F. Verbunt, J.W. Hartman, D. Bhattacharya, R.A.M.J. Wijers and G. Nelemans

Applications of Radio Pulsar Population Synthesis

Abstract

Improved population synthesis of single radio pulsars with $B \geq 10^{11}$ G confirms that the time scale on which the magnetic field of neutron stars decays is 30 Myr or more. Our simulations suggest that a fair number of neutron stars are born with velocities less than ~ 200 km/s. The derived birth rate of neutron stars near the Sun is low enough to be compatible with the assumption that all neutron stars are formed in OB associations from stars with $M > 10\,\mathrm{M}_\odot$. There is no need in our simulations for a second pulsar population (e.g., of mildly recycled pulsars) injected at $B \geq 10^{11}$ G, in addition to the neutron stars formed directly from type II supernovae.

I Population synthesis

For each prescription of the evolution of the pulse period P and its derivative $\dot{P}$ of radio pulsars, we can predict their motion in the P-$\dot{P}$ diagram. Only sufficiently luminous pulsars in an area of the sky that has been observed can be placed in the diagram. Thus, the study of pulsar evolution via population synthesis proceeds as follows (Bhattacharya et al. 1992, updated and improved by Hartman et al. 1997).

First, we assume distributions for the properties of neutron stars at birth, viz. , distributions for initial position $\vec{r}_i$ in the Galaxy, initial velocity $\vec{v}_i$, initial period P_i, and initial magnetic field $B_i \propto \sqrt{P_i \dot{P}_i}$. $\vec{r}$ and $\vec{v}$ evolve in the gravitational potential Φ of the Galaxy. The evolution of P and $\dot{P}$ is determined by $P\dot{P} = 10^{-39}(B_i e^{-t/\tau})^2$ s/G^2, where τ is the time scale on which the magnetic field decays. For $\vec{r}_i$ we choose roughly the galactic distribution of the OB stars. For $\vec{v}_i$ we use the distributions shown in Figure 1. P_i is fixed at 0.1 s. For B_i we assume a Gaussian in $\log B_i$ centered on $\log B_o$ and with width σ_B. For Φ we use the Kuijken & Gilmore (1989) potential.

We use a Monte Carlo method to choose an age $t \in (0, n\tau)$ ($n = 5, 3$ for short, long decay times) and to choose initial values $\vec{r}_i$, $\vec{v}_i$, P_i and B_i, which we then evolve to the values at age t, viz. , $\vec{r}$, $\vec{v}$, P, and B. If the P-B combination lies above the death line, we calculate the pulsar beam width from P and $\dot{P}$ (after Narayan & Vivekanand 1983),

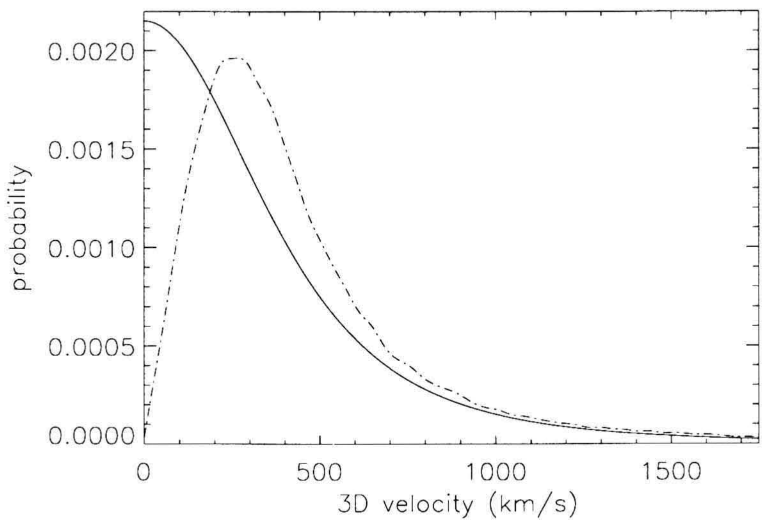

Figure 1. The initial velocity distribution of neutron stars, according to Lyne & Lorimer (1994, dash-dotted line) or to Eq. (1) (Phinney & Hansen, private communication, solid line).

