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Direct Numerical Simulation of Transitional Separated Fluid Flow Around a Circular Cylinder

Abstract

The transitional separated fluid flow around a circular cylinder is simulated numerically. The main idea of the simulation is based on the existence of large scale organized structures in the boundary layer and the near wake of the body and the possibility of modeling the dynamics of such structures using the complete Navier-Stokes equations without any turbulence models. The SMIF method is used, which is an efficient numerical method previously developed by the author with explicit hybrid non-oscillating (monotonic) finite difference scheme based on the combination of modified central and upwind difference schemes with a special switch condition and has second order accuracy and minimal scheme viscosity. It works in a wide range of Reynolds numbers. A comparison with experimental data for subcritical and critical Reynolds numbers is made.

Introduction

The transition from laminar to turbulent flow is one of the more important and less investigated phenomena in fluid dynamics. The difficulties of an experimental study of such regimes are connected with the complexity of the flow. The transitional separated fluid flow around finite bodies depends on many different factors such as the turbulence level and uniformity of the oncoming flow, the shape and smoothness of the surface of the body, etc. Therefore mathematical modeling may be one of the more effective approaches for such investigations. For the mathematical modeling it is necessary to develop an adequate physical and numerical model of the phenomena. In the past years more and more attention has been paid to the existence of an organized motion of large scale structures in laminar and turbulent boundary layers, in free shear layers, in jets, in the wake of finite bodies, etc. (Townsend, 1956; Cantwell, 1981). It is obvious that the construction of a universal model is impossible and each model (suitable for some classes of flows) should be based on the reasonable combination of determinism and chaos, possibly with taking the intermittency into account.

Our attempts are directed to the adequate reproduction of large scale (ordered, organized, coherent) structures, where the most part of the motion energy are concentrated. This investigation is based on the direct numerical simulation (without any turbulence models) using the complete Navier-Stokes equations to resolve the large scale structures (discrete model).

Foundation of the problem and numerical method

Let us consider the incompressible viscous fluid flow past a circular cylinder with radius R which is described by the Navier-Stokes equations. The transformation $x = r(z)cos\theta$, $y = r(z)sin\theta$ is used to resolve the boundary layer and the outer flow, where $r(z) = exp(z) - z - \frac{z^2}{2!} - \frac{z^3}{3!} + \eta z$ and $\eta = \eta(Re) = 4\sqrt{\frac{2}{Re}}$. Such transformation maps the investigated domain on the semistripe $\{(z, 0) : 0 \leq z \leq z_\infty, 0 \leq \theta \leq 2\pi\}$ and enables to use a finer spatial grid (with constant step size for z) in the boundary layer. Let u and v be the velocity components along z and θ respectively. The no-slip conditions on the rigid body are: u=0, v=0, and the velocities at "infinity"($z = z_\infty$) are $u = cos\theta$, $v = -sin\theta$.

The outer boundary z_∞ was taken as z_∞=5 (r_∞=105 R, where R is the radius of the cylinder). The number of grid points was 100 x 60 in z and θ directions respectively except for $Re = 10^6$ (100 x 120). So the number of the grid points in radial direction inside the boundary layer was between 10 and 15 for Reynolds numbers between 10^4 and 10^6. The influence on the solution of z_∞ and the form of the boundary conditions at infinity, the grid size and time steps were investigated in Gushchin (1985). For the calculation of fluid flows with large gradients of hydrodynamic parameters it is necessary to use an effective numerical method with finite difference schemes that have the following properties: high order of accuracy (second or higher), minimal numerical viscosity, monotony of the scheme, and applicable in a wide range of Reynolds numbers. The Splitting on physical factors Method for Incompressible Fluid flows -SMIF (Gushchin & Konshin, 1992) uses an explicit hybrid finite difference scheme which is based on the combination of a modified central difference scheme (MCDS) and a modified upwind difference scheme (MUDS) with a special switch condition and possesses all the properties mentioned above. The SMIF method was used for the calculation of 2D and 3D steady and unsteady, internal and external, homogeneous and nonhomogeneous, and also free surface fluid flows. The method is portable for convenient and parallel architectures.

The method was tested in calculations of steady and periodic separated fluid flows past a circular cylinder for small and moderate Reynolds numbers (Gushchin, 1985). The results are in a good agreement with experimental data and calculations of other authors.

Results

For the Reynolds numbers $10^2 < Re < 2 \cdot 10^3$ ($Re = \frac{U_\infty D}{\nu}$, where U_∞ is the velocity of a uniform flow at "infinity", D is the diameter of the cylinder, ν is the kinematic viscosity) the flow is periodic and laminar, but some secondary effects take place: the loss of stability in the separated boundary layer (Kelvin-Helmholtz instability in a free shear layer), secondary vortices near the body surface, and secondary separation in the vicinity of the separation point. The life time for each secondary effect arising twice per period is about one tenth of the main period.

