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Models for the Evolution of Neutron Star Magnetic Fields

Abstract

Recent observations indicate that the magnetic field of a neutron star decays significantly only when the neutron star is in an interacting binary. This paper reviews the theoretical attempts to model such evolution. Three main possibilities are discussed: the expulsion of the magnetic flux from superconducting core during propeller spin-down in companion's wind, screening of the magnetic field by accreted matter and rapid ohmic decay of crustal magnetic field as a result of heating during accretion. It is found that the screening is unlikely to be effective because of strong Rayleigh-Taylor instabilities. The spindown-induced flux expulsion as well as the crustal heating models remain promising.

I Introduction

The earliest model for the evolution of the magnetic fields of neutron stars was due to Gunn and Ostriker (1970), who suggested a simple exponential ohmic decay of the field strength with a time scale of a few million years. This was the accepted picture of field evolution for nearly two decades, despite theoretical objections raised quite early on (e.g., Baym, Pethick & Pines 1969). Of late, however, there is a growing consensus that the decay of the magnetic field of a neutron star is associated with the interaction of the neutron star with a companion star in a binary system, and in isolated neutron stars there is little evidence of field decay (see contribution by F. Verbunt et al. in this volume).

Building a physical model for the reduction of the magnetic field strength of a neutron star primarily because of interaction with a companion poses a theoretical challenge. The last few years has seen a large number of models being proposed. These models have been explored to different degrees of detail, but it would be fair to say at present that no single model has been fully developed, and no single model is consistent with all observed data. I have been asked to give an overview of these models, but in view of the limited length of this contribution I would like to confine my description to a few models which in my opinion are, or have been considered, most promising.

One of the major uncertainties surrounding the picture of the field evolution is the location of the magnetic field in the interior of the star. This issue is not unrelated to the origin of neutron star magnetic fields. If the magnetic field is a fossil remnant from the progenitor stage, the field is likely to penetrate the whole star. The major fraction of the magnetic flux in such a case would pass through the superconducting interior of the neutron star. On the other hand, if the magnetic field is generated via thermomagnetic instabilities after the birth of the neutron star (e.g., Blandford, Applegate & Hernquist 1983), then the resulting field is likely to be confined to the outer crust of the neutron star. Physical processes affecting the core field and the crustal field are different, and one therefore needs to investigate different classes of models for these two cases.

II Magnetic field in a normal n-p-e core

Some attention has been devoted in the last few years to the evolution of the magnetic field permeating the neutron star core, assuming that the matter there is in a normal (i.e., non-superfluid) state. Although this may not be the situation in a real neutron star, the results obtained from this exercise are instructive, and a number of concepts have applications also in the case of a superfluid interior.

The evolution of the magnetic field in the normal n-p-e core is governed by a generalized form of Ohm's law (Shalybkov & Urpin 1995):

$$\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} = \frac{\mathbf{G}}{2en_{\mathrm{e}}} - \frac{\mathbf{u}_{\mathrm{n}}}{c} \times \mathbf{B} + \mathbf{R} \cdot \mathbf{j}, \tag{1}$$

where $\mathbf{G}$ is a force term including pressure and partial pressure gradients. If this term has a non-zero curl, then magnetic field can be spontaneously generated via battery effect (Biermann 1950). However, in a neutron star this effect is unimportant (Shalybkov and Urpin 1995). $\mathbf{u}_{\mathrm{n}}$ is the drift velocity of neutrons, and $\mathbf{R} \cdot \mathbf{j}$ stands for

$$\mathbf{R} \cdot \mathbf{j} = R_{\|} j_{\|} + R_{\perp} j_{\perp} + \frac{R_{\mathrm{H}}}{B} \mathbf{j} \times \mathbf{B}, \tag{2}$$

where the parallel and perpendicular components are in reference to the direction of the magnetic field $\mathbf{B}$, and R_{H} is the Hall resistance. $\mathbf{R} \cdot \mathbf{j}$ includes the effects of Ohmic diffusion, Ambipolar diffusionand Hall effect (Goldreich and Reisenegger 1992; Shalybkov & Urpin 1995). Other symbols in equation (1) have their usual meaning.

