3059 K	3194 K

Both models included dust, with the maximum a_{rad}/g set at 1.0; the temperature function describing grain formation was adjusted to give most growth in the range 1200-1000 K. The parameter sets above are consistent with the Iben (1984) R-L-M relationship mentioned previously, assuming solar metallicity. There was no indication that the results obtained were in any way special to these specific models, apart from effects that would apply to any models of similar mass.

The first question, naturally, is whether the basic modeling method that has proved satisfactory for low-mass stars is also satisfactory for those of higher mass. The answer is yes. One additional effect does appear to become significant at large masses and was added in an approximate way, namely radiation pressure on water molecules. This will be described below.

Consider now the results for a 5-$M_\odot$, 800-d model. Figures 1, 2, and 3 show velocity, density, and temperature as functions of the radius. Compared to similar plots for stars of lower mass there are few significant differences. The maximum velocity jump across the shocks is a little larger here, but that is to be expected, since the escape velocity is a little greater.

The most noteworthy structural difference in the 5-$M_\odot$ model is a lower density in the wind region; the effect is larger at 10-$M_\odot$. As a consequence the mass loss rates are very low. It proved almost impossible, in fact, to find parameters or conditions that would give $\dot{m}$ for any 10-$M_\odot$ model higher than about 10^{-8} $M_\odot$ yr^{-1}, which is far below the values commonly observed for supergiants.

Clearly something was wrong with the models. The density was falling too far in the inner atmosphere, where its rapid exponential decrease occurs, forcing too low a density in the wind region. Why?

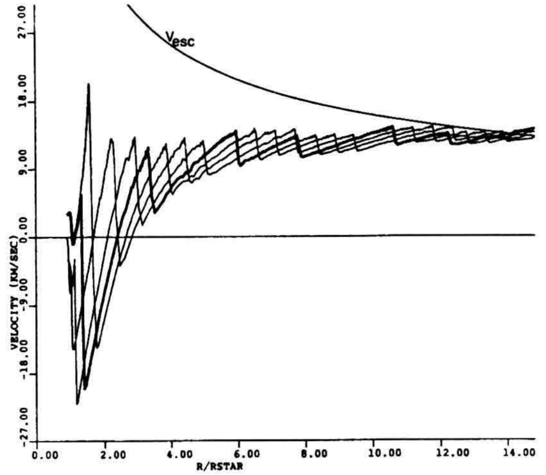

Figure 1. Radial velocity as a function of radius at phases 0.00, 0.25, 0.50, and 0.75 for the 5-$M_\odot$, 800-d model, including dust. Driving period = 800 d = fundamental. Piston velocity amplitude = 3.0 km s^{-1}. Phase 0.00 is shown bold.

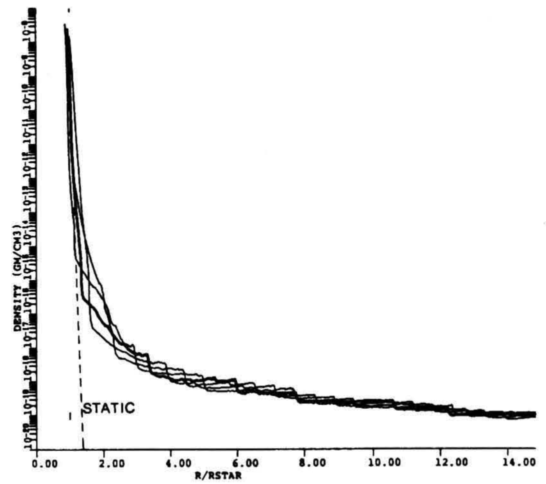

Figure 2. Density as a function of radius. Same model as Figure 1.

