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On Secondary Destabilisation of an Attachment Line Boundary Layer in Compressible Flow

Abstract

The generalised Hiemenz model is used to describe incompressible flow in the infinite swept attachment line boundary layer. This base flow is perturbed using local spatial LST and the resulting secondary instability problem is solved at high Reynolds numbers, extending earlier work. Issues raised by compressibility are discussed next; in the absence of a simplifying theoretical assumption, such as that of Görtler & Hämmerlin, we proceed to design a DNS in order to study the instability in question. The algorithm is outlined and first results on its successful application on a relevant compressible model problem are presented.

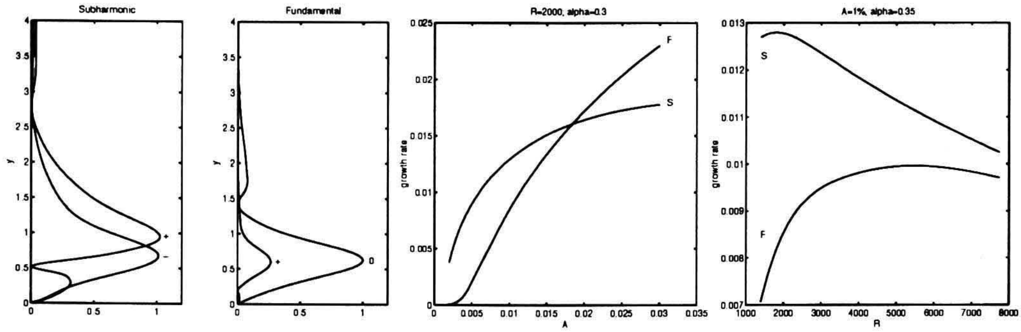
Introduction

The investigation into secondary stability was initiated as an attempt to explain the frequencies appearing in the spectrum, distinct from the linearly unstable ones, at conditions favouring linear growth (Poll, private communications; Poll *et al.*, this volume). Although not sharply defined as such, these frequencies appear in the parameter space region where harmonics of the primary linearly growing wave are expected. A secondary instability analysis is thus called for to shed light into this problem. Within the framework of the classic, in Blasius flow, Floquet analysis (Herbert, 1988) it is impossible to introduce three-dimensional streamwise (chordwise) periodic disturbances superimposed upon the linearly growing spanwise eigenmode in a theoretically self-consistent manner. The reason is, of course, the parity of the linear eigenfunctions in the attachment line problem; these have been assumed, in theoretical analyses, to inherit the symmetry of the base flow (Görtler-Hämmerlin assumption, henceforth referred to as GH) while the incompressible DNS of Spalart (1988) has demonstrated that this assumption may be justified in the linear regime. Invoking the GH assumption to study secondary instability in the attachment line, however, results in the elimination of the dependence of the system of equations on the streamwise coordinate which, in turn, prohibits the introduction of streamwise periodic disturbances in the present problem in a formal manner.

The inability to introduce streamwise periodic disturbances into the stagnation region, while being consistent with the currently available theoretical tools, does not preclude the actual presence of such waves in an experiment. The conjecture is then made that *if* present, streamwise periodic waves will amplify according to Floquet secondary theory. Further, use is made of the observation (Arnal, 1992; Theofilis & Poll, 1994) and theoretical prediction (Theofilis, 1995) that the classic Orr-Sommerfeld model is an adequate approximation to the infinite-swept attachment line boundary layer linear stability problem at high Reynolds numbers. Comparisons between results obtained using the set of assumptions exposed and experiments, currently performed, will test the validity of the present approach. The many analogies between attachment line flow and the Blasius boundary layer, alongside the success of Floquet theory in Blasius flow (Kachanov & Levchenko, 1984; Herbert, 1988) renders this model as the first candidate to be investigated.

