W.J. Solomon° G.J. Walker° & J.P. Gostelow▷

Transition Zone Predictions for Rapidly Varying Flows

Abstract

A new transitional flow model incorporating recent experimental data for the influence of free-stream pressure gradient on both the inception rate of turbulent spots and their subsequent spreading rate is reviewed. It gives good estimates of transition length for steady flows with marked spatial variations in free-stream conditions, and successfully predicts departures from the standard Narasimha intermittency distribution (or 'subtransitions') accompanying sudden changes in streamwise pressure gradient. A quasi-steady application of the new transition model is used in a preliminary attempt to describe the periodically unsteady boundary layer development during wake-induced transition on an axial compressor blade. Experimental observations show the relaxation of non-turbulent flow behind a wake-induced turbulent strip to be an important factor in turbomachine blade design; data for the length of relaxing flow from three independent transition experiments are compared.

Introduction

This paper is concerned with the prediction of transitional flow in cases where the boundary conditions change rapidly in space and/or time. The problem of rapid spatial variations is first examined. A new model making due allowance for the influence of pressure gradient on spot inception rate, and assuming spot spreading rate to be controlled by the local pressure gradient, is shown to give reasonable estimates of transition length under such varying conditions and to predict a priori the departures from Narasimha's standard intermittency distribution (or 'subtransitions') accompanying sudden changes in streamwise pressure gradient.

Transitional flow behaviour in the presence of rapid temporal variations is then examined with particular reference to the problem of periodic wake-induced transition observed on turbomachine blades. Some observations of wake-induced turbulent spot development on axial compressor blades are compared with wind tunnel measurements of triggered turbulent spots in a self-similar decelerating flow. An initial attempt to model the periodic flow on the compressor blade is made by applying the new model for spatially varying flow in a quasi-steady manner. The paper concludes by suggesting future directions for research.

Spatial variations – the influence of changing pressure gradient

Previous work

A comprehensive review of basic work on laminar-turbulent transition has been given by Narasimha (1985). The problem of modelling boundary layer development through the transition zone is examined in detail by Dey & Narasimha (1988), who also discuss the experimental data on transition published prior to that date. The early experimental surveys generally suffer from a lower level of accuracy and a lack of consistency in defining the extent of the transition zone.

The more recent survey by Gostelow *et al.* (1994), covering a wide range of pressure gradients and free-stream turbulence levels, has overcome this problem by fitting the universal distribution of Narasimha to measurements of intermittency γ within the transition zone. The fitted curve is then extrapolated to $\gamma = 0$ and $\gamma = 1$ to define the onset and completion of transition, respectively. This procedure has resulted in more reliable correlations for transition length, in terms of the boundary layer conditions at transition onset, which are presented in various forms by Gostelow *et al.* (1994).

It should be noted, however, that such simple correlations are only valid for cases in which the pressure gradient is slowly changing and conditions remain approximately similar through the transition zone. They may break down in cases such as the suction surface of an aft-loaded aerofoil, where transition may commence in a region of accelerating flow and end in a region of strong deceleration. As observed also by Narasimha *et al.* (1984), such sudden changes in the magnitude and sign of pressure gradient may cause marked deviations from the universal intermittency distribution (termed 'subtransitions').

Intermittency models for spatially varying flow

Narasimha *et al.* (1984) noted that a logarithmic plot of the intermittency distribution often exhibited a piecewise linear behaviour when subtransitions occurred. However, this does not provide a method of predicting the intermittency distribution a priori for flows with arbitrary pressure distributions.

The intermittency model of Chen & Thyson (1971), based on the turbulent spot theory of Emmons (1951), purports to describe the influence of changes in pressure gradient through the transition zone. However, as noted by Walker (1989) it only allows for the changes in turbulent spot convection rates with local changes in free-stream velocity. The more important effects of changing breakdown physics and spot spreading rates with streamwise pressure gradient variations are not accounted for.

