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Subharmonic K-Regime of Boundary-Layer Breakdown

Abstract

The paper is devoted to an experimental study of the nonlinear stages of the laminar-turbulent transition of a 2-D boundary-layer flow close to the Blasius flow. The possibility of a realization of some phenomena typical for the K-regime of boundary-layer transition initiated by subharmonic resonance at very late but still deterministic stages of the N-regime of transition are investigated. A novel disturbance source was used for the generation of instability waves of the necessary frequency and spanwise wavenumber spectrum. In the main part of the measurements (case I) the generator introduced a large-amplitude 2-D fundamental instability wave and a pair of low-amplitude oblique subharmonics with spanwise wavenumbers $\pm\beta_{1/2}$. The phase angle between the fundamental and the subharmonic waves was chosen to be favourable for the subharmonic resonance. In this case the transition process is found to begin with a rapid resonance growth of the subharmonic modes typical for the N-regime of transition. However at late stages of the disturbance development the local behaviour of the perturbations at the tips of the Λ-structures is very similar to the K-regime of breakdown with the formation of coherent structures associated with spikes in the time-traces of the hot-wire signal. A row of consecutive spikes appeared coinciding with the streamwise spacing of the subharmonic wavelength but their properties were found to be qualitatively the same as those usually observed in the K-regime.

Introduction

The nonlinear stage of the laminar-turbulent transition process in a boundary-layer is the last phase before the final randomisation and breakdown to turbulence. In contrast to the preceding linear stage many aspects of the last stage are still unclear. In a relatively short region where the disturbances reach amplitudes of the order of 1-2% of the free-stream velocity the flow transforms rapidly from a deterministic laminar flow into a stochastic turbulent one. Recent progress in the understanding of the nonlinear phase is associated with the recognition of the importance of resonance phenomena (for an overview see Kachanov, 1994).

Two main regimes of the 2-D boundary-layer breakdown have been identified. First the K-regime (after Klebanoff *et al.*, 1962) and second the N-regime (new or Novosibirsk). The former is induced by a fundamental wave with an amplitude modulation in spanwise direction leading to the formation of 'peaks' and 'valleys'

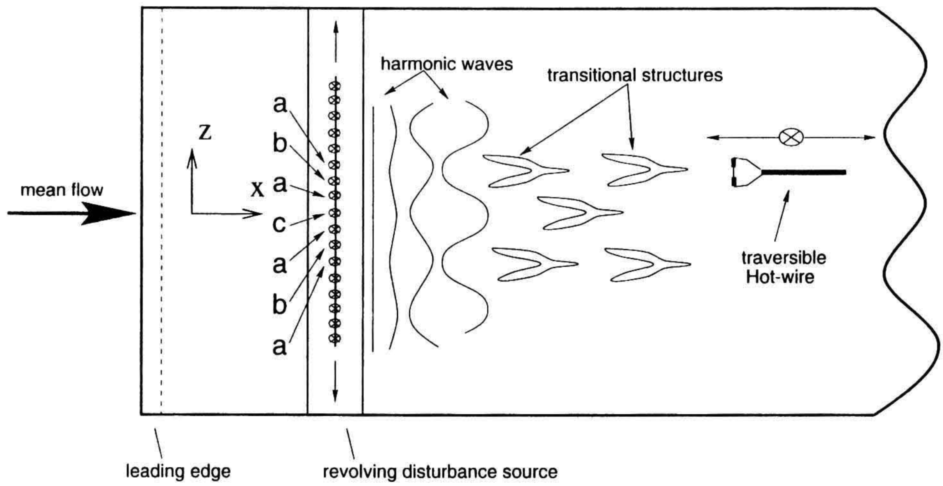

Figure 1: Sketch of the flow phenomena in the axisymmetric measuring section.

in the disturbance distribution and the formation of Λ-vortices in an aligned order (Saric *et al.*, 1984). The most characteristic feature of the K-regime is the appearance of spikes, i.e. of low speed flashes, at late stages attributed to ring-vortices snatched away from the tip of the Λ-vortices (see Hama & Nutant, 1963; Borodulin & Kachanov, 1993; Rist & Kachanov, 1995).

