Frank M. Selten and J.D. Opsteegh

Toward an Optimal Description of Atmospheric Flow

Abstract

In models of the global atmospheric circulation, the vorticity field is usually expanded into series of theoretical orthogonal functions, so-called spherical harmonics. However, the atmosphere continuously generates coherent structures, which perhaps are better represented by Empirical Orthogonal Functions (EOFS) than by spherical harmonics. EOFS are calculated from observations and describe optimally fast the most energetic structures in the circulation. Therefore, we propose to project the vorticity equation onto the dominant EOFS. This approach is evaluated in the context of a barotropic model of the atmosphere.

Introduction

An unsolved problem in atmospheric modelling is how to properly describe the evolution of the turbulent atmosphere with a finite number of 'modes' or variables. It is inevitable that by using a finite number of modes the accuray of this description is limited. The richness of circulation structures is too large to ever allow for an exact representation. A description that is not exact introduces errors in the predicted evolution of the circulation. In practice, models of the atmosphere always have some kind of parametrization to represent the interaction between the neglected scales of motion and the resolved scales. How to properly account for this effect is known as the closure problem. It is clear that a particular closure formulation will depend on which circulation structures are resolved and which are not. In this context the choice of a proper basis in phase space may be extremely important.

The most widely used descriptions are the representation of the atmospheric fields on a particular grid or the expansion of the fields in spherical harmonics, the so-called spectral representation. All features smaller than the grid size or smaller than the smallest wavelength in the spectral expansion are neglected. One could ask whether these descriptions are optimal choices. The evolution of the large scale circulation is far from random. Over and over again the same kind of structures emerge and disappear. Rather than decomposing the flow field in spherical harmonics, a description of the flow in terms of these 'coherent' structures, could have important advantages. A localized coherent structure like a blocking ridge or a travelling cyclone projects onto many spherical harmonics, from the smallest to the very large wavenumbers. The more localized a structure, the flatter the corresponding Fourier spectrum and the more Fourier modes are involved. Since many modes are needed to describe one localized structure, the expansion is highly redundant. The spectral modes occur more or less in fixed combinations corresponding to the coherent structures of the flow. Also the interaction between two localized structures, say a blocking ridge and a travelling cyclone, is described by numerous interactions among the constituent spectral modes. Truncation of the expansion into spherical harmonics leads to a misrepresentation of this interaction. This error in the small scale details are of a large scale structure can not be described with the same parametrizations as the small scale coherent structures that play a role in cascading energy towards the viscous region. Finding a suitable closure formulation in spectral space is therefore far from trivial. A description based on the coherent structures of the atmospheric circulation could be far more efficient with respect to the number of variables needed and also might be more attractive with respect to the closure problem, as the small scales that are tied to the evolution of a large scale pattern are explicitly resolved.

How to construct such a description is far from clear. A suitable candidate is a spectral expansion in Empirical Orthogonal Functions (EOFs), since these functions optimally describe the most energetic structures in the circulation. In probability theory one refers to this expansion as the Proper Orthogonal Decomposition or Karhunen-Loève expansion. Lumley (1967, 1987) proposed its application to turbulent flows to identify coherent structures. For a review on its applications in turbulence research, see Aubry (1991). Preisendorfer (1988) contains an extensive overview of the theory and use of the EOFs in meteorology and oceanography. Several studies have been published in which models of the atmosphere are formulated in terms of EOFs (Rinne and Karhilla 1975; Schubert 1985, 1986). These studies are encouraging for the EOF modeling approach, but were restricted to the short-term behaviour of severely truncated EOF models. Furthermore, the EOFs were calculated from observations and subsequently inserted into a model of the atmosphere. The model tendencies differ from observed tendencies not only because the equations are not exactly solved due to the truncation error but also because the model equations are not exact. To investigate the effect of the truncation error, Selten (1993) did the entire analysis in the context of a model atmosphere. A two-level, quasi-geostrophic, hemispheric model was used, formulated in spherical harmonics and truncated at T5. The model consists of 30 coupled ordinary differential equations describing the evolution of the expansion coefficients. EOFs were calculated from a long model integration and evolution equations for the EOF amplitudes were derived by a Galerkin projection of the model equations onto the EOF-basis. The short and long-term behaviour of several EOF models were examined for various truncation limits. In agreement with the study by Rinne and Karhilla, he found that good short term predictions are possible with only a small number of EOFs (5 in his case). For a correct long term behaviour, at least 26 EOFs needed to be included. An attempt was made to parametrize the effect of the neglected components, but the results were rather disappointing. The main shortcoming of the T5 model was an unrealistic spectral energy distribution. The shortest wave was dominant and the wave amplitudes were too large compared to the zonal flow. Furthermore, the stationary wave component was too small compared to the travelling component. Therefore we decided to evaluate the EOF model approach in the context of a more realistic model with much more degrees of freedom.