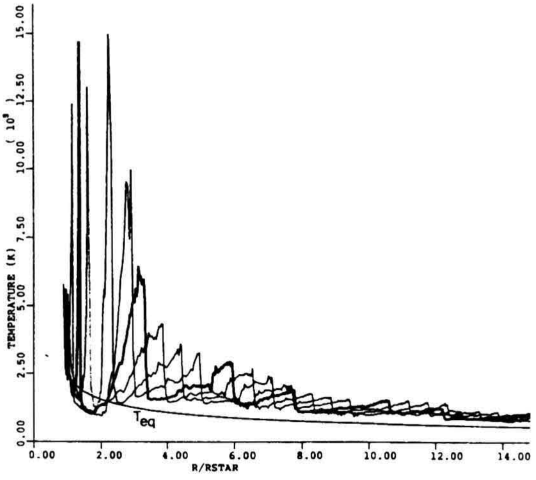

Figure 3. Gas kinetic temperature as a function of radius. Same model as Figure 1.

For increasing stellar mass and size, the radial extent of that exponential region increases; the density scale height there also increases, but not as rapidly. As a result, that region includes more scale heights, and the density falls by a larger factor in more massive stars. What could prevent this? Elitzur, Brown, and Johnson (1989) pointed out that radiation pressure on water molecules could be sufficient to have important effects in very large cool stars with low surface gravity. They sought a system in which radiation pressure on such molecules could be entirely responsible for driving mass loss, which requires rather extreme conditions. It is not necessary for that single mechanism to do everything, however. It would be quite sufficient for it to increase the scale height by a modest amount in the right region. Water opacity becomes important below about 2000 K, which is the right range to affect the exponential density decrease in my models. It need only increase the effective scale height there by 20-30% in order to increase the density in the wind region by a few orders of magnitude and give large mass rates. Could it do so?

To test this possibility, I added a term representing radiation pressure on water to the hydrodynamic acceleration equation in my code, using a temperature dependence like that described by Elitzur et al. It worked exactly as expected. The mass loss rates for 5 and $10\text{-}M_\odot$ models were easily increased to appropriate values, with no strange side effects. This crude first trial looks quite promising, but it proves nothing, of course -- except that it would be foolish to ignore this in the future. Complete time-dependent calculations for radiation pressure on molecules in dynamic atmospheres will be extremely difficult, but they *must* be undertaken at some reasonable level of approximation. And it will be well to keep this in mind as still another example of how complex the interactions can be in dynamic atmospheres.

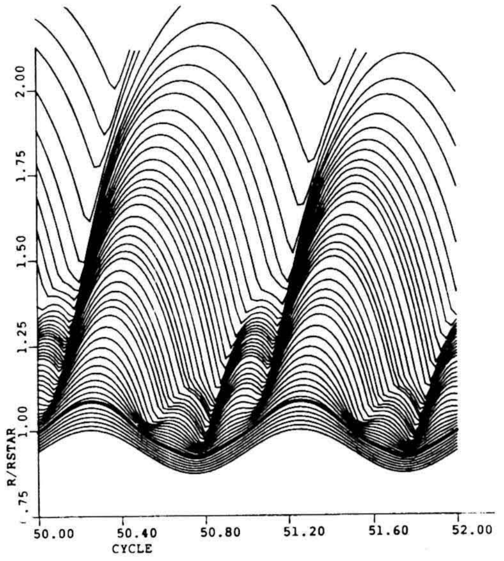

Figure 4. Radius of selected shells as a function of phase for the same model as Figures 1-3. The line at smallest radius is the piston. The bold line is the photosphere.

Figures 4 and 5 show radius as a function of time for selected zones of the $5\text{-}M_\odot$ model driven at its fundamental and first overtone periods, respectively. Figure 6 shows the average power delivered by the piston, as a function of the driving period. For this and other more massive models the ratio of the acoustic cutoff period to the pulsation periods is smaller than for low-mass models. The same principles are at work, but the results are different and more complex. The possible consequences for observed behavior of supergiants are intriguing.

Because the fundamental period (800 d) in this case is over 3 times the acoustic cutoff period (about 255 d), there are three shocks per cycle -- one main shock forming at about phase 0 and propagating outward, plus two intraperiod shocks. The light curve must be correspondingly complex. Moreover the first overtone period (400 d) is now enough larger than the cutoff value to give fairly strong reflection, so atmospheric damping of overtone pulsation should not be very great. This suggests that double mode pulsation (fundamental plus first overtone) should occur. If so, the superposition of two oscillation modes with incommensurate periods should complicate the atmospheric structure and the light curve still more. There is, in fact, a real possibility that chaotic behavior may result.