In compressible flow the issue of attachment line instability is further complicated by the lack of information on experimental results which would validate the base flow (*e.g.* Reshotko & Beckwith, 1955) to be used at the stagnation region. Even less is available on the experimental validation of the various theories put forward for the linear stability of the compressible problem. From a theoretical point of view, one combines numerical solutions to the inviscid problem in the free-stream with first-order boundary layer theory near the wall, followed by linear analysis of the resulting profiles (*e.g.* Malik & Beckwith, 1988). It is well known, however, Mack (1984, for a review), that the linear stability results obtained using slightly different base flows can be profoundly different due to the presence of derivative terms in the stability equations. In our opinion it is of little practical importance to go even further and analyse the secondary stability of compressible attachment line boundary layer along these lines before the issues raised above are addressed in a satisfactory way.

In order to proceed we choose to embed the question of attachment line secondary instability within the framework of DNS. The design requirements for a DNS of the STAgnation Region (STAR) are stated and the potential of the algorithms used to meet successfully these requirements is demonstrated. Linear and nonlinear instability results are presented for a flow problem which exhibits inviscid instability, that of a free shear layer. Aside from the good documentation available there are more reasons for selecting this flow model for validation of the code. First, from a physical point of view, it is well known that the compressible flat-plate boundary layer flow is susceptible to inviscid instability through the action of the generalised inflection points developing in the base profile (Mack, 1984). While incompressible attachment line instability is viscous in nature, the analogies between the eigenvalue spectra of this flow and the Blasius boundary layer gives rise to the conjecture that inviscid instability at the attachment line itself will come in play in compressible flow. Second, crossflow instability, active in the stagnation *region* at all Mach numbers, is inviscid in nature (Reed & Saric, 1989). Finally, a purely practical reason exists in order to focus on the free shear layer, namely that best use of available computing resources can be made by developing the DNS on a model problem which exhibits instability associated with large growth rates.



(a) (b) (c)

Figure 1: (a) Spatial structure of secondary eigenfunctions; $\bar{R}$=2000, Branch II, A = 1%; The effect of primary amplitude (b) and Reynolds number (c) on the secondary growth rate.

Review of incompressible results

Extending the approach used by Theofilis & Poll (1994) for subharmonic instability to studying fundamental disturbances as well, we use spatial LST to monitor a linearly unstable primary wave developing along the spanwise direction, which becomes unstable to three-dimensional streamwise periodic disturbances. The full body of results of this study is presented by Theofilis (1996); here a summary is presented. The spatial structure of secondary eigenfunctions at a particular set of conditions is shown in Fig. 1a. It may be seen that the secondary perturbations possess a structure analogous to that of the secondary waves found in Blasius flow; most of the activity is confined within the boundary layer, with the characteristic double-peak appearing in the subharmonic perturbation.

The effect of primary amplitude and Reynolds number on the secondary growth rate is presented in Figs 1b and 1c respectively. A number of conclusions may be drawn from these results. First, it is seen that both types of secondary instability (and presumably also detuned modes, although not studied here) may be present. As the amplitude of the primary disturbance superimposed upon the base flow $A \rightarrow 0$ the secondary growth rates are also seen to approach zero, as the case is in Blasius flow. At low A subharmonic instability is seen to be more powerful a mechanism than its fundamental counterpart. As A grows, much as in the Blasius boundary layer, fundamental instability takes over; the cross-over point is found to be $A \approx 1.8\%$. For a fixed low value of A, on the other hand, increasing the Reynolds number $\bar{R}$ results in the two instability mechanisms approaching a single one. While $\bar{R} \rightarrow \infty$ studies have not been performed, the trend obtained suggests that large $\bar{R}$ secondary instability is inviscid in nature; this prediction awaits theoretical verification.

Issues raised by compressibility

Even in incompressible flow there is no theoretical justification for the use of the GH assumption in the late linear and early nonlinear stages, although it still produces optimal results compared to other approaches in the linear regime (Arnal, 1992, and this volume). With compressibility taken into account it is not possible to use a GH-type of approach any more. The coupling of flow variables through density results in terms explicitly dependent upon the streamwise (chordwise) coordinate x appearing in the equations.

A secondary stability study based on PSE (Bertolotti, 1991), on the other hand, is physically meaningful when considering mild growth of the boundary layer. In the attachment line problem this may only be the case in compressible flow along the spanwise direction; in the incompressible regime under the GH approximation the parallel boundary layer set up along the attachment line prohibits PSE from improving the results obtained by spatial LST. However, the questions raised regarding the formal introduction of the third dimension into the compressible problem remain.

