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Masses of Neutron Stars and Black Holes

Abstract

In this paper I give a brief review of the various methods that have been proposed and used to determine masses of neutron stars and black holes in binary stars. I summarize the results obtained from analyses of relativistic effects observed in binary radio pulsars, and from analyses of the orbital parameters of X-ray binaries, supplemented by additional constraints on the orbital inclination and the mass ratio.

I Neutron stars and black holes in X-ray binaries

The first X-ray source which was shown to be a member of a binary star, Cyg X-1, was a strong black-hole candidate as well. In the words of the discoverers: "The mass of the companion probably being larger than about 2 $M_\odot$, it is inevitable that we should also speculate that it might be a black hole." (Webster & Murdin 1972); "This raises the distinct possibility that the secondary is a black hole." (Bolton 1972).

Following the discovery of the binary X-ray pulsar Cen X-3 (Schreier et al. 1972) and many other similar systems, and of X-ray bursters (Grindlay et al. 1976; Belian et al. 1976), research in X-ray binaries in the 1970's was dominated by systems in which the accreting compact object is a neutron star. But research in black holes did not disappear altogether. The discovery of strong rapid variability of the X-ray flux of Cyg X-1 (Oda et al. 1971; see also Oda 1976, for a review of early work on Cyg X-1) led to the idea that such variability is a telltale sign of an accreting black hole, which might be used to distinguish them from accreting neutron stars. On the basis of this idea Cir X-1 was long considered a black-hole candidate. However, neutron star X-ray binaries can also show rapid variability, as was strikingly illustrated by the transient V 0332+53, which was initially suggested as a possible black-hole system, but later shown to be an X-ray pulsar (Stella et al. 1985), whose pulse amplitude happened to be relatively weak compared to that of the red-noise component in the PDS. Also Cir X-1 was shown to be a neutron star when it emitted type I X-ray bursts (Tennant et al. 1986).

Ostriker (1977) suggested that black-hole X-ray binaries (BHXB) might be distinguished by the shape of their X-ray spectra. This idea was put on a firm footing by White & Marshall (1984) who showed that in an X-ray color-color diagram, derived from the HEAO-1 A-2 sky survey the two sources, then known to contain black holes (Cyg X-1, in its soft state, and LMC X-3) were located in the extreme upper-left corner, i.e., their spectra were extremely soft. A few years later, McClintock & Remillard (1986) measured the mass function of the transient source A 0620–00 (which also had a very soft X-ray spectrum during its outburst in 1975) after it had returned to quiescence, to be 3.18 ± 0.16 $M_\odot$. This immediately (see below) showed that the compact star in this system is too massive to be a neutron star, and gave some confidence in the idea that X-ray spectra may be an efficient way to select BHXBs.

In spite of the fact that some X-ray spectral characteristics of black holes, and rapid variability are also seen in some neutron stars, their combined presence, in particular in X-ray transients, has remained strikingly effective in singling out black holes.

As implied in the above discussion, the main argument that the compact object in a particular X-ray binary is a black hole, is that neutron star masses cannot exceed a certain maximum value. This assumption rests on very general considerations, e.g., that sound cannot travel faster than light, on the basis of which Nauenberg & Chapline (1973) and Rhoades & Ruffini (1974) concluded that any neutron star, independent of the equation of state (EOS) of high-density matter, must have a mass $\lesssim 3.4\,M_\odot$. Rotation of the neutron star (ignored in the above analyses) does not increase the mass limit by more than 20% (Shapiro & Teukolsky 1983). Detailed modeling of neutron stars, for a wide range of equations of state, leads (see Figure 1) to upper mass limits between ~ 1.5 $M_\odot$ (very soft EOS) and ~ 2.5 $M_\odot$ (very stiff EOS) (see, e.g., Arnett & Bowers 1977; Datta 1988; Cheng et al. 1993; Cook et al. 1994; Engvik et al. 1996; Glendenning 1998).

The fact that compact objects with dynamical mass estimates exceeding ~ 3 $M_\odot$ cannot be neutron stars, is not equivalent to their being black holes, as defined by the particular space-time structure described by Schwarzschild and Kerr metrics, which are characterized, in particular, by the absence of a hard surface. This has led to the extensive use of the term "black-hole candidate" for these objects. Of course, detection of X-ray pulsations or X-ray bursts immediately disqualifies a compact star as a black hole, but positive evidence for the absence of a hard surface has been very hard to obtain. This should not come as a surprise, since a nominal ($M = 1.4\,M_\odot$, $R = 10$ km) neutron star is just 2.5 times larger than its Schwarzschild radius, and one may expect the accretion flow to be very similar to that of a black hole of comparable mass. The energy release at the neutron star surface, which is absent for a black hole, might lead to observable differences in spectra and variability, but unless the origin of the spectra and variability of X-ray binaries is much better understood than it is nowadays, the conclusion that a black hole has been found on the basis of such phenomena must be considered weak at best. This difficulty is illustrated by the fact that the spectral and variability characteristics of atoll sources are very similar to those of black holes in their low state (see Van der Klis 1995).