Parameters	Experiment	Numerical results
Sh	0.179	0.18
L/D	4.22	3.5-4.0
$\overline{P}(\pi)$	1.019	1.0
θ_s	103°	95°
L_s/D	1.0	0.7
$\overline{C}_d$	1.237	1.0

Table 1: $Re = 1.4 \cdot 10^5$ Comparison with experimental data of Cantwell & Coles (1983).

For the Reynolds numbers $2 \cdot 10^3 < Re < 1.5 \cdot 10^5$ the characteristic features are the following:

- The flow is three dimensional.
- The flow in the boundary layer is unsteady but still laminar.
- Outside the boundary layer the flow is turbulent with large-scale structures in the wake.

Let us compare the numerical results with the experimental data of Cantwell & Coles (1983) for large subcritical Reynolds numbers ($Re = 1.4 \cdot 10^5$), where the boundary layer is still laminar. For this purpose we make some additional treatment of the obtained solution. Let $\overline{f}(x) = \frac{1}{kT}\int_{t_0}^{t_0+kT} f(x,t)dt$ be the time-mean over k periods T of the function $f(x,t)$ (velocity components, pressure, etc.), where t_0 is an arbitrary time moment in the well developed periodic flow regime. Then $f(x,t) = f(x,t) - \overline{f}(x)$ is the deviation (fluctuation) of the function $f(x,t)$ with respect to the time-mean value $\overline{f}(x)$. Time-averaging of periodic flows allows us to find time-mean flow patterns and flow characteristics (drag coefficients, angles of separation, length of the separated zone, pressure in the front and rear stagnation points of the cylinder), the amplitudes of these characteristics near the time-mean values and their dependence on the Reynolds number in the considered range of Reynolds number. Such mathematical treatment of the results allows us to calculate the second and higher order moments, and brings together computational and laboratory experiments. In Table 1 the quantitative comparison of some frequency, geometric, local and integral characteristics is shown. Here $Sh = \frac{D}{U_0 T}$ is the Strouhal number; l/D is the distance passed by the vortex during the period T; $\overline{P}(\pi)$ is the pressure in the front stagnation point; θ_s is the angle of separation; L_s/D is the length of the "recirculation" zone calculated from the rear critical point of the time-mean flow; and $\overline{C}_d$ is the total drag coefficient.

Table 2 compares the extreme values of some second moments (the energetic characteristics of the oscillating motion) and their coordinates in the investigated domain. Here only the periodic part is taken (in table 2 total values for the second moments from the experiment are shown in brackets, Cantwell & Coles,

Parameters	Experiment	Numerical results
$\overline{u'^2}$	0.085 (0.22)	0.075 (0.22)
x/D	1.0	1.0 (0.7)
y/D	± 0.45	± 0.45 (± 0.35)
$\overline{v'^2}$	0.25 (0.43)	0.22 (0.48)
x/D	1.7	1.6 (0.6)
y/D	0	0 (0)
$\overline{u'v'}$	± 0.055 (± 0.12)	± 0.07 (± 0.11)
x/D	1.3	1.3 (0.8)
y/D	± 0.4	± 0.35 (± 0.35)

Table 2: $Re = 1.4 \cdot 10^5$, comparison with experimental data of Cantwell & Coles (1983).

1983). There are a few local extrema in the investigated domain in the numerical calculations. In table 2 only one local extreme value is shown for the second moment, and in brackets the absolute extreme values are shown. As may be seen from tables 1 and 2 the numerical results and experimental data are in a good agreement.

Thus from the previous analysis the following conclusion may be drawn: the large scale structures in the near wake are adequately reproduced by our numerical solution of the Navier-Stokes equations provided sufficient resolution is used to represent the molecular mechanism, which is responsible for the separation from the surface of the cylinder. In the near wake this mechanism is not so important. It means that near the solid surface it is necessary to use a fine enough grid. This grid may be more coarse outside the boundary layer but it must be fine enough for the resolution of separated vortices.

This numerical approach was used to calculate the transitional separated regimes for critical Reynolds numbers between $2 \cdot 10^5$ and $5 \cdot 10^5$. Here it is advisable to use a finite-difference grid near the body, which allows us to reproduce the large scale vortex structures typical for the boundary layer.

It is well known that for large subcritical Reynolds numbers $Re < 2 \cdot 10^5$ laminar separation of the boundary layer takes place and the angle of separation is $\theta_l \approx 100°$ (calculated from the rear critical point). The turbulization of the separated boundary layer takes place in the near wake. At supercritical Re ($> 4 \cdot 10^5$) the separated boundary layer is turbulent. The angle of separation in this case is $\theta_t \approx 70°$. Both for laminar and turbulent separations the periodic motion with one main frequency takes place in the wake: for the laminar case the Strouhal number is Sh=0.2 and for the turbulent case Sh=0.28. In the last case this is probably connected with the decrease of the distance between the separation points, the vortices are shed more often and the wake becomes narrower.