Design requirements for a STAR DNS

Our approach to solve the theoretical problems discussed has been to use DNS for the stagnation region. The fundamental requirement for a STAR DNS is that it solves for the flow in the *vicinity* of an attachment line. Physically this implies inclusion of the currently little understood region of interaction between attachment line and crossflow instability. In so doing, DNS has the potential to provide initial conditions for the theoretical study of crossflow instability further downstream in the chordwise direction; these conditions appear to be essential for the convergence of PSE methods near the attachment line, as recently experienced in the ATTAS experiments (Stolte *et al.*, 1995).

From a numerical point of view, as with any DNS, the long-time integrations suggest use of low-dispersion, low-dissipation schemes for spatial differentiation. Schemes of this nature currently include spectral single- or multi-domain and Padé 3/4/6 or 5/6/5 compact finite-differences. The ability to interchange numerical differentiation schemes has been found to be useful for diagnostic purposes by Pruett *et al.* (1995). Time-integration may be performed by a member of the $O\left((\Delta t)^3\right)$ family of Runge-Kutta schemes derived by Wray (1986) which ensure optimal memory use, given that the time-step Δt is low because of spatial discretisation requirements and the CFL-related restriction.

Finally, the issue of the outflow boundaries in a STAR DNS has to be addressed. In the incompressible limit the generalised Hiemenz base flow ensures that a strictly parallel boundary layer is set up in the spanwise direction; this can be treated numerically as homogeneous and a Fourier expansion may be utilised in this direction. The streamwise direction, on the other hand, is one of strong acceleration of the flow; as such it lends itself to application of the spatial concept (Spalart, 1988). This may be accomplished by any of the techniques available, namely the sponge layer (Israeli & Orszag, 1981), the fringe technique (Spalart, 1988) and its windowing derivative model, the relaminarisation-zone technique (*e.g.* Eissler & Bestek, 1996) or the buffer domain approach (Streett & Macaraeg, 1989).

Application of the ideas discussed and assessment of their performance is presented next. It should be noted that the modular construction of the DNS code permits study of a variety of flow problems with minimal additional effort, concentrated mainly on the provision of base flow and the associated LST- or PSE-based initial conditions.

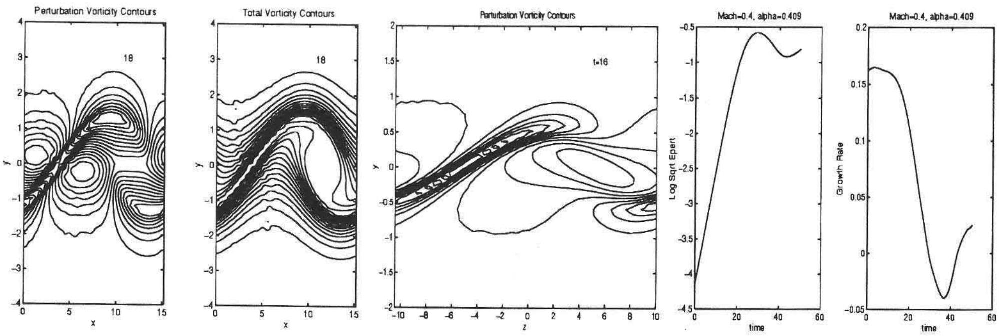
DNS validation results

The argumentation on the choice of the compressible free shear layer as the model base flow problem to validate the DNS approach has been presented earlier. Further, the Crocco-Busemann integral is utilised to obtain the base flow temperature pertinent to the hyperbolic tangent model for the base flow velocity; alternatively the Lock profile has also been used, or the temperature has been kept constant across the layer. The inviscid instability of this flow to three-dimensional linear disturbances is obtained by spectral collocation solution of the Lees-Lin system (Mack, 1984) and is fed as initial condition into the DNS at low amplitude.