Solomon *et al.* (1995) extended the Chen-Thyson model to incorporate the latter effects, using data of Gostelow *et al.* (1994) for the dimensionless spot breakdown rate $N = n\sigma\theta_t^3/\nu$ and data compiled by Gostelow *et al.* (1995) for the spot spreading half-angle α and spot propagation parameter σ, as shown in Fig. 1. A slight modification adopted here is that the spot breakdown rates for

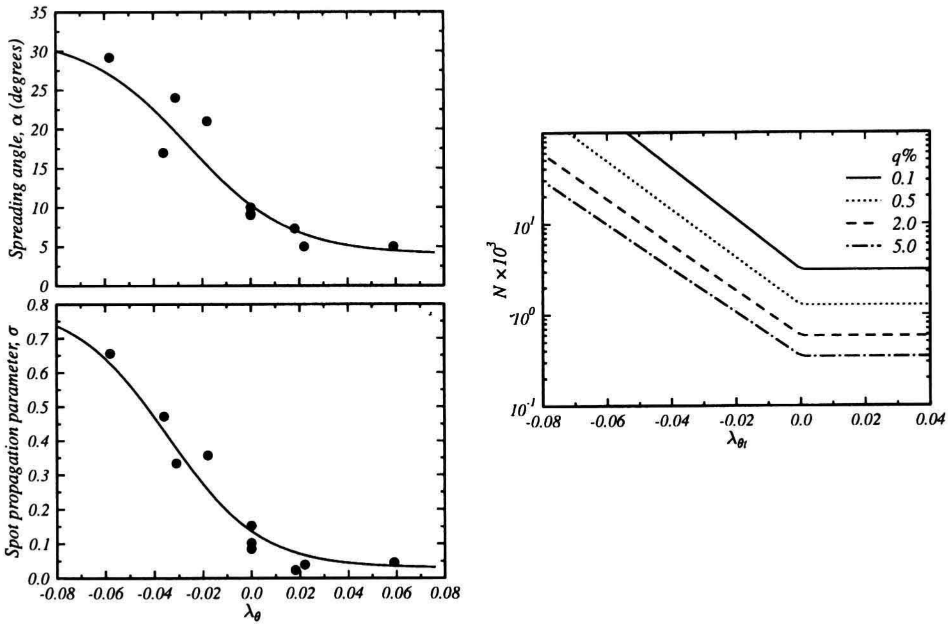

Figure 1: Correlations for spot spreading half angle α, spot propagation rate σ and generation rate N. Experimental data from various sources compiled by Gostelow *et al.* (1995).

negative pressure gradients have been assumed constant at their zero pressure gradient values. The latter change was made because the small amount of data available for accelerating flow made extrapolation dubious.

These data correlations are expressed analytically as

$$\sigma = 0.03 + (0.37/(0.48 + 3.0\exp(52.9\lambda_\theta))) \tag{1}$$

$$\alpha = 4 + (22.14/(0.79 + 2.72\exp(47.63\lambda_\theta))) \tag{2}$$

$$N = \begin{cases} 0.86 \times 10^{-3}\exp(2.134\lambda_\theta \ln(q) - 59.23\lambda_\theta - 0.564\ln(q)) & \text{if } \lambda_\theta \le 0 \\ 0.86 \times 10^{-3}\exp(-0.564\ln(q)) & \text{if } \lambda_\theta > 0 \end{cases} \tag{3}$$

where λ_θ and q are respectively the the local Pohlhausen pressure gradient parameter and free-stream turbulence intensity values.

