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Scale-Dependent Merging of Baroclinic Vortices

Abstract

The influence of stratification on the merging of geostrophic vortices in a two-layer stratified flow is investigated by numerical simulations. The vortices are initialized in relative vorticity (RVI) or potential vorticity (PVI) in the upper layer. The strong influence of the stratification observed in the RVI case is interpreted in terms of competitive effects between repulsing hetonic interaction and attracting barotropic vortex shape influence.

Introduction

Vortex merger is seen as a prototype mechanism for the evolution of two-dimensional turbulence which gives rise to long-lived, intense coherent vortices (Basdevant *et al.*, 1981; McWilliams, 1984). The barotropic merging process has received considerable attention through experimental, theoretical and numerical investigations (Brown and Roshko, 1974; Overman and Zabusky, 1982; Melander *et al.*, 1988).

The case of rotating stratified fluid is even more complicated and, as yet, little is known and a forteriori understood. For the two-layer case, Griffiths and Hopfinger (1987) (hereafter GH) observed, in laboratory experiments, that the merging conditions were strongly dependent on the stratification. For the same configuration, however, Polvani, Zabusky and Flierl (1989) (hereafter PZF), using contour dynamics numerical simulations, found no dependency of the merging on the stratification. Verron *et al.* (1990) showed the importance of initial conditions in the merging process and introduced the concepts of "Relative Vorticity Initialization" (RVI) and "Potential Vorticity Initialization" (PVI) (see also Verron and Valcke, 1994).

In this paper, after an introduction to the quasigeostrophic model to be used, we present our results in the PVI case and in the RVI case. Then we show how the enhanced merging tendency for a selective range of vortex scales can be interpreted as a competition between an attracting barotropic shape effect and a repulsing "hetonic" interaction. Finally, on the base of observations of real eddies, we speculate on the applicability of each initialization scheme.

Quasigeostrophic model

A standard model for the two-layer vortex dynamics is provided by the following quasi-geostrophic set of equations for the potential vorticity Q_i (Pedlosky, 1979):

$$\frac{DQ_1}{Dt}=\frac{D}{Dt}\left[\omega_1+\frac{1}{2}\lambda^{-2}(\psi_2-\psi_1)\right]=V_1 \qquad (1a)$$

$$\frac{DQ_2}{Dt}=\frac{D}{Dt}\left[\omega_2-\frac{1}{2}\lambda^{-2}(\psi_2-\psi_1)\right]=V_2 \qquad (1b)$$

The subscript 1 indicates the upper layer, 2 the bottom layer. ω_i is the relative vorticity and ψ_i the stream function. The internal Rossby radius λ represents the stratification and writes $\lambda=\sqrt{g'H}/(\sqrt{2}f_0)$ when the two layers are of equal depth H. The Coriolis parameter is supposed to be a constant f_0. g' measures the density jump, $\Delta\rho$, between the two layers such that $g'=g\Delta\rho/\rho$ where ρ is the reference mean density. The dissipative terms V_i are needed, in our finite-difference numerical code, to dissipate the enstrophy which tends to accumulate at the small scales not resolved by the model. They take the form of a high-order viscosity term $V_i=-A_4\nabla^4\omega_i$.

Potential Vorticity Initialization

If one considers the upper vortices as equal circular patches of uniform potential vorticity Ω, i.e. having a profile of the Rankine type (designated R_a), the initial conditions for the set of equations (1) may be written as follows:

$$Q_{10}=\sum_{k=1,2} R_a^k \qquad (2a)$$

$$Q_{20}=0 \qquad (2b)$$

The summation $k=1, 2$ on R_a^k indicates a pair of Rankine vortices. Their radius is denoted R and their initial distance centre to centre is d. This provides us with the so-called Potential Vorticity Initialization (PVI).

