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Axisymmetrization of Warm Oceanic Vortices

Abstract

Previous studies have shown that in many cases eccentric vortices become axisymmetric. In this work we report the results of a numerical experiment performed with the shallow-water, reduced-gravity model for elongated warm eddies. The evolution of these eddies is invariably toward less-eccentric states and, often, includes the shedding of a small portion of mass. An attempt is made to characterize this process by defining an intensive quantity.

Introduction

Warm eddies are common and important features in the oceans, because their properties are both distinct and persistent. Satellite imagery allows us to witness the generation and evolution of many oceanic vortices. For example, it has been observed that Gulf Stream meanders often break off to form long-lasting eddies, which carry away warm waters to colder regions. The evolution of these eddies, in turn, plays a significant role in determining the characteristics of wide areas in the ocean.

In this work we analyze the evolution of oceanic eddies similar to those mentioned above, by utilizing a primitive equation numerical model. Such a model is used because: one, we are able to study a series of eddies with very different eccentricities and strengths; and two, the ensuing states of the initially prescribed model eddies can be easily tracked. We expect a wide variety of eddy-behaviors, since previous works (e.g. Ripa, 1987) have shown that many of these vortices are unstable, particularly the most eccentric ones. Previous studies using approximate models (e.g. McCalpin, 1987) indicate that *weak* elliptical vortices evolve toward a circular state, a process known as axisymmetrization. But, do elongated eddies (unstable, but not necessarily weak) become axisymmetric in order to achieve a more stable state? If so, it would be desirable to be able to characterize this process.

The next section outlines the model used and the form of the initial conditions.

The model

The shallow-water, reduced-gravity ocean model is written here as a pair of Lagrangian momentum equations:

$$\frac{Du}{Dt}=fv-g'\frac{\partial h}{\partial x}, \tag{1}$$

$$\frac{Dv}{Dt}=-fu-g'\frac{\partial h}{\partial y}, \tag{2}$$

where u and v are the velocity components in the x and y directions, respectively, t is time, f is the constant Coriolis parameter, g' is the reduced gravity, and h is the thickness of the active layer. The latter vanishes at the eddy's edge and, since resolving this frontal line is crucial for this study, we will use a particle-in-cell (PIC) method to solve (1)–(2). This requires one more pair of Lagrangian equations, namely

$$\frac{Dx}{Dt}=u, \tag{3}$$

$$\frac{Dy}{Dt}=v, \tag{4}$$

making it [system (1)–(4)] a four-by-four system for each of the N particles' positions and velocities (x_i, y_i, u_i, v_i, $i=1, 2, 3, \ldots, N$). The initial particle distribution yields the initial h-field and their prescribed velocities should correspond to the initial velocity fields [see (5)–(7) ahead]. The numerical integration of (1)–(4) by the PIC method assures conservation of mass; thus the continuity equation $h_t+(hu)_x+(hv)_y=0$ complements (1)–(2), but it is not explicitly solved. The reader is referred to previous works for details and examples of the use of the same PIC model as employed here (e.g. Pavia and Cushman-Roisin, 1990).

An exact solution of the above described system, consisting of an elliptical clockwise rotating, anticyclonic vortex, was first given by Cushman-Roisin *et al.* (1985) and called the Rodon solution. Ripa (1987) transformed variables and equations to a frame rotating at the same rate as the eddy, Ω, so that the h contours become stationary, thereby obtaining a steady version of the Rodon. We will use another form of the latter version of the solution to prescribe the initial conditions of the model, namely:

$$u_0=\left[1+\frac{a}{b}\left(\frac{f-\Omega}{\Omega}\right)^{1/2}\right]\Omega y, \tag{5}$$

$$v_0 = \left[1 + \frac{b}{a}\left(\frac{f-\Omega}{\Omega}\right)^{1/2}\right]\Omega x, \tag{6}$$

$$h_0 = H\left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2\right], \tag{7}$$

where a and b are the minor and major semiaxes, respectively, and

$$H = \left(\frac{ab}{2g'}\right)(f - 2\Omega)[\Omega(f-\Omega)]^{1/2}, \tag{8}$$

is the center depth of the eddy. This vortex is fully characterized by two intensive parameters which are the aspect ratio, $r = a/b$, and the ratio of the clockwise rotation of the vortex to the Coriolis parameter, $S = \Omega/f$ [or by the Rossby number, $Ro \equiv S + (S - S^2)^{1/2}$]; as well as one extensive parameter (say the total volume), and three geometrical parameters: the position of the center of mass and the initial orientation. The instability of elliptical eddies to infinitesimal perturbations for the whole range of values of the parameters

$$0 < r \leqslant 1 \qquad 0 < S < 0.5, \tag{9}$$

was studied by Ripa (1987).