A number of important evolutionary time scales can be isolated in this context (Goldreich & Reisenegger 1992):

$$t_{\mathrm{ohmic}} \sim 2 \times 10^{13} \frac{L_6^2}{T_8^2} \left(\frac{\rho}{\rho_{\mathrm{nuc}}} \right)^3 \mathrm{yr}$$

$$t_{\mathrm{ambip}}^{\mathrm{s}} \sim 3 \times 10^{11} \frac{T_8^2 L_6^2}{B_{12}^2} \mathrm{yr}$$

$$t_{\text{ambip}}^{\text{ir}} \sim \frac{5 \times 10^{15}}{T_8^6 B_{12}^2}(1 + 5 \times 10^{-5} T_8^8 L_6^2) \text{ yr}$$

$$t_{\text{Hall}} \sim 5 \times 10^{10} \frac{L_6^2}{B_{12}} \left(\frac{\rho}{\rho_{\text{nuc}}}\right) \text{ yr}$$

where L_6 is the scale length of the field distribution in units of 10^6 cm, T_8 is the temperature in units of 10^8 K and B_{12} is the field strength in units of 10^{12} G. These time scales are for ohmic diffusion, solenoidal ambipolar diffusion irrotational ambipolar diffusion and Hall drift respectively. In ambipolar diffusion, compared to the solenoidal component of field movement, the time scale is much longer for the irrotational component, because the attendant drift of charged particles is suppressed by chemical imbalance. This drift can therefore take place only in the time scale over which chemical balance can be restored. The estimate of $t_{\text{ambip}}^{\text{ir}}$ above assumes restoration of chemical equilibrium via the modified URCA process. Ohmic diffusion and ambipolar iffusion cause dissipation of magnetic energy, while Hall drift only rearranges the magnetic field configuration. Goldreich & Reisenegger (1992) conjectured that Hall drift may work like a turbulent cascade, creating very small current loops which would eventually undergo quick Ohmic decay. Hall-cascade-assisted ohmic dissipation may indeed be a very important process operating in the inner crusts of neutron stars.

III Magnetic field in the superconducting interior

The interior of the neutron star is believed to contain a mixture of superfluid neutrons and superconducting protons. The proton superconductor is estimated to have a coherence length much smaller than the London penetration depth, causing it to exhibit a type II behavior. Magnetic flux passing through the core must therefore be carried by quantized Abrikosov fluxoids in the proton superconductor (see reviews by Srinivasan, this volume, Bhattacharya & Srinivasan 1995 and Sauls 1989). The total number of such fluxoids in the neutron star interior would be $\sim 10^{31}(B/10^{12}$ G). The angular momentum of the core neutron superfluid, on the other hand, is carried by Onsager-Feynman vortices, the number density of which is proportional to the angular speed of the superfluid. The number of neutron superfluid vortices in the core is $\sim 2 \times 10^{16}(P/1 \text{ s})^{-1}$. As the neutron star spins down, these vortices migrate outward, and are shed upon reaching the boundary of the superfluid.

Muslimov and Tsygan (1985) first recognized that there is likely to be a strong interpinning between the proton fluxoids and the neutron vortices. The physical nature of this pinning was elaborated upon, and its magnitude was estimated by Sauls (1989). Srinivasan et al (1990) suggested that this may be the main mechanism for the expulsion of flux from the superconducting interior—as the star spins down, the neutron vortices migrate outward, carrying with them the fluxoids. The magnetic flux is then deposited in the crust, where it can decay due to ohmic processes. In an isolated neutron star spinning down by dipole torque, the net spindown is small, and only a modest amount of flux is expelled. A neutron star in a binary can spin down to a very long period in the phase of interaction with the stellar wind of the companion, and hence a large amount of field decay can occur.