In this case, PZF found that the critical merging distance, d_c, which is the greater distance between the vortex centres below which the vortices merge, was insensitive to stratification. In our study, however, further consideration of viscous effects led us to conclude that the concept of the critical merging distance is sometimes misleading and/or inapplicable. Because of the small numerical viscosity A_4 introduced in the model, it was observed that vortices eventually merge for all initial separations d.

Additional simulations with different values of A_4 showed that for $d/R\leqslant 3.3$ the value of A_4 has little influence on t_c/T, the time of merging non-dimensionalized by the vortex turnover period, $T=2\pi/\Omega$, which is always less than 5. These mergings are real "convective" mergings. For $d/R \geqslant 3.6$, t_c/T increases drastically and is strongly dependent on the value of A_4. On a convective timescale, the vortices are stable and the merging is induced by the viscosity on a longer viscous timescale. In these cases, we classified the interaction as non-merging from an inviscid point of view.

Taking into account the temporal dimension of merging evolution, the critical distance of merging can therefore be understood as the boundary between merging and no merging on a convective timescale. However, we have found that in practice this distinction is difficult to make. For this reason, we have decided to consider the time required for merging, t_c/T, as the relevant vortex interaction factor.

Concerning the influence of the stratification, our simulations showed that, in the PVI case, t_c/T is independent of the stratification. This result is analogous to PZF's conclusion about an independent critical distance of merging.

Relative Vorticity Initialization

If we now assume that the bottom layer is at rest and that the initial vortices are defined in relative vorticity as two Rankine profiles of radius R in the upper layer, the initial flow fields will be $\psi_{20}=0$ and ψ_{10} given by

$$\nabla^2\psi_{10} = \sum_{k=1,2} R_a^k$$

Consequently, the initial potential vorticity in the two layers is

$$Q_{10} = \sum_{k=1,2} R_a^k - \frac{1}{2}\lambda^{-2}\psi_{10} \tag{3a}$$

$$Q_{20} = +\frac{1}{2}\lambda^{-2}\psi_{10} \tag{3b}$$

This is the Relative Vorticity Initialization (RVI) as introduced by Verron *et al.* (1990).

Figure 1 shows the Q_{10} profiles. These profiles are dependent on the background stratification expressed by λ/R. In particular, we see that as λ/R diminishes, the vortex acquires an increasingly pronounced "skirt" shape because of the vorticity stretching term, $1/2 \cdot \lambda^{-2}\psi_{10}$, coupling the two layers.

In the RVI case, merging was found to be strongly dependent on background stratification, as can be seen in Figure 2. It shows the isocurves for the time of merging $t_c/T = 5$, 15, 30, 45 and ∞ as a function of the initial distance d/R and of the stratification parameter λ/R.

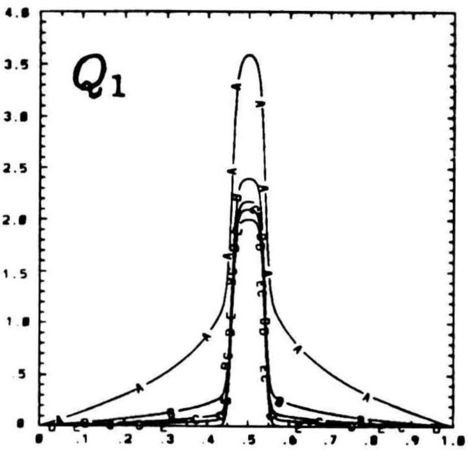

Fig. 1. Initial upper-layer potential vorticity (s^{-1}) in the RVI case for $\lambda/R = 1$ (A), 2 (B), 3 (C), 4 (D), ∞ (E).