The N-regime is initiated by the interaction of a 2-D fundamental wave and a pair (or more) of oblique subharmonic waves (Kachanov *et al.*, 1977; Kachanov & Levchenko, 1984). The phase angle between them has to be favourable for a parametric resonance which leads to a rapid amplification of the subharmonic amplitude and to the formation of Λ-vortices in a staggered order. The breakdown of the Λ-vortices in the N-regime is characterised by a gradual broadband filling of the disturbance spectrum but the formation of spikes as in the K-regime has not been observed experimentally (e.g. Corke & Mangano, 1989). However, Laurien & Kleiser (1989) indicated the possibility that the breakdown of Λ-vortices in the N-regime can probably occur in the same way as in the K-regime if the initial amplitude of the subharmonic is high enough (for discussion see also Kachanov, 1994).

Experimental set-up and procedure

The experiment was conducted in the Laminar Wind Tunnel of the Hermann-Föttinger-Institute in Berlin. It is a closed-circuit tunnel with an axisymmetric test section made of Plexiglas tubes with an inner diameter of 441 mm and a total length of 6000 mm. The boundary-layer of the nozzle is blown out and the boundary-layer under investigation develops downstream at the elliptic leading edge of the test section. All measurements are made at the inner wall of the Plexiglas tube as illustrated in Fig. 1.

A ring shaped disturbance source (based on the experimental experience of Gaponenko & Kachanov, 1994) was inserted between the measurement sections to excite the flow. The source consists of a slit in the wall with 0.5 mm width and 260 mm length in spanwise direction. The slit is connected to loudspeakers via 32 plastic tubes with a spanwise spacing of 8 mm. Time-periodic volume fluctuations are produced by an excitation system consisting of a micro-computer, DA-converters, amplifiers and loudspeakers. The ring with the disturbance source can be rotated to vary the spanwise position (z-direction) relative to the hot-wire probe.

A constant-temperature hot-wire anemometer with linearizer was used to measure the streamwise component of the time-mean and fluctuation velocities U and u. The hot-wire of 5 μm diameter and 1 mm active length is mounted on an x-y traverse. The hot-wire signal is sampled and analysed with a Tektronix Fourier analyser in a way that phase locked ensemble averaged or instantaneous time-series triggered by the excitation signal were stored. The amplitude and phase values of the waves were determined by a Fourier transformation of the time series.

The free-stream velocity U_∞ was fixed at 7.2 m/s resulting in a Reynolds number of $Re_{\delta 1}$=785 at the position of the disturbance source (x=547 mm, $\Delta x = 0$ mm). The mean flow distribution $U(y)$ in the boundary-layer was close to that of a Blasius profile despite the slight favourable pressure gradient due to the growth of the boundary layer in the test section. The free-stream turbulence intensity was below Tu_∞=0.08% in a frequency range between 0.1 and 1000 Hz.

The boundary-layer was excited through the slit in the wall with a two-dimensional fundamental wave and/or a pair of three-dimensional oblique subharmonic waves. The fundamental frequency was f=62.5 Hz corresponding to a nondimensional frequency parameter $F = 2\pi f\nu/U_\infty^2 = 115.5 \cdot 10^{-6}$ at the position of the source. The subharmonic spanwise wavenumber was
$\beta_{1/2} = \pm 2\pi/32$ mm $= \pm 0.196$ rad/mm.

Four cases of excitation were investigated:

Case I : A 2-D fundamental instability wave with large amplitude A_1 and phase angle φ_1 and a pair of oblique subharmonics with low amplitudes $A_{1/2}$, phase angle $\varphi_{1/2}$ and spanwise wavenumber $\pm\beta_{1/2}$. The phase angle between the fundamental and the subharmonic waves was chosen to be favourable for subharmonic resonance according to earlier measurements (Kachanov & Levchenko, 1984).

Case II : Only the pair of oblique subharmonic waves with the same spanwise wavenumber $\pm\beta_{1/2}$ and the same low amplitude as in case I.

Case III : Only the 2-D fundamental wave with the same amplitude as in case I.