The intermittency distribution from the new model is given by

$$\gamma = 1 - \exp\left[-n\int_{x_t}^{x_i} \frac{\sigma}{\tan(\alpha)}\left(\frac{dx}{U}\right)\int_{x_t}^{x_i} \tan\alpha \; dx\right] \tag{4}$$

where n is the spot generation rate $(m^{-1}s^{-1})$ and $U(x)$ is the free-stream velocity distribution. Assuming the turbulent spots are triangular in planform, with

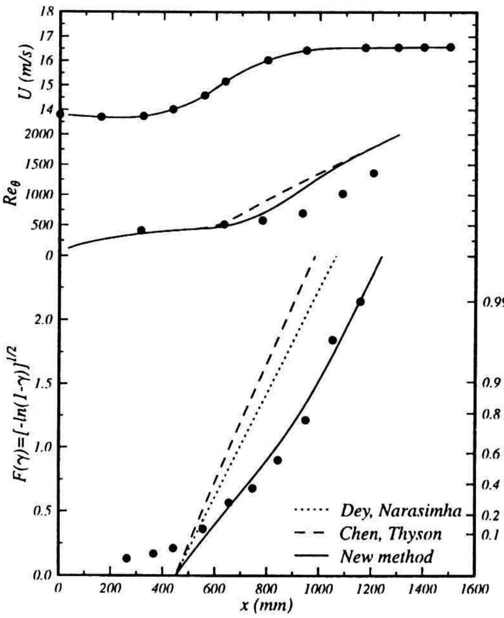

Figure 2: Calculation of transition in flow with a changing pressure gradient. Experimental data from Narasimha *et al.* (1984) - case DFU1.

leading and trailing edge celerities of aU and bU respectively, the propagation parameter is given by $\sigma = \tan\alpha(b^{-1} - a^{-1})$.

The model retains Narasimha's concentrated breakdown hypothesis, with the spot generation rate being determined by the boundary layer parameters and streamwise pressure gradient at transition onset. As seen from Fig. 1, the application of positive pressure gradient may alter the generation rate by an order of magnitude. The spot spreading angle and propagation parameter are assumed to vary continuously through the transition zone, according to the local value of λ_θ obtained from a purely laminar boundary layer calculation.

Fig. 2 shows the results of the new model for the flow DFU1 reported by Narasimha *et al.* (1984). Case DFU1 involves an increasing acceleration over the forward part of the transition region; this is subsequently relaxed to near zero pressure gradient at the end of transition. The new model successfully describes the the observed intermittency distribution; in particular, it predicts the marked kink or 'subtransition' in the logarithmic intermittency plot of $F(\gamma)$ around $x = 900$ mm. The comparative predictions by the methods of Dey & Narasimha (1988) and Chen & Thyson (1971) exhibit a linear variation in $F(\gamma)$ over the whole transition region, with slope approximating the observed behaviour in near zero pressure gradient conditions over the rearward part of the transition zone; the marked reduction in growth rate of intermittency in accelerating flow over the forward part of the transition zone is not predicted.

The predicted boundary layer growth through the transition zone is too rapid despite the intermittency distribution being reasonably well estimated. This indicates that there are deficiencies in modelling of the emerging turbulent boundary layer which remain to be addressed.

Temporal variations - periodically unsteady flows

Triggered turbulent spots

Fig. 3 shows the development of transition in a strongly decelerating laminar boundary layer subjected to periodic perturbations from triggered turbulent spots generated at a fixed position by blowing through a hole in the surface. The time-space ($t - x$) diagram uses shading to indicate ensemble-averaged values of RMS disturbance level integrated over the boundary layer height; the contours indicate ensemble-averaged values of velocity profile shape factor H (displacement thickness/momentum thickness).

The development of the triggered spot is evident from the band of increasing disturbance level which appears at the upstream limit of the figure at times between 25 and 30 ms. This is accompanied by a sudden reduction in shape factor H at the leading edge of the spot. Prior to the spot passage the unperturbed laminar layer undergoes a natural transition with a characteristic fall in shape factor and the RMS disturbance level peaking around the centre of the transition zone.

Behind the spot is the familiar relaxation zone (or calmed region) which is characterised by a low disturbance level and a slow return of H to the unperturbed laminar value. The energyzing of the surface layer associated with the passage of a turbulent spot has the very important practical implication that laminar separation will be temporarily delayed in this region. Another important consequence is the stabilisation of flow associated with the lower shape factor values in the relaxation zone; this significantly delays the natural transition process behind the spot, as evidenced by the tongue of low disturbance level extending beyond $x = 500$ mm.