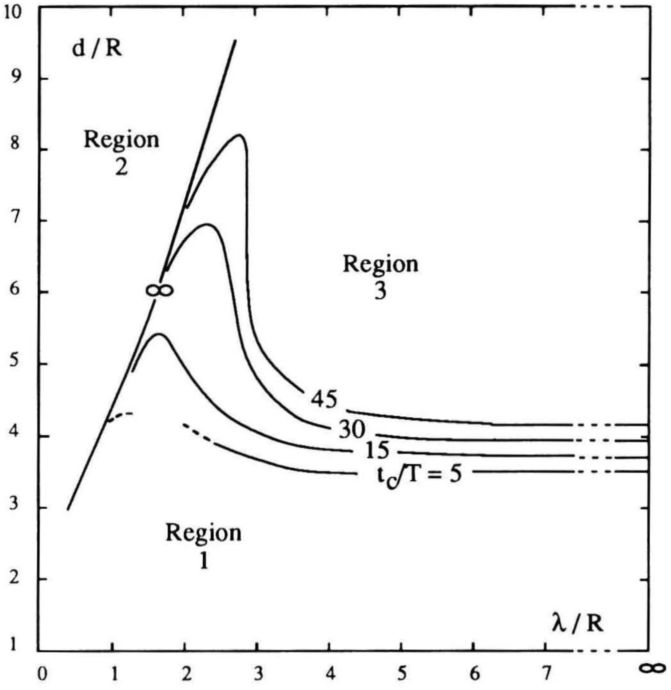

Fig. 2. Isocurves of t_c/T for the RVI simulations with $A_4/\Omega R^4 = 10^{-4}$.

Three regimes of vortex interaction may be roughly identified. Region 1 corresponds to where merging occurs on a convective timescale. The interesting point is that a marked peak subregion can be identified where merging occurs in a much shorter time than is the case for other values of λ/R. For example, for an initial distance d/R as large as 5.5, and when $\lambda/R \simeq 1.7$, merging can be obtained after a time t_c/T of around only 15, while it is 3 times longer when stratification parameter is $\lambda/R = 3.2$. Merging is therefore very much favoured for a restricted range of λ/R, between around 1.5 and 3. In Region 2, on the other hand, the vortices do not merge and the distance separating them increases with time. The merging time, t_c/T, was therefore set at ∞. In Region 3, the vortices are stable on a convective timescale.

The appearance of a peak in Region 1 and of inhibited merging in Region 2 is an important feature of vortex interaction in the RVI case. Unlike the PVI situation, stratification appears to have a marked influence on merging here. In the next sections, we propose an explanation to this phenomenon as a competitive process between attracting barotropic shape effect and repulsing hetonic effect.

Barotropic shape effect

Let us first assume, in the case of strong stratification, that the dynamical effect of the stretching term is negligible,

$$\frac{D}{Dt}\left[\frac{1}{2}\lambda^{-2}(\psi_2 - \psi_1)\right] << \frac{D\omega_1}{Dt}$$

Equations (1) will reduce to

$$\frac{DQ_i}{Dt} = \frac{D\omega_i}{Dt} = 0$$

still assuming the same initial conditions (3).

Note that the term $1/2 \cdot \lambda^{-2}\psi_{10}$, corresponding to the initial interface deformation, is kept in the initial definition of the potential vorticity profile in the two layers, but it is assumed to play no further role on the dynamics. The merging problem for the upper layer reduces to a purely barotropic problem in which the initial profile of vorticity is subject to the above initial conditions (3a). The distinction between relative and potential vorticity has no further significance. Since stratification is no longer dynamically present, λ/R is no longer a measure of the Rossby radius but it becomes simply an initial vortex shape factor.

We studied the barotropic merging problem for the upper layer as the initial vortex shape varies as in (3a). The results are presented in Figure 3. In Region 1, the vortices merge rapidly on a convective timescale and in Region 3, they do not, as before. For each particular value of d/R, t_c/T decreases as λ/R decreases.