Case IV : The same as case I but with the phase angle between the fundamental and the subharmonic waves opposite to the resonant case.

The initial conditions at 50 mm downstream of the source are shown in Fig. 2.

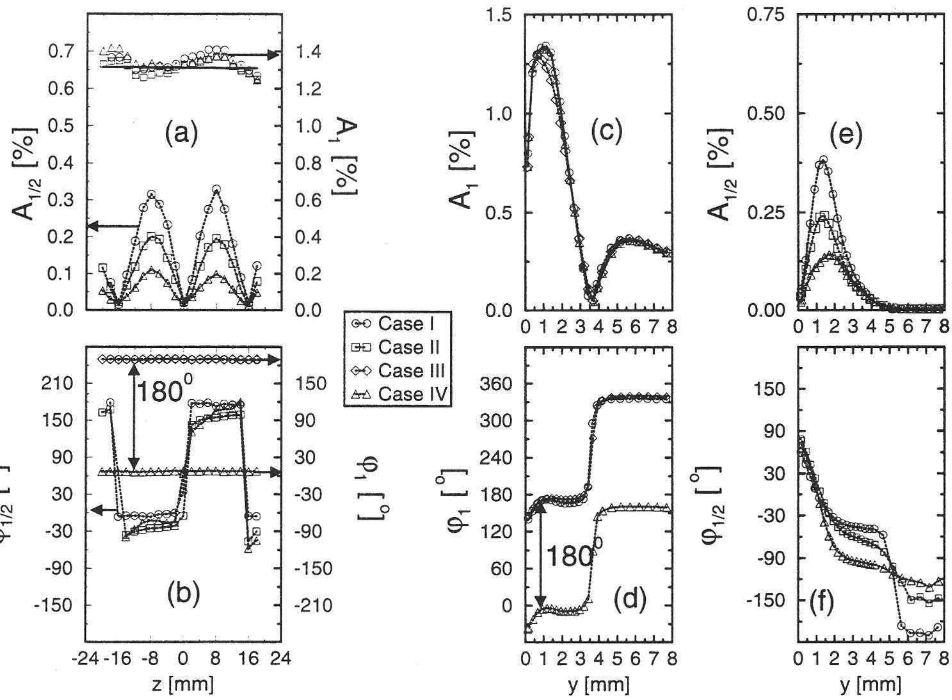

Figure 2: Initial Conditions of the excitation at 50 mm downstream of the source. (a),(b) spanwise distribution. Amplitude (c) and phase (d) of the fundamental wave. Amplitude (e) and phase (f) of the subharmonic wave.

Experimental results

The downstream development of the instability waves is shown in Fig. 3. In case I an exponential growth of the 3-D subharmonic waves was observed and the phase angles (calculated in degrees of the fundamental period) of the fundamental and the subharmonic waves show a synchronisation i.e. they have the same phase-velocity. This is the main condition for the existence of three-wave nonlinear resonance and in particular of the parametric resonant amplification (Craik, 1971; Herbert, 1984).

In case II the subharmonic amplitude decreases downstream according to linear stability theory (outside the neutral stability curve).

The fundamental amplitude in case III behaves in the same way as in case I and IV. This shows that it is not influenced by the nonlinear interaction with the subharmonic wave and plays only a 'catalytic' role in the whole process (e.g. Herbert, 1988; Kachanov, 1994).

In case IV when the initial phase shift between the fundamental and the subharmonic wave was introduced opposite to the resonant case (I) the amplitude of the subharmonic attenuates initially even faster than in case II. This is followed by a slight growth of the subharmonic amplitude probably because

