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Preliminary Measurements of the Distribution of the Velocity of a Fluid in the Immediate Neighbourhood of a Plane, Smooth Surface by J.M. Burgers and B.G. van der Hegge Zijnen – Revisited and Discussed –

Abstract

This is a brief historical excursion to revisit some hot-wire measurements in a flat-plate laminar-transitional-turbulent boundary layer which were performed before 1924. Using calculated skin-friction data, the measured mean velocity profiles are presented in inner-law coordinates and compare well with more recent measurements.

Introduction

When reading in the recently published "Selected papers" of J.M. Burgers (1995), I was intrigued by the hot-wire measurements performed by van der Hegge Zijnen and Burgers in the early twenties of this century. Since the data were presented in tables, it was easy to process them and to plot them in inner-law coordinates. These are the first hot-wire measurements ever taken in a boundary layer and, before discussing them, it seems to be appropriate to briefly set the "boundary layer scene" at Delft and in Europe at the time. J.M. Burgers was appointed as ordinary professor of "Aerodynamics Hydrodynamics and their application" at the Technical University of Delft at the age of 23 in 1918. His laboratory was functioning in the beginning of 1921 and so was an open return wind-tunnel of the Eiffel type with a test section of $4 \times 0.8 \times 0.8$ m (Fig. 1). In the same year B.G. van der Hegge Zijnen joined Burgers' group and probably designed and built the hot-wire probe and the CT-anemometer and performed the measurements in the boundary layer (van der Hegge Zijnen, 1924).

L. Prandtl had presented his boundary-layer theory to a small scientific community in 1904 and his doctoral student H. Blasius had solved the equations for the 2-D incompressible laminar boundary layer with zero pressure gradient in 1907. An important result was the Blasius skin-friction formula $c_f = 0.664/\sqrt{Re_x}$. Transition from a laminar to a turbulent boundary layer was probably discovered by Prandtl (1914) when he investigated the drag of spheres. Finally the principle of hot-wire measurements had been brought to general attention by L.V. King (1914). Burgers and van der Hegge Zijnen quote earlier

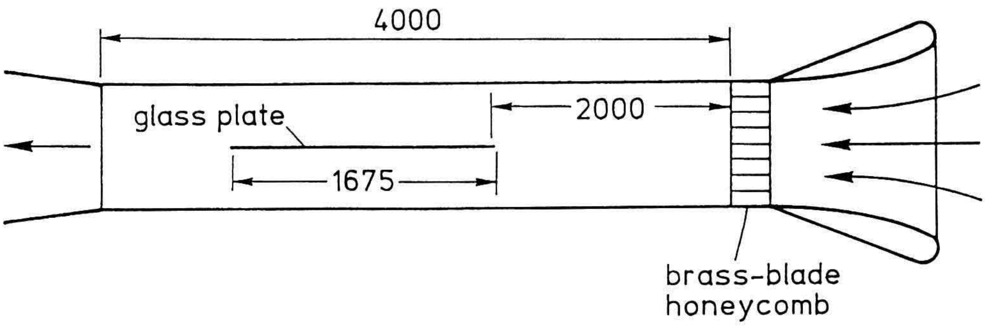

Figure 1: The closed test section of the Eiffel-type open windtunnel at Delft (length in mm).

measurements in a boundary layer than their own by Riabouchinsky (1914) "but the St. Petersburg group did not supply sufficient data for a detailed study of the flow in the boundary layer". So Burgers decided to investigate the problem in Delft and the goals were clearly stated:

- "obtaining data of the distribution of the velocity in the boundary layer;
- to ascertain if the upstream part of the boundary layer showed the laminar motion, corresponding to Blasius' formula;
- to obtain data on the change from the laminar state of motion to the turbulent one;
- to determine the gradient $(\partial \overline{u}/\partial y)_{y=0}$".

Where and how did they perform their measurements? The test plate was made of polished glass with a leading edge formed by two circular arcs (0.75 m radius of curvature). The plate did not span the tunnel width (0.40 m wide) and had a length of 1.675 m. The hot-wires were made of platinum-iridium with diameters of 200 to 15 μm and lengths between 21 and 29 mm. The wall distance was set by a micrometer screw and the near-wall distance was controlled by the wire/image method. The probes were calibrated in the irrotational flow against a Pitot-static tube. The freestream velocity u_∞ was kept constant at 8 m/s at each respective measuring station and the atmospheric pressure and the temperature were measured for each velocity profile. Only mean-velocity data are available.