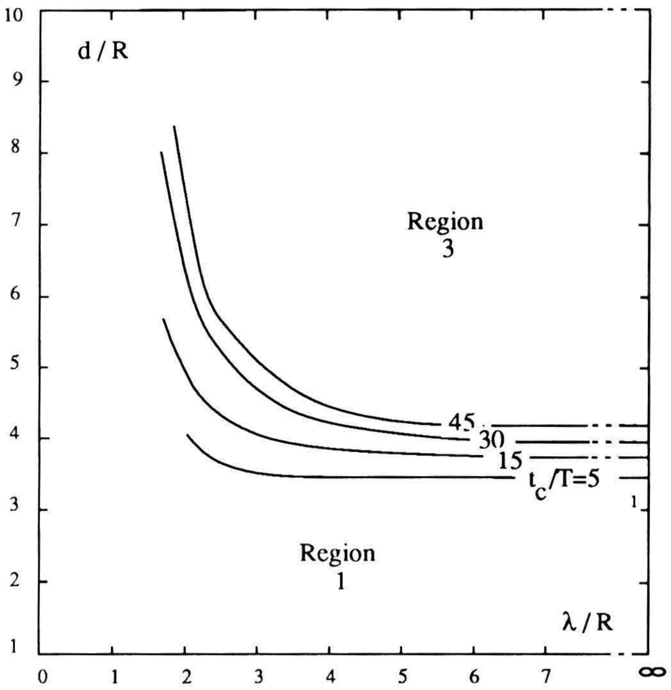

Fig. 3. Isocurves for $t_c/T=5$, 15, 30, 45 as a function of d/R and of the shape factor λ/R for barotropic simulations with $A_4/\Omega R^4=10^{-4}$.

We may therefore conclude that the more pronounced the "skirt" shape is, the greater is the tendency for vortices to merge. This is not really surprising if one considers that the influence of the vortices on each other increases as the vorticity profile extends from their core.

The interesting point is that the isocurves for $t_c/R=5$, 15, 30, 45 in Figures 2 and 3 coincide with one another almost perfectly for $3.2<\lambda/R<\infty$ and, in both cases, the tendency to merge continues to increase strongly. This means that, in the RVI case, the pure effect of the shape of the upper-layer vortices is likely to be the principal reason for the increased tendency to merge when λ/R decreases from ∞ to about 3, but no significant baroclinic dynamics is subsequently required.

Heton interaction

Let us now consider the two-layer system in the RVI case for small values of λ/R. In the expression of the RVI initial potential vorticity (3), the stretching term $\lambda^{-2}\psi_{10}$ is now relatively large. For each vortex initialized in the upper layer, there is a corresponding one of opposite sign in the lower layer associated with this stretching term. The dynamical effect of these lower-layer vortices can no

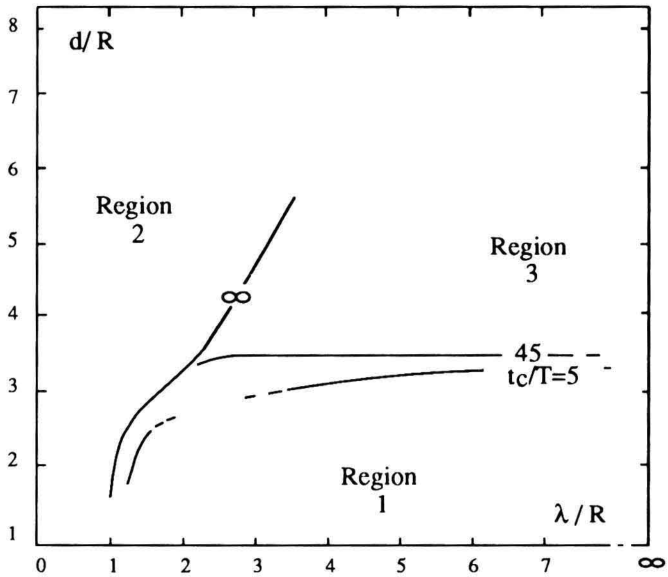

Fig. 4. Isocurves for $t_c/T = 5$, 45 and ∞ as a function of d/R and λ/R for finite-core hetons with $A_4/\Omega R^4 = 2 \cdot 10^{-5}$.

longer be neglected. The RVI situation becomes equivalent to the initialization of two vortices in the upper layer and two vortices of opposite sign in the lower layer.