Jahan Miri and Bhattacharya (1994) explored in detail the spindown process and the consequent field decay in wide low-mass X-ray binaries, systems that appear to have produced neutron stars with the lowest known magnetic fields (millisecond pulsars). They found that the field strengths, their correlation with the orbital periods, and the apparent lower limit to the field at $\sim 10^8$ G, can be explained if the ohmic decay time of the magnetic field after expulsion to the crust is around 10^8–10^9 yr. If the field decays too quickly, then the spindown torque due to the companion's wind diminishes too early to achieve enough flux expulsion. If it decays too slowly, then eventual spin-up to millisecond periods becomes impossible.

Bhattacharya and Datta (1996) investigated the ohmic evolution of the expelled field in the crust, and found that in a cold neutron star with the temperature of the inner crust of $\sim 10^6$ K, the ohmic evolution of the field is decided mainly by the resistivity arising out of impurity scattering. Ohmic time scale in the range 10^8–10^9 can be obtained with a fairly modest impurity concentration, the parameter

$$Q = \sum_i n_i (Z_i - Z_0)^2 / n_0$$

being in the range 0.01–0.1 depending on the equation of state of neutron star matter; here n_i and Z_i are the density and the charge number of the i-th species of impurity atoms, and n_0 and Z_0 that of atoms of pure matter. If the crust is heated to a temperature $\sim 10^8$ K, say by accretion, then phonon scattering makes a significant contribution to the resistivity, and field decay is hastened.

It is, however, the massive binary systems that seem to place stronger constraints on this scenario (Jahan Miri 1996). Many X-ray pulsars with long spin periods still appear to have quite strong fields $\sim 10^{12}$ G (see White, Nagase & Parmar 1995), while by the time the spin-up in heavy accretion finishes, the field has already decayed by some two orders of magnitude (e.g., pulsars B1913+16, B1534+12, J1518+4904). This constrains the required Ohmic time scale after flux expulsion very strongly, and is barely consistent at $10^{7.5}$ yr.

How is one to reconcile this constraint with that for low-mass binaries? One possibility is to attribute most of the ohmic decay of the expelled flux to the phase of heavy accretion by Roche-lobe overflow, when the crust is heated to a high temperature. A low mass binary takes a long time to evolve to this stage, before which a slow but steady propeller phase removes angular momentum from the neutron star. This is also an ideal situation for flux expulsion; if the spin-down is too quick then flux expulsion efficiency is reduced because of the drag on flux lines, as well as possible back-reaction from flux accumulated at the bottom of the crust (Jones 1988;

Ding, Cheng & Chau 1993; Jahan Miri & Bhattacharya 1994). Massive binaries may well suffer from this inefficient expulsion, being thereby unable to produce neutron stars with as low magnetic fields as in low-mass binaries. The other possibility is a progressive lengthening of effective decay time as, for example, is expected in case of a Hall cascade-mediated decay.

While this remains an attractive model for the field decay in neutron stars, a number of unsolved problems still shroud the fluxoid dynamics. Foremost of these is the collective effects on their motion. The drag on fluxoids, for example, is strongly dependent on collective effects. If large groups of fluxoids act as macroscopic, collective entities then an important restriction applies on their motion: they cannot move faster than the rate allowed by Ohmic diffusion (see Ruderman, this volume on how to turn this into an effective drag coefficient). If they do, then they must carry the charged component with them, causing a chemical imbalance which would suppress the irrotational part of the flow, akin to ambipolar diffusionGoldreich & Reisenegger 1992). On the scale of the width of a single fluxoid, however, the Ohmic diffusion time is too small to be important for dynamics. It is therefore very important to understand the collective effects, to which end not much progress has been made so far. Another important question is whether the proton superconductor does exhibit Type II properties. Some recent calculations have revised the estimate of the proton energy gap downwards (e.g., Wambach, Ainsworth and Pines 1991) almost into the Type-I regime. If the superconductor does happen to be of Type I, then quantized fluxoids no longer exist, and the field may be trapped into macroscopic regions of normal matter sandwiched between superconducting layers. There is no clear picture of how the magnetic flux would behave under such condition, and whether accretion can have any effect on its evolution.

