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Nonlinear Rossby Waves and Vortices

Abstract

Interaction between planetary waves and vortices are analysed by decomposing into vortex and wave flow fields. The longtime evolution of a strong vortex is considered by assuming a quasi-stationary distribution of the potential vorticity inside the vortex core. The radial structure of the symmetric circulation in the vortex is described providing zero azimuthal velocity at the core boundary. The model describes weakening of the vortex, decreasing of its size, shrinking of the core and the development of a shielding annulus around the vortex core, depending only on the meridional position of the vortex center. The results are compared with numerical simulations using a semi-spectral quasigeostrophic model.

Introduction

Large-scale vortex structures, remaining coherent during many turnaround times, have been recognized to be typical in quasi-two-dimensional planetary flows. Such intense vortices, e.g. tropical cyclones in the atmosphere, frontal rings and lenses in the ocean, the Great Red Spot of Jupiter and other giant eddies occurring in the atmospheres of the outer planets, maintain their identities while traveling over distances much larger than their typical size. Thus, providing highly anisotropic transport of trapped fluid with different physical properties, coherent vortices are of fundamental interest for understanding the general atmospheric and oceanic circulations.

Over the past decade, observational programs, theoretical analyses, numerical simulations and laboratory experiments have improved increasingly our understanding of the structure and dynamics of planetary vortices. The results are summarized in books and reviews, e.g. Khain & Sutyrin, 1983; Kamenkovich *et al.* 1986; Flierl, 1987; Korotaev, 1988; McWilliams, 1991; Hopfinger & van Heijst, 1993; Nezlin & Sutyrin, 1994. In particular, substantial progress has been made on interaction of intense vortices with highly dispersive Rossby waves being generated due to the background gradient of the potential vorticity in planetary flows.

In analytical studies of steadily propagating anticyclones without Rossby wave radiation (Nycander & Sutyrin, 1992) or of the initial acceleration of a monopolar vortex on the beta-plane (Sutyrin & Flierl, 1994), a symmetric circulation was prescribed. The problem was reduced to calculating the asymmetric circulation, the so-called beta-gyres, which modifies the vortex translation. Here we consider the nonlinear feedback between the vortex and the generated Rossby waves, thus allowing for the description of a change in the symmetric vortex structure due to the meridional drift of the vortex centre.

Decomposition into vortex and wave flows

We consider a localized vortex on the beta-plane using the equivalent-barotropic quasigeostrophic approximation which is a generic model for the quasi-two-dimentional planetary flows. A basic property of an inviscid flow is the material conservation of the potential vorticity (PV) in fluid particles. In the absence of a flow, PV depends only on the meridional coordinate, y, i.e. there are no closed isolines of PV. If a fluid perturbation of PV is strong enough, an area with closed isolines of PV exists. This area is considered to be a vortex core where Rossby waves propagating along the PV isolines are trapped.

To describe an interaction between the vortex and the Rossby waves we decompose the velocity into the vortex flow, $\vec{\mathbf{V}} = \vec{\mathbf{k}} \times \nabla \Psi$, corresponding to PV inside the vortex core, Q, and the wave flow field, $\vec{\mathbf{w}} = \vec{\mathbf{k}} \times \nabla \phi$, induced by the PV perturbation outside the core, ξ. Thus, inside the core the wave streamfunction, ϕ, obeys $\nabla^2 \phi - \phi = 0$, while the evolution of Q is described as follows

$$\frac{\partial Q}{\partial t} + \vec{\mathbf{V}} \cdot \nabla Q = -\vec{\mathbf{w}}^* \cdot \nabla Q, \tag{1}$$

$$\nabla^2 \Psi - \Psi = Q - Y_0 - r \sin \theta. \tag{2}$$

Here polar coordinates (r, θ) moving with the vortex center are used, $\vec{\mathbf{w}}^* = \vec{\mathbf{w}} - \vec{\mathbf{c}}$ is the wave velocity relative to the vortex center, defined as an extremum of PV, $\vec{\mathbf{c}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{w}}_0$ is the drift velocity and $Y_0 = \int c_y dt$ is the meridional displacement of the vortex center. In the nondimensional Eqs. (1)-(2), the radius of deformation and the Rossby wave speed are used as spatial and velocity scales, respectively.

Outside the core $\nabla^2 \Psi - \Psi = 0$, whereas the wave perturbation of PV obeys

$$\frac{\partial \xi}{\partial t} + \vec{\mathbf{w}}^* \cdot \nabla \xi + \vec{\mathbf{w}} \cdot \nabla (r \sin \theta) = -\vec{\mathbf{V}} \cdot \nabla (\xi + r \sin \theta), \tag{3}$$

$$\nabla^2 \phi - \phi = \xi. \tag{4}$$

This decomposition clearly shows that outside the core Rossby waves are forced by the vortex circulation, whereas inside the core the advection of PV by the vortex flow is accompanied by the feedback due to the wave field.