This is analogous to an initial configuration of two hetons as described by Hogg and Stommel (1985) (hereafter HS). HS originally defined a heton as a pair of opposite-sign point vortices in opposite layers. In the present case, the major difference is that the vortices have a finite core with specific shape defined by equations (3) instead of being point vortices.

To understand better the concept of hetons, we investigated the interactions between two equal finite-core hetons in which all vortices (positive or negative) are of the Rankine type and have same intensity. The results of this particular study, shown on Figure 4, are presented with more details in Valcke and Verron (1993).

In Region 1, the vortices merge on a convective timescale, while in Region 3 they do not. In Region 2, the behaviour of the vortices is typical of a hetonic configuration. Initially, each heton is split by the other, but, once the two vortices of each heton are horizontally separated, the main coupling occurs across the interface between these two vortices of opposite-sign. The hetons begin to self-propel in opposite directions.

The most interesting conclusion concerns this Region 2: for $\lambda/R < 2$ approximately, the divergence motion of the hetons is strong enough to increase the distance between the same-layer vortices even when d/R is such that they would normally undergo convective merging. Their tendency to merge is therefore very much countered.

These results lead us to conclude that the divergence behaviour typical of Region 2 in the RVI case can be explained by the heton effect. For a weak stratification, the dynamical effect of the lower-layer vortices inhibits merging of the upper-layer vortices.

These considerations lead to the general conclusion that, as far as merging is concerned, the RVI behaviour, and in particular the increased tendency to merge for $1.5 < \lambda/R < 3$, is the result of two competing effects, one of *attraction* and one of *repulsion*. The attracting effect results from a pure barotropic shape influence and tends to promote merging as λ/R decreases because of the growth of the vorticity "skirt". The repulsing effect is caused by a heton-specific type of baroclinic interaction which, in a weakly stratified regime, tends to make the same-layer vortices diverge from each other. The peak area in the curves is the result of these competing effects.

Real vortices

Relatively little is known about the detailed structure of eddies in the ocean which, undoubtedly, have horizontal and vertical structures much more complicated than the simple representations discussed above.

A relevant question is now whether the "real" eddies have a signature in potential vorticity all along the vertical, as in the RVI case, or only at the surface (for example only above the thermocline), as in the PVI case.

It is first almost certain that the laboratories eddies created by GH are not totally PVI. In fact, their merging showed a strong dependency on the stratification which contradicts PVI results. However, it is not possible to assert that they are totally RVI even if their merging behaviour is qualitatively similar to our RVI results.

Observations by Olsen (1980) of Gulf Stream rings show that the ring can be clearly identified by an anomaly in potential density and potential vorticity in the area between the surface and about 1500 m. The interesting point here is that the velocity field, also clearly stronger above 1500 m, shows a reversal in sign below 1000 db. The author suggests that this feature, reminding one of the hetonic configuration associated with the RVI, could be common in the Gulf Stream cyclonic rings.

Arhan and Colin de Verdière (1985) presented a detailed analysis of field measurements taken during the Tourbillon Experiment in a region of the North East Atlantic. The relative vorticity of the eddy observed was of the same sign throughout the whole water column but was clearly intensified above the main pycnocline (850 m). The eddy potential vorticity seemed to be confined to the upper 1000 meters. Based on their findings, one is tempted to suggest that the eddy observed corresponds more to a PVI situation.

It is difficult, from the above examples, to reach a conclusion on the relevant "initialization" for real eddies, especially on their deep structure because of the lack of observations. However, it is likely that the physical mechanisms of eddy generation act, not only to induce potential vorticity anomalies above the thermocline, but also to alter the deep potential vorticity. Complex interactions may therefore results, from which PVI behaviour with regard to merging might be an illustrating prototype.

Acknowledgements

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