and from this we decide on a probability basis whether the pulsar beam is directed to the Earth. If so, the luminosity L is found from P and B via a model luminosity L_m (according to Prószyński & Przybicień 1984), and a distribution around this with minimum luminosity $\log L_{min} = \log L_m - a$ and a width determined by $1/b$ (after Narayan & Ostriker 1990). From $\vec{r}$ we calculate the position in the sky of the simulated pulsar, and its dispersion measure DM (after Taylor & Cordes 1993), and with L its flux S at Earth, to check whether any of four investigated surveys would have detected the pulsar. We then choose the age of the next pulsar, and repeat the procedure, until we have 2000 simulated detections. Whereas pulsar birth and motion are simulated throughout the Galaxy, we limit the detections to pulsars which have a projected distance $d_{\rm proj}$ on the Galactic plane to the Sun of less than 4 kpc.

II Comparison with observations

We compare the simulated distributions of P, B, L and DMsin b_g—where b_g is the galactic latitude—with the observed ones, by means of Kolmogorov-Smirnov tests. By varying $\log B_o$, σ_B, a and b we find that good simulations of the observed properties of radio pulsars can be produced only for decay times $\tau \geq 30$ Myr. The best simulations are found for $\tau \simeq 100$ Myr, and have $\log B_o(G) \simeq 12.34$, $\sigma_B \simeq 0.34$, $a \simeq 1.5$,

$b \simeq 3.5$. The problem with the models with short decay times $\tau < 30$ Myr is that they have difficulty producing pulsars with long periods and strong fields. If one tries to solve this by enhancing the detection of long-period pulsars via a change in the luminosity law (specifically, by reducing a), then the simulations show far too many high-luminosity pulsars.

The two assumed velocity distributions both give acceptable results, but there is a hint in our simulations that models with the Lyne-Lorimer velocity distribution produce too many pulsars at large distance from the plane but with short characteristic age. This may suggest that the actual distribution of initial velocities has a fair number of pulsars at low velocities, $v_i < 200$ km/s, say, as is the case for the Phinney-Hansen adaptation of the Paczyński (1990) function

$$p(u)du = \frac{4}{\pi}\frac{du}{(1+u^2)^2}, \qquad \text{where} \qquad u \equiv \frac{v_i}{\sigma_v} \tag{1}$$

which has $\sigma_v \simeq 600$ km/s.

Hartman (1997) calculates the transverse velocities for young (< 3 Myr) detected radio pulsars in our population synthesis for both Lyne-Lorimer and Phinney-Hansen initial velocity distributions. Comparing them with those derived from the proper motion measurements of 25 young pulsars from Lyne & Lorimer (1994), and taking into account the errors in the measurements (sometimes comparable to the measured value) he finds that the Phinney-Hansen distribution describes the observations better (Figure 3).

III Progenitor mass

Our simulations show that some 60% of the pulsars detected at $d_{\rm proj} < 4$ kpc were also born with $d_{\rm proj} < 4$ kpc, i.e., most pulsars detected near the Sun were born near the Sun (see Figure 2). This indicates that our simulations provide better constraints on the local birthrate, $\sim 2\,{\rm kpc}^{-2}\,{\rm Myr}^{-1}$, than on the Galactic birthrate. Indeed, models with very different pulsar birth rates near the Galactic Center produce very similar results near the Sun. The local birthrate is sufficiently low that all neutron stars can be produced in OB associations from stars with $M \geq 10\,{\rm M}_\odot$. Blaauw (1985) argued that the local birthrate of neutron stars was about $20\,{\rm kpc}^{-2}\,{\rm Myr}^{-1}$, which required all nearby stars with masses down to $5\,{\rm M}_\odot$ to turn into neutron stars. The main reason for this different conclusion is that Blaauw assumed a short decay time, $\tau < 5$ Myr, and thereby concluded that all detectable pulsars were produced in the last few Myr; in our models with long decay times, the detected pulsars can be produced over a much longer time interval, and the local birth rate is correspondingly lower.