In the next section, the evolution of a series of widely diverse eddies will be investigated by means of a numerical experiment.

The numerical experiment

The main numerical experiment consists of a series of runs with eddies whose initial conditions correspond to all cases varying S from $S = 0.05$ to $S = 0.45$, every 0.05, and the aspect ratio from $r = 0.1$ to $r = 0.9$, every one tenth. These 81 cases include a wide range of the parameter space (9). Each run is time-integrated until $t = 500f^{-1}$, which is considered long term in all cases. The resulting h-field was fitted with a second order polynomial of the form

$$h \approx A - Bx^2 - Cy^2 + 2Dx + 2Ey + 2Fxy, \tag{10}$$

which, after some manipulation of the six coefficients, yields the $x-y$ coordinates of the center of the eddy, its orientation (θ), the maximum depth (H), and the minor (a) and major (b) semiaxes (mean radius [$R = (ab)^{1/2}$] and aspect ratio [$r = a/b$] can be calculated from these last two). The theoretical rotation rate of the vortex, assuming the eddy passes adiabatically through a series of Rodon-like states, is obtained using the numerical values of R and H to solve for Ω in (8). In doing so we make the transformation $\Omega = f(1 - \cos\alpha)/2$, $0 < \alpha < \pi/2$, so that (8) becomes $H = (Rf)^2 \sin(2\alpha)/8g'$. This theoretical rotation rate is compared with the measured one $d\theta/dt$.

We consider this procedure (10) to be successful if its root-mean-square (*rms*) error is less than 20%. Larger *rms* errors indicate that the original eddy has broken up into two or more parts, so that the resulting h-field is not appropriately fitted by (10). The results, therefore, must be discarded. Not surprisingly, this is the situation for several of the most eccentric eddies, while the less eccentric ones showed the smaller *rms* errors. Furthermore, in the cases with *rms* error of less than 20% the difference between the parameters of the original eddy (prescribed) and those of the final (fitted) eddy seemed proportional to the eccentricity of the original eddy. This difference is mainly in aspect ratio, r, and to a lesser extent in S. A general result is that in all cases the evolution was toward greater r (all eddies "axisymmetrize" somewhat), and expel mass (the estimated volume of the final eddy was smaller than the volume of the original one). In contrast, changes in S seemed to depend on the sign of the potential vorticity, q; i.e. for $q<0$, the value of S increased and for $q \geqslant 0$, S decreased, changed little or not at all.

A couple of typical examples include:

a. Evolution of a highly elongated eddy

The evolution of an eddy whose initial parameters are $r=0.2$ and $S=0.15$ is examined at $\Delta t = f^{-1}$ intervals; i.e. the least-squares procedure (10) is performed every $ft=1$. This particular case is chosen because it exhibited one of the most dramatic parameter changes during the main experiment.

To help us visualize the nature of the step-by-step parameter change, we define an intensive quantity

$$\mu \equiv q_0 V^{1/2} = f(S, r), \tag{11}$$

where q_0 and V are the potential vorticity of the center and the volume of the eddy, respectively. We expect this quantity to be approximately conserved because the system conserves potential vorticity, and we assume the eddy to conserve most of its mass. If potential vorticity and all the mass is conserved in one single blob, and if the eddy were to change its position in parameter space, then it should move along a particular isoline of μ. The evolution may be divided into two stages. During the first stage, characterized by rapid changes, the eddy departs from its original isoline, and apparently does not conserve μ. This is so because the least-squares procedure overestimates the size of the eddy when it undergoes distortion and shedding of mass. During the second stage, the eddy regains an almost elliptical shape and, despite high-frequency inertial oscillations, the trajectory closely follows its μ-isoline. This is a particularly dramatic case, which we examined in detail in order to check the results of the numerical experiment.

b. Evolution of a stable eddy

The evolution of an eddy with $r=0.6$ and $S=0.05$ is similarly examined, it moves relatively little within the parameter space. The small movement is mainly toward greater r with even smaller change in S. In contrast with the previous case, this eddy is selected because its initial position in parameter space is located within the stable region of all the stability diagrams considered. [The eigenperturbations of the eddy's linear stability analysis are n-order polynomials, all eddies are stable for $n < 3$, and Ripa's (1987) diagrams include $n > 3$ up to $n = 6$. The corresponding stability diagrams for $n = 7$ and $n = 8$ present new instability areas suggesting that perhaps the high Ro region is full of higher-order instability tongues; but the eddy-parameters of this case remain outside of them.] From time series of r and other parameters we observe that besides a small axisymmetrization, inertial oscillations are the only action taking place. Other minor changes are more likely due to numerical dissipation than stability reasons.

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