Evolution of the symmetric circulation

Now we separate the vortex flow into symmetric and asymmetric parts, denoting by $\langle\ \rangle$ the azimuthally averaged values

$$\vec{\mathbf{V}} = \langle \vec{\mathbf{V}} \rangle + \vec{\mathbf{v}}, \qquad \Psi = \langle \Psi \rangle + \psi, \qquad Q = \langle Q \rangle + q.$$

Thus, the evolution of the symmetric vortex inside the core obeys

$$\frac{\partial \langle Q \rangle}{\partial t} = -\langle (\vec{\mathbf{v}} + \vec{\mathbf{w}}^*) \cdot \nabla q \rangle = \langle J(q, \psi + \phi^*) \rangle, \tag{5}$$

where $\phi^* = \phi + c_x r \sin\theta - c_y r \cos\theta$ describes the flow relative to the vortex center and J denotes the Jacobian.

The evolution of the asymmetric vortex circulation inside the core is described by

$$\frac{\partial q}{\partial t} = -\frac{\partial}{\partial \theta}[\Omega q - \Gamma(\psi + \phi^*)] + J(q, \psi + \phi^*) - \langle J(q, \psi + \phi^*) \rangle, \tag{6}$$

$$\Omega = \frac{\partial \langle \Psi \rangle}{r \partial r}, \qquad \Gamma = \frac{\partial \langle \mathrm{Q} \rangle}{r \partial r}$$

Here Ω is the angular rotational velocity and Γ is defined from the radial PV gradient.

For a strong vortex with a characteristic rotational frequency $\Omega_0 \gg 1$ there are three different time scales: the turnaround time $\simeq \Omega_0^{-1} \ll 1$, the typical wave time $\simeq 1$ and the vortex evolution time $\simeq \Omega_0$. Fast fluid rotation prevents growth in the amplitude of the vortex asymmetry and wave flow which remain at the order of unity ($q, \xi \simeq 1$). Unlike a linear wave packet, a strong vortex is long-lived since it changes its intensity on the order of unity at the wave time scale, which is much smaller than the vortex intensity, Ω_0 (Sutyrin, 1987).

In order to describe the long-time vortex evolution we introduce the slow time, $\tau = t\Omega_0^{-1}$, assuming that the flow does not depend on the fast turnaround time, $t\Omega_0$. Thus, considering the leading order terms, denoted by the capital letters in Eqs. (5)-(6), we conclude that $\langle Q \rangle = \langle Q \rangle_{init}$ and $\Omega q = \Gamma(\psi + \phi^*)$, such that using Eq. (2), we obtain

$$(\nabla^2 - 1)\langle \Psi \rangle = \langle Q \rangle_{init} - Y_0(\tau) \tag{7}$$

$$\left(\nabla^2 - 1 - \frac{\Gamma}{\Omega}\right)\psi = \frac{\Gamma}{\Omega}\phi^* - r \sin\theta \tag{8}$$

Eqs. (7)-(8) allow for an explicit expression of the vortex flow through the meridional displacement Y_0 and ϕ^* in the core by taking into account the assumption of no PV perturbation outside the core and provided the core boundary is known.

According to Eq. (7), the radial profile of the symmetric PV inside the core changes independently on the radius due to the appears meridional displacement. As a result, the jump in the symmetric PV appears at the core boundary. Such an annulus with opposite radial PV gradient may lead to instability and to the appearance of a tripolar structure as shown in recent numerical simulations by Hesthaven *et al.* (1993). Rotating and oscillating satellites produce increased mixing and smoothing of PV near the core boundary.

In the leading order expansion we neglect this effect on the symmetric circulation and write the solution of Eq. (7) inside the core $r<r_c$

$$\langle \Psi \rangle = Y_0[1-r_c K_1(r_c)\, I_0(r)] - K_0(r)\int_0^r I_0(r')\,\langle Q\rangle_{init} r'dr'$$

$$-I_0(r)\int_0^{r_c} K_0(r')\langle Q\rangle_{init} r'dr'. \tag{9}$$

Outside the core $\langle \Psi \rangle = F(Y_0, r_c)\, K_0(r)$, where

$$F = Y_0 r_c I_1(r_c) - \int_0^{r_c} I_0(r')\,\langle Q\rangle_{init} r'dr' \tag{10}$$

Thus, the symmetric circulation depends only on the meridional position, Y_0, and the core radius, r_c

$$V_\theta = -Y_0 r_c K_1(r_c)\, I_1(r) + K_1(r)\int_0^r I_0(r')\langle Q\rangle_{init} r'dr'$$

$$-I_1(r)\int_r^{r_c} K_0(r')\,\langle Q\rangle_{init} r'dr' \qquad r<r_c \tag{11}$$

To satisfy Eq. (3) outside the core, the azimuthal velocity of the vortex must be zero in the leading order. This approximation has been used by Korotaev (1988) to describe the near stationary wave field outside the core and the associated vortex translation in the next order. Here we use the assumption $V_{\theta c}=0$ at the core boundary for estimating the core radius by setting $F=0$ in Eq. (10). Such an approach allows for a description of the long-time evolution of the symmetric vortex structure during its meridional displacement, Y_0.