We implement a hemispheric barotropic model with forcing and dissipation, formulated in spherical harmonics and truncated at T21. The number of degrees of freedom is 231. The model has a realistic climate and a variability comparable to the variability of observed 10-day running mean 500hPa height fields. The aim of the present study is to investigate whether a model based on the first few dominant EOFs can adequately describe the longterm dynamics of the T21 model. Further we want to study the potential for a proper closure of the reduced system in which small scale details of large scale structures are explicitly resolved.

The experiment

The barotropic model was integrated for 30 years and the statevector was archived once every day. This dataset can be depicted as a cloud of points in the 231-dimensional phase space. We refer to this set of points in phase space as the attractor of the model, since, starting from a random point in phase space, every trajectory after some finite time, traces this same set of points. The attractor is not an isotropic and homogeneous set of points. There are directions in phase space in which the model makes large excursions. These directions correspond to preferred circulation patterns. In some regions in phase space, the density of points has a local maximum. These regions correspond to regimes, quasi-stationary circulation patterns which persist for some time. EOFs are designed to describe optimally fast the anisotropic and inhomogeneous structure of the attractor. Suppose we project the set of points onto a unit vector in phase space. The first EOF points into the direction in phase space which maximizes the projection. The second EOF points into the direction in phase space which maximizes the projection under the restriction that it must be orthogonal to the first and so on. In this way, a new set of basis vectors is constructed which optimally fast describes the attractor. Note that this set depends on the choice of an inner product in phase space. The calculation of EOFs can be written in the form of an eigen problem. The EOFs are the eigenvectors of the covariance matrix and the corresponding eigenvalues are the mean squared projections onto the EOFs.

We calculated EOFs from the 30 year dataset. The EOFs are ordered with respect to their eigenvalues. The EOF with the largest eigenvalue comes first. The eigenvalue spectrum is steep, indicating the existence of preferred circulation patterns. The first eigenvalue is very large and the corresponding EOF is almost equal to the climate. The other EOFs have virtually zero mean amplitudes and thus describe deviations from the mean flow, represented by the first EOF. As explained in Selten (1993), the climate is retained in the expansion to allow for dynamical interactions between the climate component and the anomalous EOFs. It turns out that 99Z of the dataset projects onto the first EOF. The fractional mean squared error in the representation of anomalies is less than 5Z when the expansion includes the first 32 EOFs. Thus the state vector can efficiently be approximated by a spectral expansion into the first few dominant EOFs. A visual inspection of the EOF patterns reveals that as the eigenvalues become smaller, the scale of the structure tends to become smaller, more isotropic in the zonal direction and the patterns tend to shift toward the equator in the meridional direction. Having constructed a basis in phase space, evolution equations for the expansion coefficients are obtained by a Galerkin projection of the barotropic vorticity equation onto this basis. This procedure yields a coupled system of ordinary differential equations describing the evolution of the expansion coefficients. The general form of this system is given by

$$\frac{da_i}{dt} = \alpha_i + \beta_{ij} a_j + \gamma_{ijk} a_j a_k, \qquad i = 1, ..., T \tag{1}$$