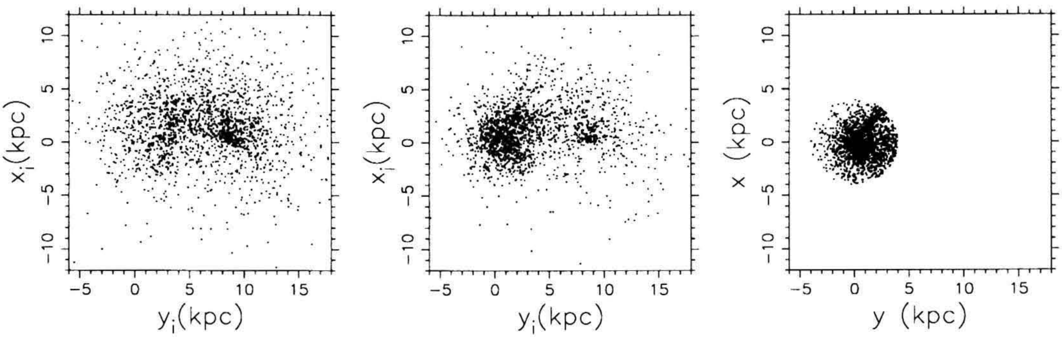

Figure 2. Positions in the Galaxy of neutron stars for a model with $\tau = 100\,\mathrm{Myr}$. The Sun is at (y,x=0,0), the Galactic Center at (8.5,0). The left frame shows the birth place of neutron stars that now have $d_{\mathrm{proj}} < 4\,\mathrm{kpc}$, the middle frame the birth place of pulsars detected at $d_{\mathrm{proj}} < 4\,\mathrm{kpc}$, and the right frame the current position of these detected pulsars. 1000 pulsars are plotted in each frame.

IV Other arguments against field decay

A kinematic argument for a long time scale for field decay, can be made as follows (Lorimer 1994, 1996, Hartman & Verbunt 1995). Pulsars which started to move away from the Galactic plane between 60 and 100 Myr ago currently are close to the Galactic plane again, having completed about half an oscillation. For $\tau \geq 100\,\mathrm{Myr}$ a large characteristic age $\tau_c \equiv P/(2\dot{P})$ corresponds to a large real age, and one predicts that pulsars with $\tau_c \sim 100\,\mathrm{Myr}$ are detected closer to the Galactic plane than pulsars with $\tau_c \sim 50\,\mathrm{Myr}$. For shorter decay times $\tau \leq 10\,\mathrm{Myr}$ one would predict that the average distance to the plane is a monotonically increasing function of τ_c. The observations suggest that detected pulsars with $\tau_c \sim 100\,\mathrm{Myr}$ are closer to the Galactic plane than pulsars at shorter τ_c, and thus that the decay time of the magnetic field is long.

The detection of a nearby pulsar with very low luminosity and with characteristic age $\tau_c \simeq 1.6 \times 10^8$ yr also suggests a long decay time (Tauris et al. 1994): if the pulsar were only a few Myr old, the detection of a nearby specimen would be extremely unlikely unless the birthrate of such low-luminosity pulsars were uncomfortably high. A high real age for this pulsar does not lead to such problematic conclusions.