In the category of spin-magnetic field coupling falls another set of models by Ruderman (1991a,b,c). In this picture crustal plate tectonics causes the magnetic poles to migrate to the (rotational) equator as the star spins down. At the equator the poles may come very near each other due to flux-line tension, and thereby annihilate a part of the dipole moment. On spin-up, the poles are pushed to the rotational poles of the star. The magnetic moment increases if the magnetic poles migrate to opposite rotational poles, and decreases further if they migrate to the same rotational pole.

IV Screening of the magnetic field by accreted matter

Suggestions that the accreted matter on the neutron star surface can reduce its externally visible magnetic moment by diamagnetic screening have been in the literature for a long time (e.g., Bisnovatyi-Kogan and Komberg 1974), and has been developed beyond a mere suggestion more recently (Romani 1990,1993). The basic idea is that the accreted matter arrives at the magnetic poles, guided by the magnetic field. At the pole an accretion column builds up till the pressure at the bottom of the column exceeds the confining magnetic pressure. At this point matter begins to flow sideways, dragging the field lines with it. Upon reaching the equator from both poles field reconnections occur, causing an eventual burial of the original field under the newly acquired layer of accreted matter. Once the screening is accomplished, the buried field can be driven deeper by further accretion. This mechanism, if it works in practice, would not care if the original magnetic field resides in the core of the star or in the crust.

However, whether this mechanism works at all is in serious doubt, mainly because of the matter flow being strongly susceptible to Rayleigh-Taylor instability. To stretch the vertical field lines in the horizontal direction, the flow must create a horizontal component of the magnetic field of magnitude similar to the original vertical field. The pressure of this horizontal magnetic field therefore nearly equals the hydrostatic pressure of the accretion column. Under this condition, the Rayleigh-Taylor instability grows very rapidly: at the burial depths estimated by Romani (1993) (density at burial depth $\rho \sim 10^6 B_{12}^{-1.5}$ g cm^{-3}) the overturn time works out to be a few microseconds, much shorter than the "flow time" (time needed for the accretion column to be replenished with accreted matter). The accreted matter is therefore likely to just move through the magnetic field, continuously creating horizontal extensions which overturn, reconnect and disappear, leaving the original vertical field essentially undisturbed in the end (Konar 1996).

V Magnetic fields confined to the outer crust

Over the last few years, a number of papers have investigated the ohmic decay of magnetic field initially confined only to the outer crust (Sang & Chanmugam 1987; Urpin and Muslimov 1992; Urpin & van Riper 1993; Geppert and Urpin 1994; Urpin & Geppert 1995,1996). Initial magnetic field configuration such as this may be expected if the field is generated after the birth of the neutron star via thermomagnetic instabilities (Blandford, Applegate and Hernquist 1983). The evolution of this field is determined by two major parameters: (i) electrical conductivity σ which is a function of the density, temperature and impurity strength Q in the crust, and (ii) the scale length of the field distribution. The smaller the scale length, the faster the Ohmic diffusion. An initially tightly confined field distribution near the surface diffuses to larger scales, into regions of higher density in the interior. This phase results in a power-law decay of the field strength at the surface (see Bhattacharya 1995 for a review). In practice, in an isolated neutron star the crustal temperature is high enough for some field decay (by $\lesssim$ an order of magnitude, depending on the initial penetration depth of the field configuration) to occur in the first $\lesssim 10^6$ yr or so, beyond which the decay proceeds very slowly. This early phase of decay might be responsible for the field strengths of very young pulsars associated with supernova remnants being somewhat larger than the average field strength of the majority of isolated pulsars which are a little older.