Wake-induced transition

Fig. 4 shows a similar time-space contour plot for the process of transition on the suction surface of a C4 section blade in an axial compressor stator. t^* is time normalised by the rotor wake passing period; s^* is surface distance normalised by the blade chord. The shading indicates ensemble-averaged intermittency obtained from a chordwise array of surface hot-film sensors; it changes from white to black as γ increases from 0 (fully laminar) to 1 (fully turbulent).

The dark wedges commencing around $s^* = 0.1$ at intervals of $t^* = 1$ correspond to transitional and turbulent flow wedges induced by passing free-stream disturbances from the wakes of upstream rotor blades. These are interspersed with laminar or transitional flow regions extending rearward to around $s^* = 0.9$.

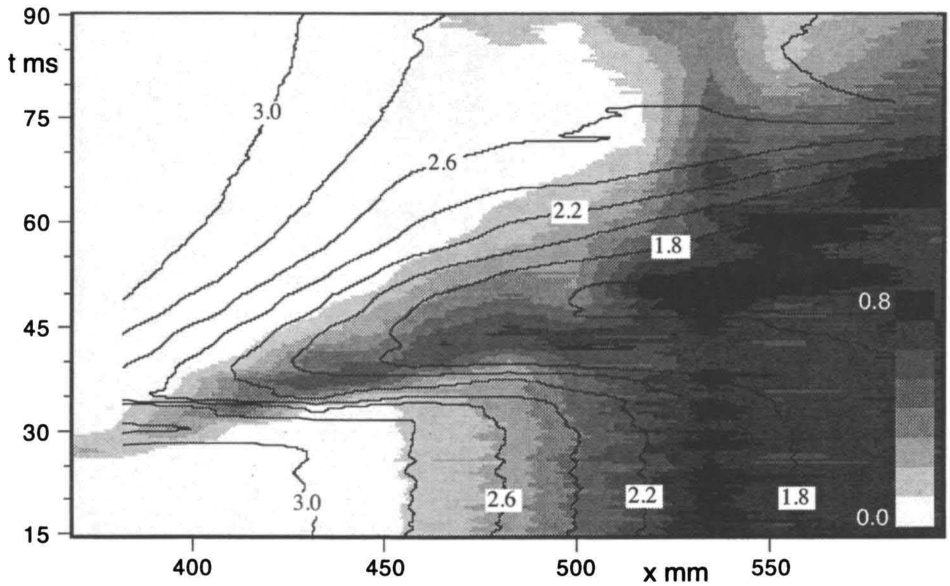

Figure 3: $t - x$ diagram for a triggered spot in an adverse pressure gradient. Shading indicates RMS disturbance level integrated over the boundary layer height. Contours give the shape factor H.

The turbulent strip which should have originated around $t^* = 2$ is missing because the corresponding rotor blade was removed to investigate the influence of changing wake frequency on the unsteady flow behaviour. In the absence of this strip, the turbulent onset following the passage of the preceding strip moves forward to around $s^* = 0.5$ when the following wake-induced transitional strip arrives. The inter-wake turbulent onset would probably asymptote to a fixed streamwise location, given a sufficiently long time interval between successive rotor wake passages (possibly around $s^* = 0.4$ where laminar boundary layer separation is predicted by a steady flow boundary layer calculation).

These results mirror those of the triggered spot observations, and clearly indicate the existence of marked flow stabilisation caused by a preceding wake-induced turbulent strip. The altered boundary layer profile behind the wake-induced turbulent strip is able to maintain laminar flow regions over the majority of the blade surface, whilst preventing intermittent laminar separation provided that the rotor wake passing frequency is sufficiently high. Full optimisation of axial turbomachine blade design clearly requires a detailed understanding of these unsteady phenomena and an ability to incorporate them in engineering design calculations.