V Variance of DM

Small-scale structure in the galactic electron distribution leads to variance in the dispersion measures. To investigate this, we perform a thought experiment in which we compare a homogeneous, smooth electron distribution with one in which all electrons are enclosed in spheres, all of which have the same size and electron density in them

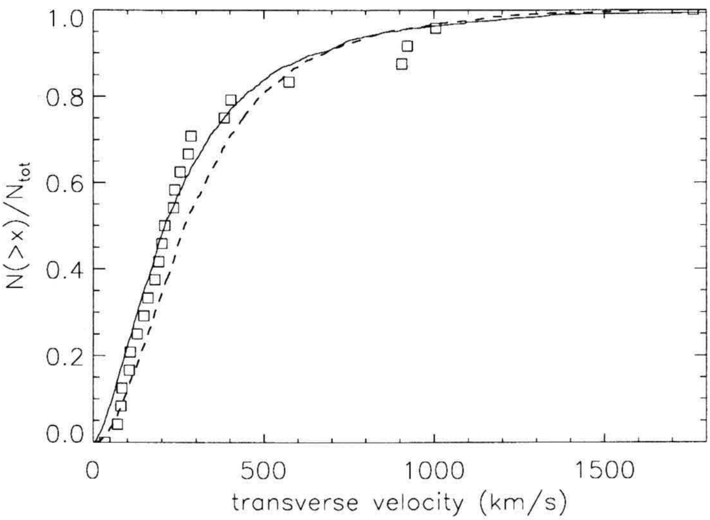

Figure 3. The cumulative transverse velocity distribution of 25 young pulsars from Lyne & Lorimer (1994) (open squares), together with simulated samples for initial distributions according to Eq. (1) (solid line) and to Lyne & Lorimer (1994) (dashed line), where measurement errors have been taken into account.

(Nelemans et al. 1997). In the inhomogeneous model, the average dispersion measure at a given distance d is the same as the dispersion measure for this distance in the smooth model, $\overline{\mathrm{DM}} = \mathrm{DM}_s$, but the dispersion measures are spread around this average with a variance given by $\sigma_{\mathrm{DM}} = (\,\overline{\mathrm{DM}}\frac{9}{8}dm_1\,)^{1/2}$ where dm_1 is the expected dispersion measure for passage through one sphere. This relation is essentially the result of the Poissonian variance in the number n of clouds encountered along the line of sight, for which $\sigma_n = \sqrt{n}$.

We implement this variance in our simulations as follows. From the actual simulated position and the Taylor & Cordes (1993) model we calculate the expected dispersion measure $\mathrm{DM}_s \equiv \mathrm{DM}_{\mathrm{TC}}$. We then choose a simulated dispersion measure from a Gaussian distribution around $\mathrm{DM}_{\mathrm{TC}}$ with width $\propto \sqrt{\mathrm{DM}_{\mathrm{TC}}}$. This new dispersion measure is then combined with the Taylor & Cordes model to get the derived distance and luminosity of the simulated pulsar. A smooth model for the dispersion measures leads to a peak in the simulated distributions at the maximum dispersion measure in each direction, caused by the pulsars above the electron layer. The implementation of the inhomogeneous electron distribution removes this peak, in agreement with observation (Figure 4). However, the new simulations still predict too many pulsars at $|\mathrm{DM}\sin b| \leq 5$; the Kolmogorov-Smirnov tests are not sufficiently sensitive to detect this, but we do consider it as an unsolved problem of our simulations. Many pulsars in

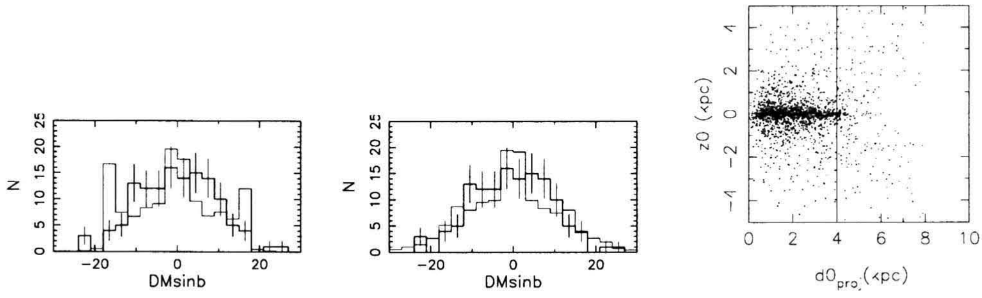

Figure 4. The observed (thick line with error bars) and simulated (thin line) distributions of DM $\sin b$ for simulations that do not and that do include variance in DM (left and middle, respectively). Right: actual distances projected on the plane $d0_{\rm proj}$ as a function of actual distance from the plane $z0$ for pulsars detected in the simulation that includes variance in the dispersion measure. The derived projected distances $d_{\rm proj}$ for all these pulsars is less than 4 kpc.