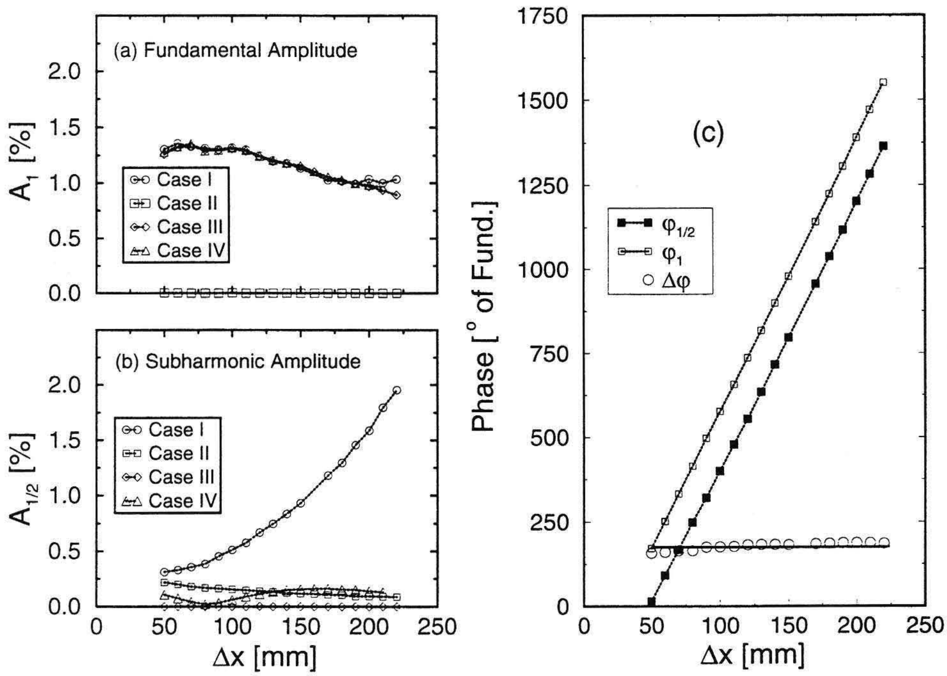

Figure 3: Downstream development of the instability waves at a spanwise position where the subharmonic amplitude has a maximum and at y/δ_1=0.7. (a), (b) Amplitudes. (c) Phase angles.

of an amplification of a very small but remaining resonant component of the subharmonics.

The resonant growth of the subharmonic waves results in an intensive spanwise modulation of the flow with formation of 'peaks' and 'valleys' with a spanwise distance $\pi/\beta_{1/2}$. In Fig. 4 the typical Λ-shaped disturbance structure can be seen in a staggered order characteristic of the N-regime or subharmonic type of boundary-layer breakdown. The Λ-vortices are inclined to the wall and show strong vorticity concentration in the "legs" and at the "tip". Thus until this stage of the transition development we find all typical attributes of the N-regime of breakdown which are well known from previous experimental and theoretical studies. At the same time, the subsequent development of disturbances turned out to be qualitatively the same as in the K-regime. In particular as shown in Fig. 5, in the peak positions the first, second, third and so on 'spike', and other phenomena typical for the K-regime (Klebanoff*et al.*, 1962) were observed further downstream. No significant differences in the disturbance behaviour near the peak position were found between this late stage of the N-breakdown and the corresponding stage of the K-breakdown except for the order of the Λ-structures and the frequency of their passing by. Consequently, we can speak of a "convergence" of mechanisms of the N- and K-breakdown at late stages of disturbance development, meaning a qualitative similarity of the local process near the planes of symmetry of the Λ-structures.