Discussion of data

Burgers & van der Hegge Zijnen could not measure the skin friction and so determined it from the wall slope of the mean-velocity profile. These data (•) are

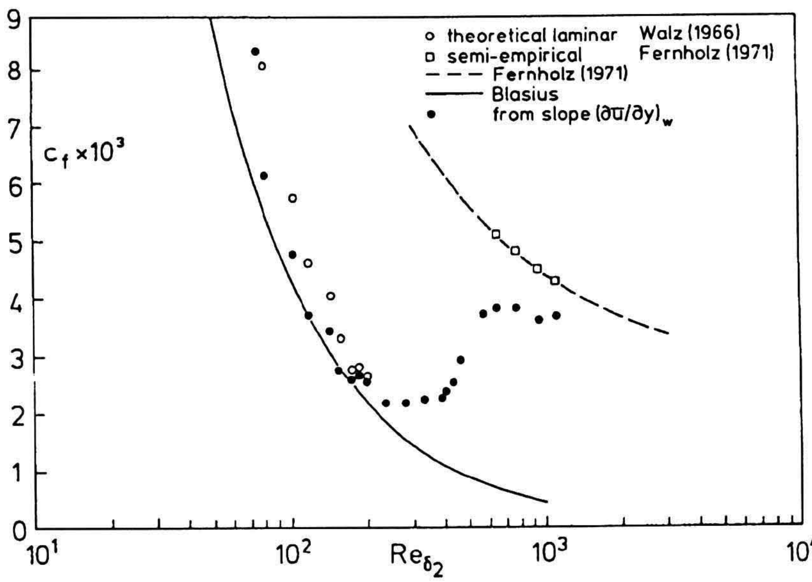

Figure 2: Comparison of skin-friction data determined from the wall slope $(\partial\overline{u}/\partial y)_w$ and from semi-empirical relations in a zero-pressure gradient 2-D boundary layer. Data from Burgers & van der Hegge Zijnen (1924).

plotted as $c_f = 2\overline{\tau}_w/\rho U_\infty^2$ against $Re_{\delta_2} = U_\infty \delta_2/\nu$ in Fig. 2 where δ_2 is the momentum loss thickness, $\overline{\tau}_w$ the mean skin friction and U_∞ the free stream velocity. They are compared in the upstream region with the skin-friction formula of Blasius for a zero-pressure gradient laminar boundary layer. They were also recalculated from the velocity profiles (o) by the skin-friction formula of Walz (1966) which is of the form

$$c_f = \frac{3.452}{Re_{\delta_2}}(H_{32} - 1.515)^{0.716}, \tag{1}$$

and thus can take account of the pressure gradient dp/dx in the streamwise direction. Since the data lie above the curve of Blasius a favourable pressure gradient must have been present (this is also discussed by Hansen, 1928). There is still no reliable calculation method for the skin friction in a transitional boundary layer; so we can only show Burgers' "slope data".

The downstream boundary layer is apparently turbulent and the "slope method" becomes rather inaccurate. Skin friction (⊗) was therefore also determined from a semi-empirical relationship (Fernholz, 1971) and compared with the Delft data which are definitely low. The skin-friction data, determined as given in the legend of Fig. 3, were used to plot the mean-velocity profiles in inner-law scaling in Fig. 3. It is astonishing how well the measurements agree with the linear law in the viscous sublayer and how they show the typical be-

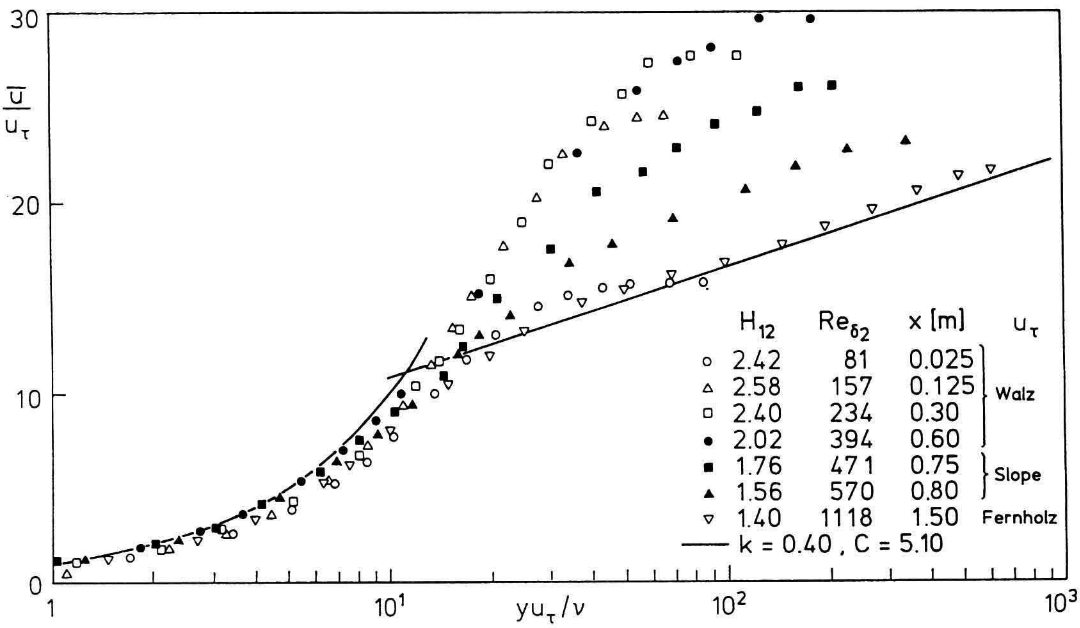

Figure 3: Mean velocity profiles in a zero-pressure-gradient 2-D boundary layer in inner-law scaling. Data from Burgers & van der Hegge Zijnen (1924).