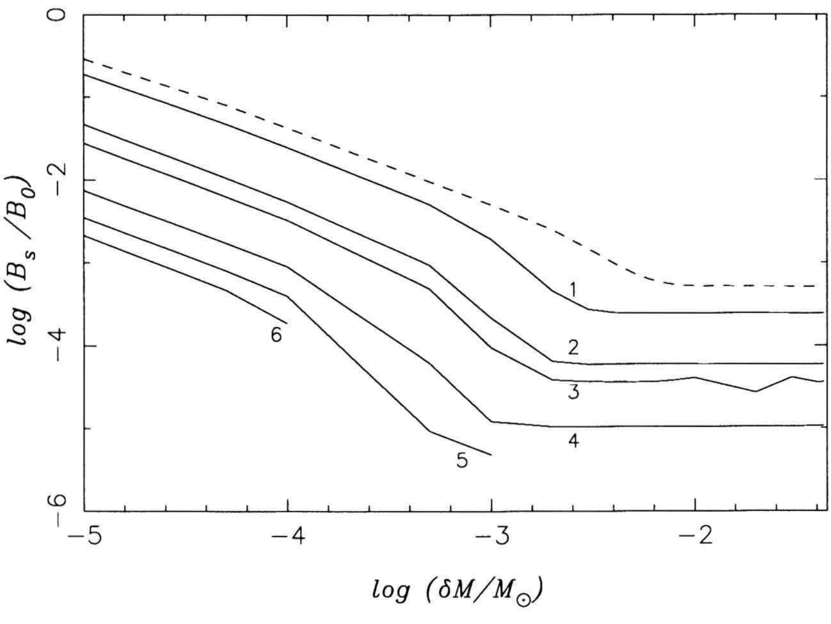

Figure 1. Evolution of the magnetic field of a neutron star undergoing accretion. The initial field is assumed to be confined to the outer crust. Accretion causes heating, enhancing ohmic diffusion rate, and also pushes current-carrying layers to higher densities (and hence higher conductivities), thereby slowing down the decay. The net result is an initial rapid decay followed by freezing. The figure plots the surface field strength versus the amount of mass accreted. Curves 1 to 6 correspond to accretion rates of 10^{-9}, 2×10^{-10}, 10^{-10}, 10^{-11}, 10^{-12} and 10^{-13} $M_{\odot}$/yr. The crustal temperatures as a function of accretion rate were obtained from a fit to the results of Zdunik et al. (1992). The dashed curve corresponds to 10^{-9} $M_{\odot}$/yr, but for a factor-of-two lower temperature of $10^{8.5}$ K. The evolution is shown for a maximum of 10^{9} yr. Taken from Konar & Bhattacharya (1997).

A substantial decay of the magnetic field would, however, take place once the crust is heated by accretion. The work of Urpin and Geppert (1995,1996) showed that the decay due to even mild accretion could be quite substantial. Konar and Bhattacharya (1997) developed this further and included the effect of the inward convection of current-carrying layers due to accreted overburden: all of the original crustal matter is pushed into the core if the total accreted mass exceeds $\sim 10^{-2}$ $M_{\odot}$. The effect of accretion is therefore found to be to first hasten the field decay, and then to *freeze* it at a residual value as most of the flux enters the core. The higher the accretion rate, the stronger is the residual field. Figure 1 shows an example of such evolution. As can be seen, the field strength depends not only on the net accreted mass, but also on the accretion rate.

VI Summary

To sum up, the present situation on modeling neutron star magnetic field evolution appears to be as follows.

If the magnetic field is trapped in the superconducting interior of the neutron star, expulsion is a must for field decay to take place. Spindown-induced flux expulsion remains the most promising mechanism for this. Many uncertainties still remain in the dynamics of magnetic flux under this condition, and reliable quantitative predictions cannot be made unless some of these are settled.

In the crust, Hall-cascade assisted ohmic decay appears very promising. Detailed investigation of this would be very useful.

Burying of the magnetic field by accreted matter has serious difficulties. This picture is unlikely to explain the low fields of recycled pulsars.

If the field is entirely in the crust to start with, it is expected to show ~ 1 order of magnitude decay in the early life of isolated neutron stars. On accretion, heating hastens decay while migration of currents to higher densities stabilizes the field.

It has been popular in recent literature to parametrize the amount of field reduction as a simple function of the total mass accreted. There is no physical model which can accomplish this. The closest is the scenario where the field is originally confined to the crust and the decay happens due to heating on accretion. Even in this scenario, however, the final field has a major dependence on the accretion rate itself, in addition to the net accreted mass.

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