Identical indices in a term imply a summation with the index running from 1 to the truncation limit T. The interaction coefficients are symmetric i.e. $\gamma_{ijk} = \gamma_{ikj}$. In case of spherical harmonics, the majority of the interaction coefficients, γ_{ijk}, are zero. For EOFs this is not the case. The derivation of the system (1) is unique and straightforward in case of spherical harmonics. When EOFs are used in the expansion, ambiguities arise in the derivation of (1). The reason is that spherical harmonics are eigenfunctions of the Laplace operator and EOFs are not. Choices have to be made in the derivation of the reduced system (1) which lead to different interaction coefficients and affect the integral constraints obeyed by the reduced system. It is not possible to derive a reduced system which conserves both energy and enstrophy. In this study a formulation is used which conserves only energy in the absence of forcing and dissipation. The behaviour of the EOF model (1) is evaluated at a truncation limit of 20 EOFs. The choice of this limit is rather arbitrary at this point. A thorough investigation of the behaviour as a function of the truncation is left for a future study. Both the ability to predict the evolution of the circulation of the T21 model two weeks ahead in time and the ability of simulating the T21 climatology and variability is investigated. The performance of EOF[20] model is compared with the performance of a T20 version of the T21 model. The T20 version still has 210 variables.

Results

It turned out that the mean prediction skill of the EOF[20] model of the 100 two-weeks forecasts was much better than the skill of the T20 model, both measured by anomaly correlations and RMS errors. The mean anomaly correlation of the T20 forecasts dropped below 0.6 at day 7, whereas the EOF[20] model is skilful upto day 11. Apparently, a truncation in spectral space as opposed to a truncation in EOF space, seriously affects the evolution of the dominant circulation structures.

To simulate the T21 climatology, the EOF[20] model and the T20 model were integrated for 3000 days. From the last 2000 days, the mean field, the variance pattern and the mean squared amplitudes (msa) of the EOFs were calculated. The EOF model showed a substantial climate drift and a too large variability with distinctly different spatial characteristics. The msa spectrum showed an almost white distribution. The explanation for this effect is the blocked flow of energy through the EOF spectrum due to the truncation. In the mean, energy enters the model through the first EOF, which is virtually the climatological mean circulation. Energy is subsequently transported through the spectrum and is most efficiently dissipated near the end of the spectrum due to the scale selective damping. By truncating the system, energy is accumulated near the truncation limit. Also the T20 model climate differed much from the T21 climate and although the variance had about the correct magnitude, the spatial distribution was quite different. Clearly, a closure assumption is needed to account for the systematic effect of the neglected interactions.

The proposition was made, that the mean effect of the neglected interactions is merely to damp the resolved modes and that it can be adequately described by a linear damping. An extra linear damping was introduced for each component to compensate for the accumulation of energy due to the neglected interactions. The extra damping balances exactly the mean error in the energy tendency of the reduced model. The strength of the extra damping was objectively calculated for each EOF and turned out to be almost the same for each EOF. The same closure assumption applied to the T20 model did not result in reasonable values for the extra damping. Both EOF models and the T20 model were integrated again with the inclusion of the extra damping. From the integration with the EOF[20] model the climatology, the variance pattern and the spectrum of msa of the EOFs were calculated. The correspondence with the T21 model was very good. The T20 model with the same closure assumption was not capable to simulate the T21 model climate and variance.

Conclusion

The main conclusion of this study is that by using EOFs as a Galerkin basis, a significant reduction in the number of variables is possible. The reason is that EOFs efficiently describe the dominant circulation patterns and that a closure is possible for the systematic effect of the neglected interactions. In a subsequent study it is tried to model the instantaneous effect of the neglected scales as well.

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Royal Netherlands Meteorological Institute
P.O. Box 201, 3730 AE De Bilt
The Netherlands