haviour in a transitional and for the last profile (∇) in a turbulent boundary layer (e.g. following the logarithmic law of the wall). Fig. 4 presents the development of the relevant boundary-layer parameters, such as the shape parameter H_{12}, the Reynolds number Re_{δ_2}, and the skin-friction parameter c_f against the streamwise distance x. The different flow regimes can be determined clearly but again it seems to be appropriate to quote Burgers and van der Hegge Zijnen (1924) on the transition regime:

> "... the character of the motion changes at $x = 0.75$ m. ... The conclusion is allowed that here the two manners of motion are present at the same time: the laminar one at $x < 0.75$ m, the turbulent one at $x > 0.75$ m. .... It was observed that the needles both of the galvanometer and of the ammeter were very quiet during the measurements at the former part of the glassplate ... . The fluctuations showed a maximum in the region of transition; in this region they were so intense, that they could be observed by the eye from the glittering of the wire. Probably this phenomenon has to be ascribed to the formation of large vortices or irregular waves, which mark the breaking down of the laminar motion. ... These fluctuations gave the greatest disturbances for values of y between about 0.25 and 1 mm" (corresponding with $y^+ = 5.8$ and 23 at $x = 0.80$ m which is very close to the critical layer).

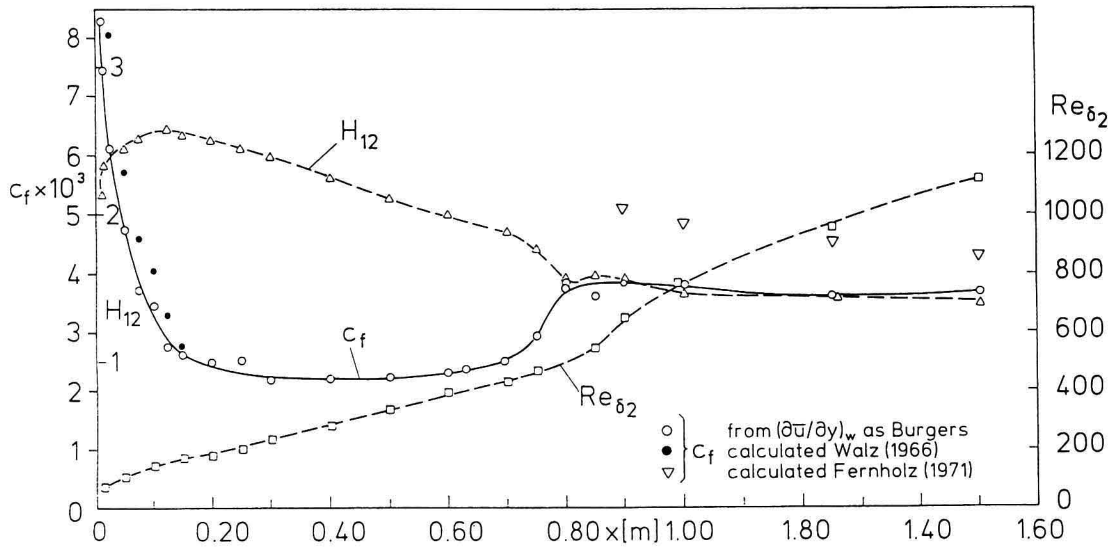

Figure 4: Streamwise development of the Reynolds number Re_{δ_2}, the shape parameter H_{12} and the skin-friction coefficient c_f in a zero-pressure-gradient 2-D boundary layer. Data from Burgers & van der Hegge Zijnen (1924) (lines are for visual aid only).

The first transition diagram $\delta/(\nu x/U_\infty)^{1/2}$ against Re_x was presented by Hansen (Aachen, 1928) who also used and discussed the measurements from Delft.

Finally one should mention some conclusions of Burgers and van der Hegge Zijnen about the velocity distribution and about transition:

i) For values of $y < 0.2$ mm the mean velocity u becomes approximately a linear function of y.

ii) In the turbulent part values of the mean velocity u can be represented for values of $y > 1.25$ mm by a power law

$$u/u_\infty = (y/\delta)^{1/7} . \tag{2}$$

iii) The region of transition marks itself by an inflection of the curve of the boundary layer thickness δ.

iv) The transition from the laminar state of motion to the turbulent one is influenced by the magnitude of the disturbance of the air current.

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