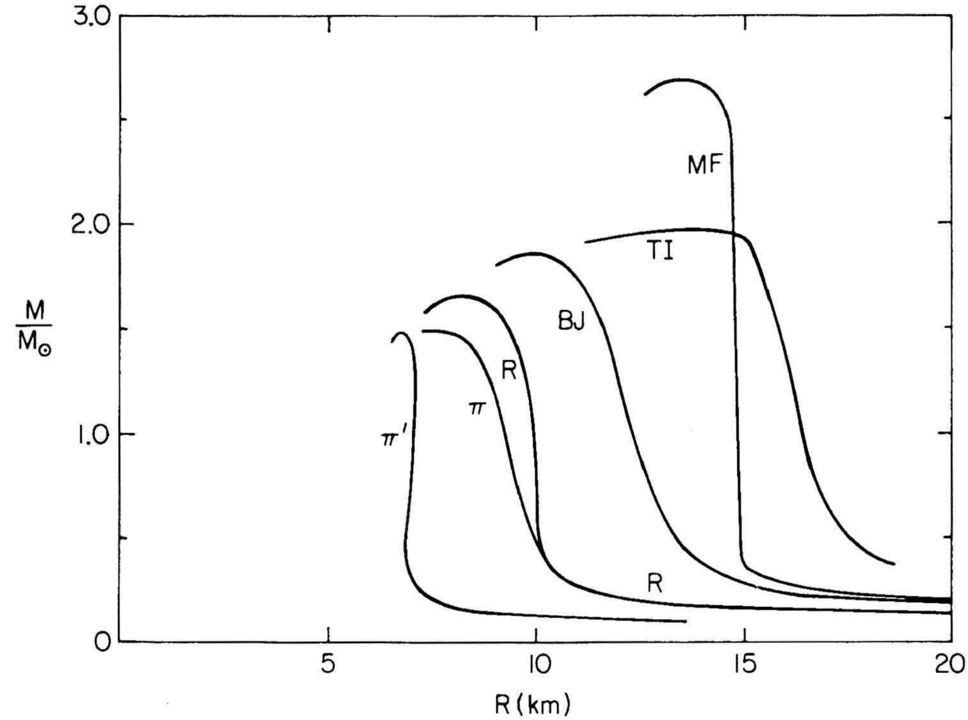

Figure 1. A selection of theoretical mass-radius relations for neutron stars, for several assumed equations of state (from Baym & Pethick 1979).

One way to infer the absence of a hard surface would be to show that at some distance from the compact object matter flows inward which does not give rise, by a large margin, to the emission of X rays from the compact object at the expected rate. Such evidence for the absence of a hard surface has recently been presented by Narayan et al. (1996, 1997) from a comparative study of the X-ray properties of quiescent SXTs with black holes and neutron stars, respectively.

Currently, ten X-ray binaries are known to contain black holes on the basis of a dynamical mass determination; seven of these are transient low-mass X-ray binaries (see Table 3). Another 17 systems are suspected to be BHXB on the basis of their X-ray spectra (White & Van Paradijs 1996). The total number of transient BHXB in the Galaxy is estimated to be of order 10^3 (Tanaka & Lewin 1995; White & Van Paradijs 1996).

II Neutron star masses

Apart from their crucial role in distinguishing black holes from neutron stars, the importance of measuring the masses of compact stars in X-ray binaries is that they may provide constraints on the properties of the high-density matter in the interior of neutron stars.

These properties are described by an equation of state (EOS), which together with the Oppenheimer-Volkov equations allows one to calculate models of the interior structure of neutron stars (see, e.g., Shapiro & Teukolsky 1983). Since neutron stars can be considered to be zero-temperature objects these models form a one-parameter sequence in which mass, M, and radius, R, depend only on the central density. For a given equation of state one thus has a unique mass-radius relation.

As will be discussed in more detail below, most neutron star masses are consistent with a value close to $1.4\,M_\odot$. From Fig. 1 it appears that at this value masses do not allow one to draw conclusions about the stiffness of the EOS of neutron star matter. For that, one would need observed masses in excess of $1.6\,M_\odot$, which would exclude the softest EOS (note that stiff equations of state are not excluded by low neutron star masses). Similarly, measurements of the gravitational redshift, z, at the neutron star surface alone are not a sensitive EOS discriminant, since both stiff and soft equations of state allow M/R ratios up to $\sim 0.2\,M_\odot\,\mathrm{km}^{-1}$ (see Fig. 1), corresponding to redshifts up to ~ 0.6.

Very accurate neutron star masses have been determined from a variety of general-relativistic effects on the radio pulse arrival times of double neutron star systems. These results will be briefly summarized in §III. Neutron star masses have been determined for six HMXB pulsars from pulse arrival time measurements, in combination with radial-velocity observations of their massive companions (see §IV). Masses have also been estimated for the low-mass binary radio pulsar PSR J1012+5307, whose companion is a white dwarf, and for the neutron stars in the LMXBs Cyg X-2 (a Z source), Cen X-4 (an SXT) and 4U 1626–67 (an X-ray pulsar).

In addition to direct measurements of mass and radius, a variety of other ways to obtain observational constraints on the EOS of neutron stars have been proposed. I will limit myself here to just mentioning them.