At critical Reynolds numbers in the transitional regime the periodicity is absent, whereas in experiments wide range of frequencies is registered. The

Parameters	θ_l	θ_t	C_d	Sh
Experiment	90°	45°	0.48	0.5
Numerical results	93°	48°	0.32	0.42

Table 3: $Re = 4 \cdot 10^5$; Comparison with experimental data of Bychkov & Larichkin (1987)

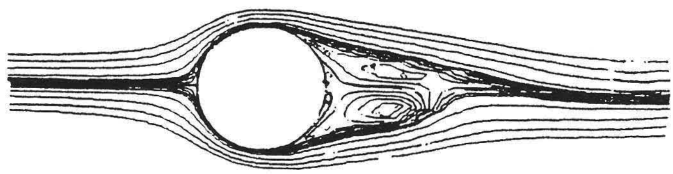

Figure 1: $\overline{\Psi} = const.$; $Re = 4 \cdot 10^5$.

existence of the bubble, its size and place may be one of the reasons for the non-periodic lift force. A flow with nonzero lift force coefficient is indeed possible (Schewe, 1986). The separated structures were observed in calculations at Reynolds numbers between $2 \cdot 10^5$ and $4 \cdot 10^5$. The time-averaged flow pattern ($\overline{\Psi}$= const, Ψ is the streamfunction) for $Re = 4 \cdot 10^5$ is shown in Fig. 1. The comparison of our numerical results with experimental data of Bychkov & Larichkin (1987) is given in Fig. 2 where the time-mean surface pressure $\overline{P}(\theta)$ is shown, and in Table 3. The presence of two "plateaus" in every semi-plane confirms the existence of two separation points on the cylinder surface as found in experiments. Here θ_l corresponds to the laminar separation point and θ_t to the turbulent separation point. There is a bubble between these two points. Curve 1 corresponds to the numerical results ($Re = 4 \cdot 10^5$), curve 2 is the laminar regime of the separation in the experiment of Fage & Falkner (1921) for $Re = 1.1 \cdot 10^5$, curve 3 is the turbulent regime in the experiment of Roshko (1961) for $Re = 8.4 \cdot 10^5$, and the circles are the transitional regime in the experiment of Bychkov & Larichkin (1987) for $Re = 4 \cdot 10^5$.

For a more detailed comparison of the spectral characteristics of the flow with experimental data of Farell & Blessmann (1983) the numerical calculations were used to investigate the spectral density of velocity fluctuations. In both calculations and experiment the dependence on time of the absolute value of the total velocity was recorded in four points in the vertical wake cross section located at $x = 2.07D$ from the centre of the cylinder. The points had different y coordinates. Then a spectral analysis of the time dependences of the absolute values of the velocity was made. On the diagrams depicted in Fig. 3 for different Reynolds numbers and different wake points the nondimensional frequency (Strouhal number) is along the abscissa axis and the spectral density of the velocity fluctuation $S(f)/\sigma$ is along the other axis. Here $S(f) = \overline{u'^2(f)}$ is the square of the velocity fluctuations for some frequency f and $\sigma = \overline{u'^2}$ is

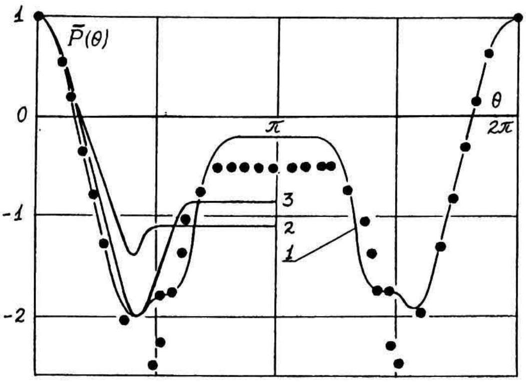

Figure 2: $\overline{P}(\theta)$-surface pressure.