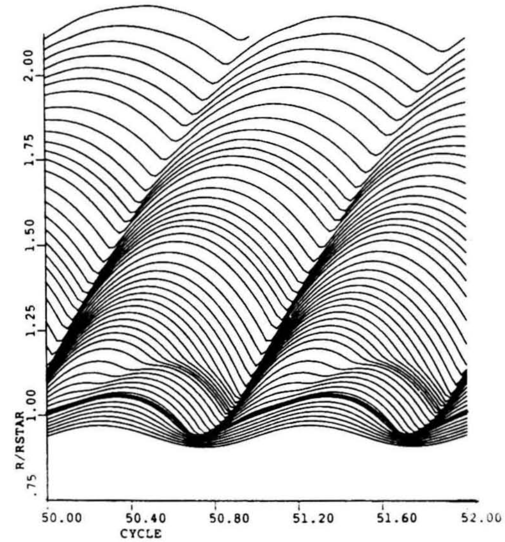

Figure 5. Same as Figure 4 except the driving period = 400 d = first overtone.

To illustrate further the complexity that can develop, results for the 10-$M_\odot$, 800-d model are plotted in Figure 7, which shows *four* shocks per cycle. No attempt has yet been made to calculate a complete emergent light curve in this case, but it probably will be complicated and a little irregular in appearance, with an apparent periodicity which is related more to the acoustic cutoff period than to the pulsation period. And if double-mode pulsation occurs, that would add more complications.

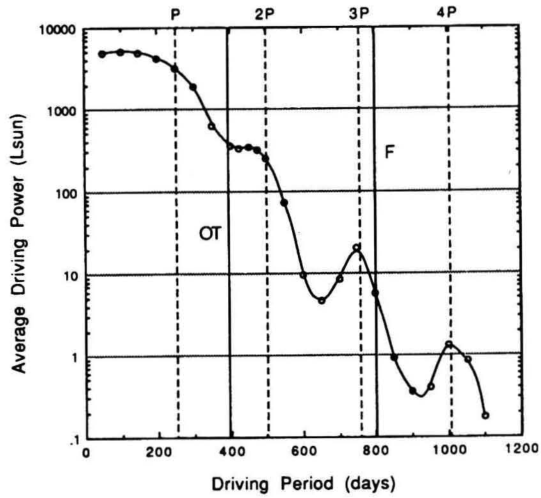

Figure 6. Average mechanical power delivered by the piston as a function of the driving period for the 5-$M_\odot$ model. Piston velocity amplitude = 3.0 km s^{-1} in all cases. P = acoustic cutoff period.

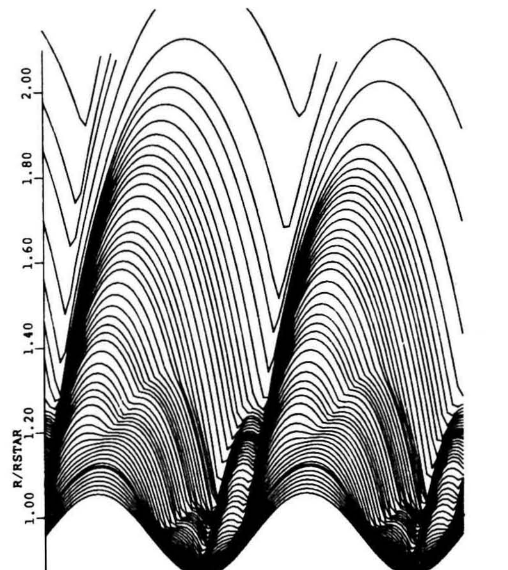

Figure 7. Same as Figure 4, but for the 10-$M_\odot$, 800-d model, including dust. Driving period = 800 d = fundamental. Piston velocity amplitude = 6.0 km s^{-1}.