our simulations with inhomogeneous electron distributions are assigned a $d_{\rm proj} < 4$ kpc even though they have rather large projected distances; this is true especially for simulated pulsars at larger distances from the Galactic plane ($|z| > 1.5$ kpc).

VI Reliability of population synthesis

To put it succinctly: the reliability of a population synthesis is as good as the input assumptions.

The form and period dependence of the pulsar beam is still the subject of debate. The derived birthrate of pulsars is directly proportional to the absolute value of the beaming factor. The relative detection probabilities as a function of P, and thereby the observed P distribution, depend on the period dependence of the beaming factor at $P > 0.1$ s. We have verified that the differences between the different proposed beaming models are not large enough to affect our conclusions.

The luminosity function is highly uncertain, and thus causes appreciable uncertainty in the simulations. It should be noted however that the simulations themselves offer good constraints. For example, if one assumes very large numbers of low-luminosity pulsars one predicts that the detected pulsar population is dominated by very nearby pulsars, contrary to observation. We also have found that the condition that the input model luminosity for detected pulsars should be the same as the output simulated one, in the dependence of L_m on P and $\dot{P}$, puts significant constraints on the form of this dependence. The uncertainties are still large, but in our estimation not large enough to impair our main conclusion that there is no evidence for field decay on short time scales.

The detection algorithms for our simulations are too simple. The sharp lower boundary to the detectable fluxes leads to the prediction that many pulsars will be detected close to the detection limit, contrary to observation. If we arbitrarily impose a minimum flux for detection of 10 mJy, in addition to the ones described for the simulations that we use, the quality of our simulations actually improves, especially with respect to the description of the flux distribution. (This extra limit removes only 6 of the 129 pulsars in the comparison sample of the real pulsars.) Bailes (private communication) stresses that the pulsar surveys are affected by radio interference, which causes pulsars to remain undetected which could have been detected in ideal circumstances. Failure to model this leads to an underestimate of the pulsar birth rate, but does not affect the conclusions about field decay, as long as the interference treats all pulsars equally, i.e., does not bias detection towards pulsars with specific periods or magnetic fields.

The various proposed velocity distributions that are compatible with the observed proper motions of radio pulsars all lead to acceptable simulations; as remarked above, there may be a hint that the number of low-velocity ($v_i < 200\,\mathrm{km/s}$) pulsars is not small. We do not consider the uncertainty in the dispersion measure to be a problem.

The success of our simulations shows that there is no evidence for the presence in the population of ordinary pulsars (i.e., single pulsars with $P > 0.1\,\mathrm{s}$) of any second population, such as mildly recycled pulsars. We have investigated how far this absence of evidence can be used as evidence for absence (Hartman, Portegies Zwart & Verbunt, 1997). Binary evolution scenarios and radio pulsar population synthesis both indicate that the fraction of recycled pulsars amongst the observed single radio pulsars is less than 1% at $B \geq 10^{11}\,\mathrm{G}$.

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Authors' Addresses

F. Verbunt, J.W. Hartman, and G. Nelemans: Astronomical Institute, Utrecht University, Postbox 80.000, 3508 TA Utrecht, the Netherlands

D. Bhattacharya: Raman Research Institute, Bangalore, 560 080, India

R.A.M.J. Wijers: Institute of Astonomy, Madingley Road, Cambridge 0HA CB3, United Kingdom