Calculations of the core radius, r_c, the radius of maximum azimuthal velocity, r_m, and the corresponding value of $V_{\theta m}$ for an initially Gaussian vortex with $Q_{init}=(6-2r^2)\exp(-r^2/2)$ are presented in the table below (the initial core radius is defined by $\Gamma(r_c)=0$):

Y_0	r_c	r_m	$V_{\theta m}$
0.0	2.21	1.00	1.20
1.8	1.68	0.77	0.83
3.0	1.25	0.64	0.56
4.0	0.97	0.51	0.35

It shows that this simple model for the slow evolution of the symmetric circulation in the leading order displays such essential features as shrinking of the core, weakening of the vortex and decreasing of its size while the meridional displacement increases.

Semi-spectral model

To compare the results with the two-dimensional solution of the initial value problem we use a semi-spectral numerical model based on a decomposition of the flow field into azimuthal modes (Sutyrin, 1989),

$$\xi = \sum_{-M}^{M} \xi_m(r, t)\, e^{-im\theta}, \qquad \phi = \sum_{-M}^{M} \phi_m(r, t)\, e^{-im\theta}.$$

Generally the translation of the vortex center, defined as the extremum of PV, is caused only by the flow with $m = 1$:

$$c_x + ic_y = \frac{r_c^2}{2} K_2(r_c) - 1 - i\int_0^{r_c} K_1(r)\, q_1 r dr - i\int_{r_c}^{\infty} K_1(r)\, \xi_1 r dr. \tag{12}$$

Here the first two terms describe the westward drift produced by the PV perturbation (the last term on the left-hand side of Eq. (8)) resulting from axisymmetrization of PV inside the vortex core. The next term arises due to the distortion of the vortex shape inside the core as it was considered by Sutyrin & Flierl (1994) for a vortex with piece-wise constant PV. The last term describes the translational feedback from the wave flow outside the vortex core.

The generation of the wave field outside the core is described by Eq. (3),

$$\frac{\partial \xi}{\partial t} = -\Omega \frac{\partial}{\partial \theta}(\xi + r \sin\theta) + J(\xi, \psi + \phi^*) + J(r \sin\theta, \psi + \phi). \tag{13}$$

By considering the dominant terms in Eq. (13), the initial development of an asymmetric spiral structure outside the core was described by Sutyrin (1987);

$$\xi = r \sin(\theta - \Omega t) - r \sin\theta, \qquad \xi_1 = ir(e^{i\Omega t} - 1). \tag{14}$$

This solution allows for explaining the meridional and zonal acceleration of a strong vortex during many turnaround times ($\Omega_0 t \gg 1$).

The initial value problem was solved numerically for the same example of an initially Gaussian vortex as considered above. This vortex is not strong ($V_{\theta m} = 1.2$) and its trajectory deviates significantly from the theory of azimuthal mode $m = 1$ perturbation after $t \approx 2\pi$ due to the development of higher azimuthal modes and the change in the symmetric circulation. Nevertheless, calculations show that results of the semi-spectral model for $M = 8$ agree well with the high resolution (512x256) pseudo-spectral solution obtained by Sutyrin *et al.* (1994) until $t \approx 40$. At this time the vortex decays and becomes essentially weaker than the generated wave flow field.

In this case of a rather weak monopole, a tripolar structure does not appear in spite of the initial presence of the annulus with the opposite radial gradient of PV. The vortex core gradually shrinks due to leaking of fluid near the separatrix in the PV distribution. However, the difference between the maximal azimuthal velocity, $V_{\theta m}$, as well as r_m, calculated from the semi-spectral model and from the simple model, presented in the table above, does not exceed 10%.

Conclusion

Decomposition into vortex and wave flows improves the understanding of their interaction. Planetary waves are generated by the vortex flow outside the core of closed isolines of potential vorticity. They produce a feedback resulting in the modification the vortex motion and as well as its structure.

For the long-time evolution of a strong vortex the dependence of the symmetric circulation on the meridional displacement is described by assuming zero azimuthal velocity at the core boundary. The model displays such characteristic features as shrinking of the vortex core, weakening of the vortex, decreasing size and developing of an annulus with opposite radial gradient of potential vorticity at the core boundary.

Numerical solutions using a semi-spectral model with only a few azimuthal modes agree well with the results of pseudo-spectral calculations with high resolution in two dimensions. Even for a rather weak initial vortex, the simple model of symmetric circulation is capable of reproducing well the vortex decay depending on the meridional displacement.

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