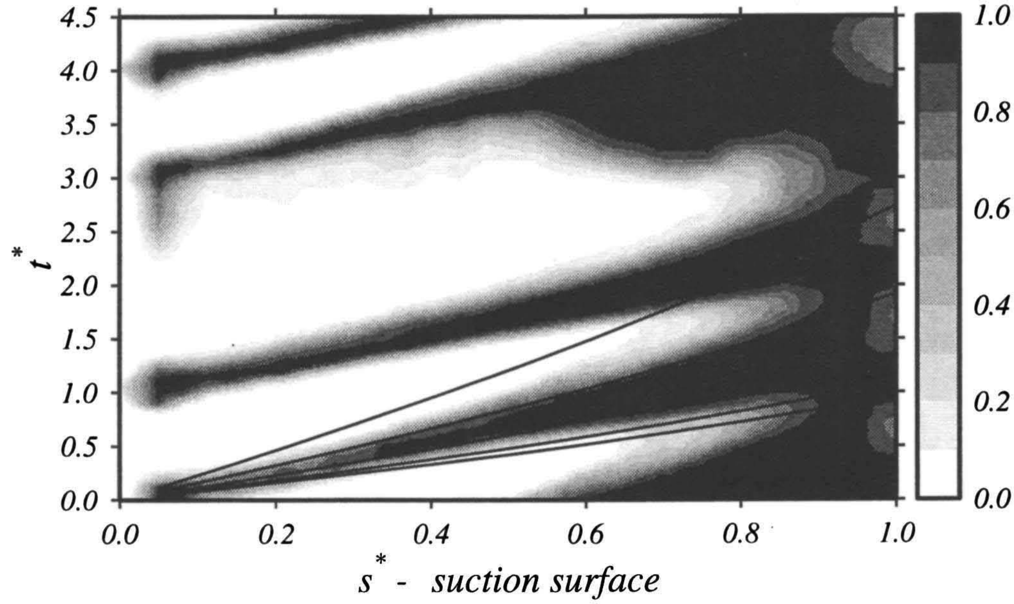

Figure 4: Wake-induced turbulent spots developing on an axial compressor blade. $t-s$ diagram of ensemble averaged intermittency. Wake induced turbulent strip at $t^* = 2$ eliminated by removal of a single upstream rotor blade. Particle trajectories for $1.0U$, $0.88U$, $0.7U$, $0.5U$, $0.35U$ overlaid.

Detailed observations of wake-induced transition have also been reported by Halstead *et al.* (1995) for blade surfaces in multi-stage axial compressors and turbines. Unsteady hot-wire measurements of boundary layer development were used to complement surface film data in these studies. Fig. 5, reproduced from Halstead *et al.* (1995), illustrates the morphology of boundary layer development on the suction surface of a stator blade in an embedded compressor stage. The behaviour is closely similar to that observed in the present study, but the existence of an alternative mode of transition in regions of the $t - x$ plane between successive wake-induced turbulent strips is more clearly evident.

Quasi-steady calculations for wake-induced transition

The transition model for spatially varying flow described above has been applied in a quasi-steady manner as a first attempt at modelling the periodically unsteady flow associated with wake-induced transition. The results are compared with experimental data of Halstead *et al.* (1995) for the boundary layer development on the suction surface of an axial compressor stator (Test Point 2B), shown in Fig. 6.