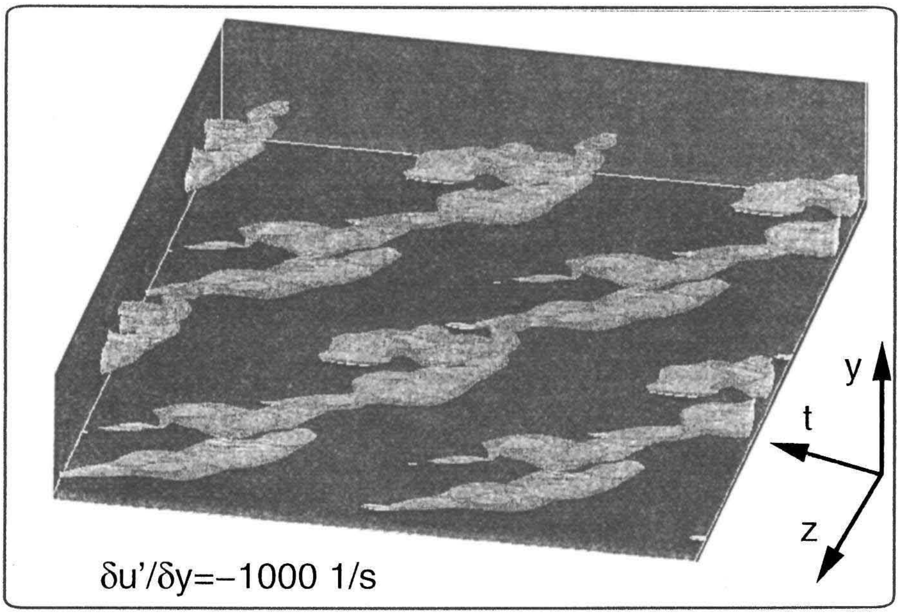

Figure 4: Iso-surfaces of one value of an estimate of the spanwise vorticity fluctuation $\delta u'/\delta y = \omega'_z$ over two spanwise and two streamwise periods of the subharmonic wave. Measured at Δx = 380 mm.

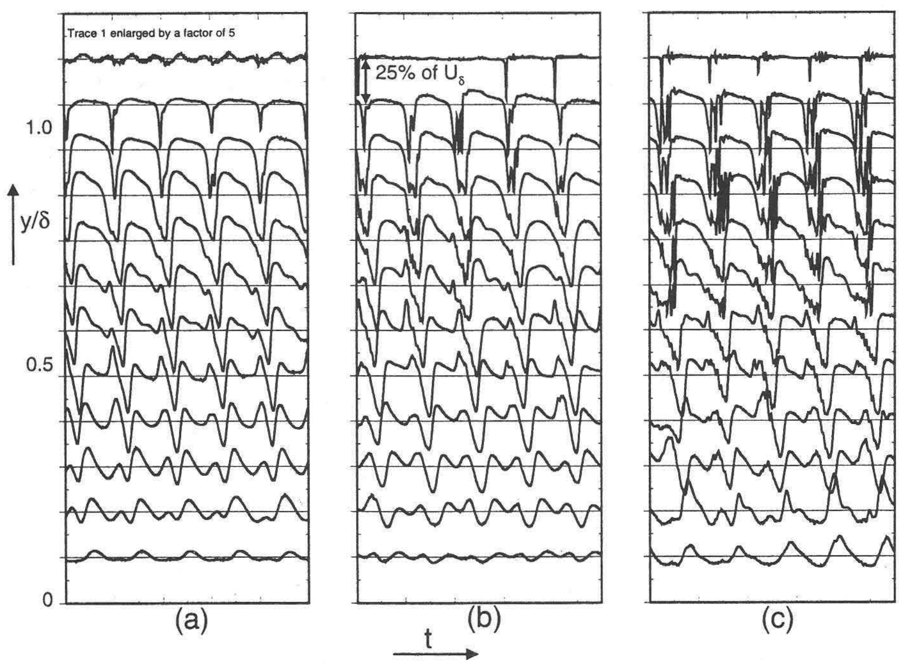

Figure 5: Time-series obtained far from the source at various distances from the wall in the peak position. Five periods of subharmonic frequency are shown. (a) Δx = 420 mm. (b) Δx = 440 mm. (c) Δx = 460 mm.

Conclusions

The N-regime of the boundary layer transition is reproduced under controlled disturbance conditions by the excitation of a 2-D fundamental wave and a pair of 3-D subharmonics. It is shown that introduced separately modes are stable or close to neutrally stable and transition (N-regime) occurs only when the conditions for subharmonic resonance are satisfied. In this case detailed hot-wire measurements show the formation of Λ-structures in staggered order with the subharmonic frequency as observed in other studies of the N-regime. However, the further disturbance development was found to be qualitatively the same as usually observed in the other regime of breakdown – the K-regime. That is namely the formation of typical Δ-wing shaped 3-D high-shear layers and, especially, the formation of a first, second, third, etc. spike in the peak position of the Λ-structures.

The results testify that at late stages of the N-breakdown local physical mechanisms of the nonlinear disturbance development are qualitatively the same as those characteristic of the K-regime of transition.

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