The compressible three-dimensional Navier-Stokes equations are then marched in time with spatial derivatives in the streamwise x and spanwise z directions calculated using Fourier collocation (temporal approach) and those in the normal y direction using a choice of Chebyshev collocation or Padé 3/4/6 compact finite-differences; for the type of instability considered and the purposes of validation of the numerical techniques suffices to integrate only the Euler part of the equations. Characteristic non-reflecting boundary conditions are applied at the $|y| \to \infty$

Table 1: Convergence history for the reproduction of the 2- and 3-D LST result by DNS. A denotes amplitude of the superimposed perturbation.

Mach=0.4, α=0.409							
	ψ=0			$\psi = \pi/6$			
	LST	DNS				DNS	
		A=10^{-2}	A=10^{-6}	NY	(8, NY, 8)	(16,NY,16)	(32,NY,32)
NY	ω_i	ω_i	ω_i		ω_i	ω_i	ω_i
32	0.155332	0.156024	0.156028	32	0.130842	0.130379	0.130380
64	0.155301	0.155275	0.155277	64	0.130305	0.129809	0.129806
128	0.155301	0.155300	0.155301	128	0.129795	0.129961	0.129961
				256	0.129930	0.129948	0.129948



(a) (b) (c)

Figure 2: Nonlinear development of (a) 2D and (b) 3D unstable inviscid mode; (c) perturbation energy and growth rate evolution with time in the 2D simulation.

boundaries (Adams, 1992). The solution is obtained exclusively in real space, derivatives being calculated by straightforward matrix multiplication as opposed to the classic FFT; the former approach has been found to be of comparable efficiency to the latter for the grids and machine architecture utilised. Both the conservative form of the governing equations, as well as the pressure equation (Pruett *et al.*, 1995) have been solved with identical results obtained including the nonlinear stages.

The quality of the DNS solution in the linear regime is assessed by monitoring the reproduction of two- and three-dimensional linear growth rates. Such a comparison is presented in Table 1 as a function of the number of nodes NY utilised in the normal direction y. Both a two-dimensional ($\psi = tan^{-1}\left(\frac{\beta}{\alpha}\right) = 0$) and a three-dimensional linear perturbation are used to initialise the DNS. The former was used in order to monitor spurious growth of three-dimensionality in our code; the perturbation eigenvector was obtained on the same grid as that used for the DNS. By contrast, the three-dimensional mode was obtained on a grid different to that on which the DNS was performed and was transferred on the latter using piecewise cubic Hermite interpolation. The agreement between LST and DNS results for the 2D mode may be seen as typical of well-resolved DNS; no spurious three-dimensional growth was detected in this simulation. The interpolation procedure was proven to be responsible for the discrepancy between LST and DNS for the 3D mode; with the DNS performed on the grid on which the LST problem is solved agreement similar to that obtained for the 2D mode is achieved.

In Figs 2a and 2b the results of two simulations starting with the 2- and 3D modes as initial conditions, respectively, are presented. The nonlinear structures characterising the vortex roll-up at the late transitional stages may be seen. The wiggles present in both simulations are a result of attempting to solve the Euler equations without any form of artificial dissipation, and not of low resolution. In a corresponding Navier-Stokes calculation these wiggles disappear. Finally in Fig. 2c we present the evolution of a 2D mode through the linear stage (growth rate obtained by the slope of the function $ln(E_{\text{pert}})^{1/2}(t)$, E_{pert} being the perturbation energy) into nonlinear saturation at large t; at this time a well-resolved analogous three-dimensional calculation would have resulted into a turbulent state having been reached.

Conclusion

The theoretical difficulties in modelling secondary stability in the attachment line problem, combined with the lack of experimental support for a specific model of base flow and its primary stability, led us to design a DNS for the stagnation region. The numerical tools to be utilised have been validated on a compressible three-dimensional model problem, physically relevant to the flow at hand.

A simplified model has been proposed for secondary destabilisation of the corresponding incompressible flow; its results are currently being compared with recent experiments and, if validated, will be used to provide intuition in the parameter ranges to be monitored by DNS. Work in this area, as well as on the remaining issues regarding the STAR DNS described, namely buffer implementation as well as base flow and its LST is currently underway.

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