Fig. 6 presents data for three separate cases:

1. Long-term time-average values of boundary layer parameters.

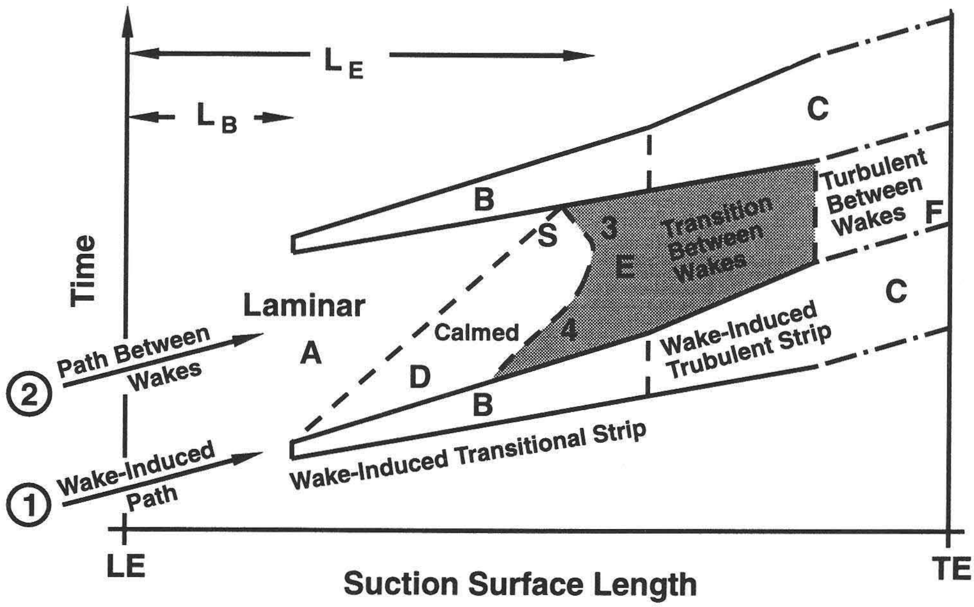

Figure 5: Unsteady boundary layer development on an axial compressor blade Halstead *et al.* (1995) - Test Point 2B.

2. Ensemble-average values along a wake-induced transition path in the $t - x$ plane (Path 1 shown in Fig. 5).

3. Ensemble-average values along a path between successive wake passages in the $t - x$ plane (Path 2 shown in Fig. 5).

The ensemble-average values fluctuate quite significantly during the wake passing cycle, the values for the wake path being around twice those for the long-term time-mean.

The boundary layer calculations have all been implemented with the time-mean surface pressure distribution reported by Halstead *et al.* (1995). The linear combination integral method of Dey & Narasimha (1988) was employed with the following modifications:

1. Calculation of the laminar component was progressed beyond separation by the artificial device of maintaining the shape factor H and skin friction coefficient C_f at their separation values of 3.70 and 0.0 respectively.

2. The initial momentum thickness for the turbulent component at transition onset was assumed equal to that of the laminar component, and the power law model for starting the turbulent calculation was limited to one streamwise step.

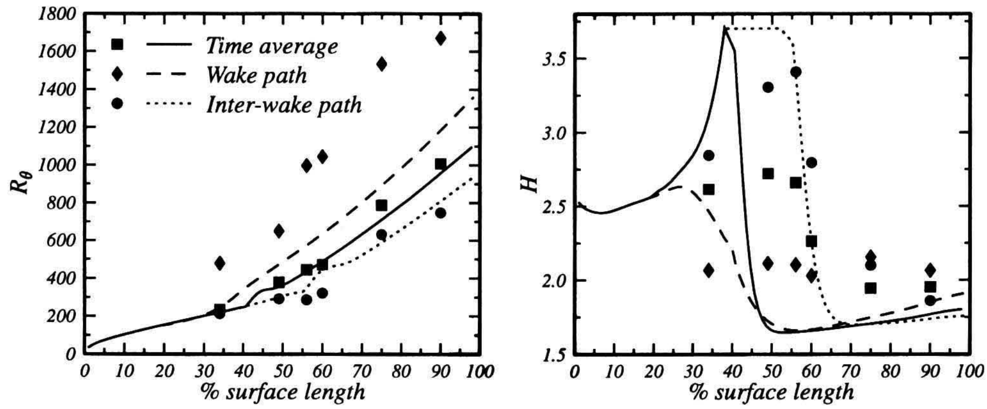

Figure 6: Quasi-steady calculation of the boundary layer development in the unsteady flow on the suction surface of a compressor blade. Symbols are experimental data of Halstead *et al.* (1995) - Test Point 2B.