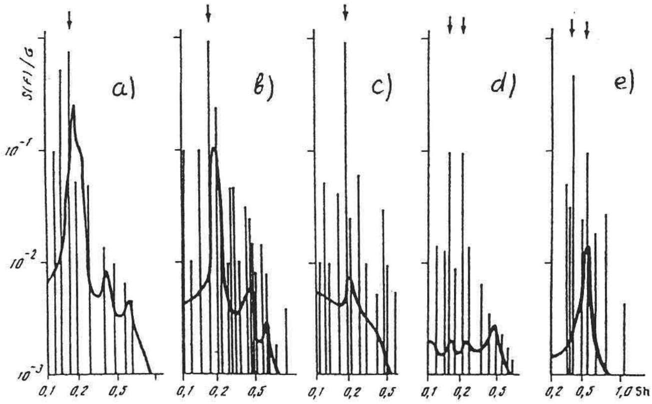

Figure 3: Spectral density of velocity fluctuation. a) $y = 0.57D$, $Re = 1.27 \times 10^5$, $Sh = 0.18$; b) $y = 0.57D$, $Re = 2.32 \times 10^5$, $Sh = 0.18$; c) $y = -0.5D$, $Re = 3.44 \times 10^5$, $Sh = 0.20$; d) $y = 0, Re = 3.74 \times 10^5$, $Sh = 0.34$, $Sh = 0.42$; e) $y = -0.2D$, $Re = 3.80 \times 10^5$, $Sh = 0.34$, $Sh = 0.42$.

the time-averaged of the squared velocity fluctuations. The experimental data are shown on diagrams by solid lines. The numerical results give us a discrete spectrum.

It is well known (and the same was observed in our calculations) that when the Reynolds number increases from 10^5 until $3 \cdot 10^5$ the Strouhal number increases from 0.18 until 0.20. When the Reynolds number increases (but $Re < Re_{cr}$) a sharp rise of Strouhal number takes place until 0.4 - 0.5. In the calculations at $Re = 3.8 \cdot 10^5$ the value $Sh = 0.42$ (Fig. 3e) was obtained. Moreover in the calculations at $Re = 3.74 \cdot 10^5$ and at $Re = 3.8 \cdot 10^5$ another additional frequency with $Sh = 0.34$ was observed (see Fig. 3d, e). The similar phenomenon was observed previously in some experiments (Bearman, 1969).

The dependences of time-mean total drag coefficient and of the Strouhal number Sh on the Reynolds number are shown in Figs 4 and 5, respectively.

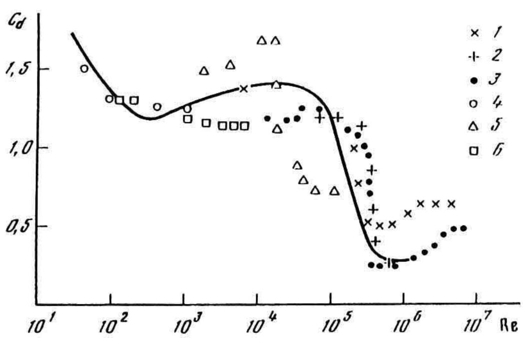

Figure 4: The dependence of the total drag coefficient $\overline{C}_d$ on the Reynolds number Re.

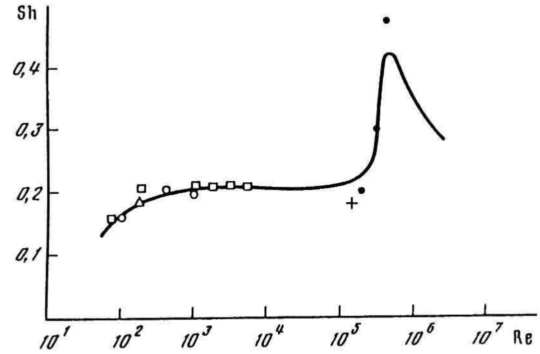

Figure 5: The dependence of the Strouhal number Sh on the Reynolds number Re.

The crisis of $\overline{C}_d$ and the sharp rise of the Strouhal number take place and are simulated numerically (without any turbulence models) for critical $Re = 4 \cdot 10^5$. As seen from Fig. 4 and 5 the numerical results denoted by the solid line are in a good agreement with experimental data (1 = Cantwell & Coles, 1983; 2 = Achenbach & Heinecke, 1981; 3 = Schewe, 1986) and calculations of other authors (4 = Jordan & Fromm, 1972; 5 = Kawamura *et al.*, 1986; 6 = Braza *et al.*, 1986). Moreover from the calculation the following formula is found for time-mean friction drag coefficient

$$\overline{C}_{d_f} = 1.879/(\log Re)^{2.58} ,$$

which coincides up to a constant multiplier with the well known Prandtl-Schlichting law (Schlichting, 1979) for a smooth flat plate for turbulent regime at $Re < 10^9$.

Conclusion

The direct numerical simulation of transitional separated fluid flow around a circular cylinder is based on the resolution of large ordered structures existing inside and outside the boundary layer. A spatial transformation of the radial coordinate was applied. The SMIF-method with explicit hybrid finite differences (second order accuracy, minimal scheme viscosity, workable in a wide range of Reynolds number, monotony) is used.

The obtained numerical results and comparison with experimental data confirm the applicability of the previously developed numerical approach for direct numerical simulation of transitional separated fluid flows.

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