The transition onset was variously computed or prescribed from experimental data, as described below. The intermittency variation through the transition zone was computed from the model for spatially varying flow described above.

The time-mean boundary layer calculation was implemented with transition onset at 38% suction surface length (SSL) as calculated from the method of Ghannam & Shaw (1980). The free-stream turbulence level of 2.2% at transition onset was estimated from measurements upstream of the blade row, with due allowance for variation in free-stream velocity up to the onset point. The predicted variation in boundary layer thickness, as indicated by the momentum thickness Reynolds number R_θ, is in surprisingly good agreement with experiment. However, this is somewhat deceptive in view of the significant fluctuations in ensemble-average values mentioned earlier and the less satisfactory predictions of velocity profile shape factor H through the transition zone.

The wake path calculation uses the experimentally observed transition onset of 19% SSL, with a corresponding free-stream turbulence level of 4.4%. The computed end of transition ($\gamma = 0.99$) at 60% SSL compares very well with the experimentally observed value of 62% SSL, indicating that the intermittency model is performing quite reasonably in this situation. However, the values of R_θ indicate that the boundary layer growth along the wake path is markedly underpredicted. This is thought to be largely due to deviations from two-dimensionality associated with relative flows normal to the blade surface in the passing rotor wakes: similar calculations for a turbine blade suction surface, where the direction of rotor wake relative flow is reversed, show an overprediction of boundary layer thickness along the wake path. There nevertheless remain significant turbulence modelling problems, as can be seen from a comparison of the shape factor data.

The inter-wake path calculation uses the experimental onset position of 52% SSL with corresponding free-stream turbulence level of 1.5%. This computation is likely to be the least reliable, as it completely ignores the marked flow relaxation effects following the passage of the preceding wake-induced transitional strip. Transition occurs somewhat too rapidly, with the predicted end of transition at 70% SSL compared with 84% SSL from experiment. The boundary layer thickness is in fair agreement, but the shape factor development again shows significant discrepancies.

There is a general trend in all three computations for the shape factor to be overestimated immediately prior to transition onset and in the forward part of the transition zone. This probably results from the combined effects of:

1. The influence of free-stream turbulence in reducing the laminar boundary layer shape factor prior to transition.

2. The neglecting of relaxation effects producing a higher than normal shape factor value for the laminar boundary layer component in the transition zone.

The shape factor falls too rapidly through the transition zone, and only slowly recovers towards experimental values in the region of fully turbulent flow further downstream. This indicates significant problems with modelling of the emerging turbulent boundary layer.

Modelling of the relaxation zone

For differential types of boundary layer calculations the extent of the relaxation zone following a turbulent spot should be predicted directly provided that an appropriate variation of intermittency is prescribed. For integral calculation methods, and for general engineering design purposes, it is necessary to provide some estimate of the relaxation time T_r needed for the laminar flow to approach its unperturbed state.

Table 1 gives data for dimensionless relaxation time $T_r = t_r U/\delta_L$ for the zero pressure gradient experiment of Schubauer & Klebanoff (1955), the triggered spot in adverse pressure gradient of Gostelow *et al.* (1995) and the wake-induced transition observations of Solomon & Walker (1995). δ_L is the total thickness of the unperturbed laminar layer surrounding a turbulent spot. There is some scatter in the individual experiments, but the values of T_r are all of order 100.

Concluding remarks

A new model for the intermittency variation in flows with rapid spatial variations has been applied in a quasi-steady manner to periodic flows involving wake-induced transition on axial turbomachine blades. The predictions of intermittency for this case are encouraging, but some significant deficiencies in modelling the boundary layer development through the transition zone remain to be addressed.

Table 1: Comparison of relaxation times $T_r = \frac{t_r U}{\delta_L}$ observed in different flows. q is local free-stream turbulence level.

Flow	T_r	q
Schubauer & Klebanoff (1955), flat plate, zero pressure gradient	100-220	0.03%
Gostelow *et al.* (1995), flat plate, adverse pressure gradient	90	0.3%
Solomon & Walker (1995), compressor blade	50-150	2.2%

One of the most important challenges is the modelling of flow relaxation following the passage of a turbulent spot. The increased shear stress within the relaxing non-turbulent flow region has the important consequences of delaying laminar separation and locally stabilising the flow. Some typical values of dimensionless relaxation time have been presented as a first step in the modelling process.

Other areas of uncertainty which remain for future investigation include improvement of the spot growth rate correlations with more experimental data and development of a correlation for the spot generation rate which is independent of the spot growth parameters and valid in favourable pressure gradients. The effects of free-stream turbulence on all of these parameters also needs to be clarified. The model could be refined to allow for changes in spot shape around separation and possibly to predict transition in the separated shear layer. Transition onset correlations or prediction methods need to be improved as does the modelling of the emerging turbulent boundary layer.

Acknowledgements

The authors gratefully acknowledge financial support from the Australian Research Council and Rolls-Royce plc.

References

Abu-Ghannam, B.J. & Shaw, R. 1980 – Natural transition of boundary layers – the effects of pressure gradient and flow history. *J. of Mech. Eng. Sc.* **22**, 213-228.

Chen, K.K. & Thyson, N.A. 1971 – Extension of Emmons' spot theory to flows on blunt bodies. *AIAA Journal* **9**, 821-825.

Dey, J. & Narasimha, R. 1988 – An integral method for the calculation of 2-D transitional boundary layers. Fluid Mechanics Report 88 FM 7, Indian Institute of Science, Bangalore.

Emmons, H.W. 1951 – The laminar-turbulent transition in a boundary layer – part I. *J. of Aerosp. Sc.* **18**, 490-498.

Gostelow, J.P., Blunden, A.R. & Walker, G.J. 1994 – Effects of free-stream turbulenc and adverse pressure gradients on boundary layer transition. *ASME J. of Turbomachinery* **116**, 392-404.

Gostelow, J.P., Melwani, N. & Walker, G.J. 1995 – Effects of a streamwise pressure gradient on turbulent spot development. ASME Paper 95-GT-303, International Gas Turbine Congress, Houston.

Halstead, D.E., Wisler, D.C., Okiishi, T.H., Walker, G.J., Hodson, H. P. & Shin, H. 1995 – Boundary layer development in axial compressors and turbines: Parts 1-4. ASME Papers 95-GT-461-464.

Narasimha, R. 1985 – The laminar-turbulent transition zone in the boundary layer. *Progress in Aerospace Science* **22**, 29-80.

Narasimha, R., Devasia, K.J., Gururani, G. & Narayanan, M. A.B. 1984 – Transitional intermittency in boundary layers subjected to pressure gradient. *Experiments in Fluids* **2**, 171-176.

Schubauer, G.B. & Klebanoff, P.S. 1955 – Contributions on the mechanics of boundary layer transition. NACA TN 3489.

Solomon, W.J. & Walker, G.J. 1995 – Incidence effects on wake-induced transition on an axial compressor blade. Proc. 12th Int. Symposium on Air Breathing Engines **2**, pp. 954-964, Melbourne, Australia.

Solomon, W.J., Walker, G.J. & Gostelow, J.P. 1995 – Transition length prediction for flows with rapidly changing pressure gradients. ASME Paper 95-GT-241 (Accepted for publication in the Transactions of the ASME).

Walker, G.J. 1989 – Transitional flow on axial turbomachine blading. *AIAA J.* **27**, 595-602.

Authors' addresses

°Department of Civil and Mechanical Engineering
University of Tasmania
Hobart, Tasmania, Australia

▷Department of Engineering
University of Leicester
Leicester, England