(i) Measurement of the limiting spin period of neutron stars (Friedman et al. 1986; Glendenning 1998).

(ii) The cooling history of neutron stars (Becker & Trümper 1998).

(iii) Measurement of the neutron star magnetic field from the energy of a cyclotron line in the spectrum of an X-ray pulsar, combined with an interpretation of its spin behavior in terms of an accretion torque model (Wassermann & Shapiro 1983).

(iv) Glitches in the spin period, and its derivative, of radio pulsars (Alpar 1998).

(v) The neutrino light curve during a supernova explosion (Loredo & Lamb 1989).

(vi) Measurements of kHz quasi-periodic oscillations in the X-ray intensity of LMXB, interpreted as Keplerian frequencies of orbits around neutron stars may provide constraints on the mass-radius relation of neutron stars by requiring that the neutron star and the innermost stable orbit around it, must fit inside the Kepler orbit (Kluzniak & Wagoner 1985; Miller, Psaltis & Lamb 1996; Kaaret & Ford 1997; Zhang et al. 1997). A detailed discussion of these kHz QPO is given by Van der Klis (1998).

(vii) Gravitational bending of X rays, as inferred from the X-ray pulse profile for a radio pulsar for which the emission geometry is determined from the radio pulse properties (Yancopoulos et al. 1994).

(viii) Measurement of the gravitational redshift of γ-ray lines emitted as a result of spallation processes on the surface of an accreting neutron star (Bildsten, Salpeter & Wasserman 1993).

(ix) Time-resolved spectroscopy of X-ray bursts, in principle, allows the derivation of constraints on the mass-radius relation of neutron stars (see Lewin, Van Paradijs & Taam 1993, for a detailed discussion). The practical value of this method is limited by the systematic uncertainty in the interpretation of the burst spectra (Van Paradijs et al. 1991) and the possibility that the burst emission is not uniformly distributed across the neutron star surface (Bildsten 1995, 1998).

III Mass determinations for binary radio pulsars

Relativistic effects

Radio pulse arrival times can be measured with exquisite accuracy (to several tens of ns), and at this level of accuracy several relativistic effects become strongly detectable. These have been conveniently described by Taylor & Weisberg (1989), based on a theoretical formalism developed by Damour & Deruelle (1986), for the case that general relativity is explicitly assumed to be valid. I refer the reader to these two papers for details, and limit myself here to a brief listing of the effects, as they have been applied so far to several binary radio pulsars.

By far the largest part of the pulse arrival time variations is the light travel time across the orbit, which Taylor & Weisberg describe by the "Roemer delay" parameter a_{R} (close to the classical semi-major axis of the orbit), which is related to the masses m_{p} and m_{c} of the pulsar and its companion, respectively, their sum M, and the orbital period P, by:

$$\frac{a_{\mathrm{R}}^3}{P^2} = \frac{\mathrm{G}M}{4\pi^2}\Big[1 + (\frac{m_{\mathrm{p}}m_{\mathrm{c}}}{M^2} - 9)\frac{\mathrm{G}M}{2a_{\mathrm{R}}\mathrm{c}^2}\Big]^2. \tag{1}$$

The orbit precesses, at a rate measured by the quantity k, defined by $\dot{\omega} = 2\pi k/P$, where k is given by:

$$k = \frac{3\mathrm{G}M}{\mathrm{c}^2 a_\mathrm{R}(1 - e^2)}. \tag{2}$$

Here ω is the angle between the line of nodes and the direction toward periastron (as measured in the orbital plane), and e is the orbital eccentricity.

The variation along the orbit of the gravitational redshift and time dilation (Einstein delay) is given by the quantity γ, related to the system parameters by:

$$\gamma = \frac{eP\mathrm{G}m_\mathrm{c}(m_\mathrm{p} + 2m_\mathrm{c})}{2\pi\mathrm{c}^2 a_\mathrm{R} M}. \tag{3}$$

Due to the emission of gravitational radiation the orbit decays; the rate of decrease of the period is given by

$$\dot{P} = -\frac{192\pi}{5\mathrm{c}^5}\,(2\pi\mathrm{G}/P)^{5/3}\, f(e)\, m_\mathrm{p} m_\mathrm{c} M^{-1/3}, \tag{4}$$

where

$$f(e) = \left[1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right](1 - e^2)^{-7/2}. \tag{5}$$

Finally, the pulses may show a measurable Shapiro delay, which reflects that near the pulsar companion the path along which the pulsar signal travels is curved. The shape of the orbital phase dependence of this delay is characterized by two quantities s and r, given by

$$s = \sin i, \quad r = \mathrm{G}m_\mathrm{c}/\mathrm{c}^3. \tag{6}$$

In the case of the Hulse-Taylor pulsar, PSR B1913+16, all above effects have been measured, each of which provides a different constraint on m_p and m_c. As shown by Taylor & Weisberg (1989), this overdetermined set of constraints leads to one consistent solution ($m_\mathrm{p} = 1.442 \pm 0.003\,\mathrm{M}_\odot$, $m_\mathrm{c} = 1.386 \pm 0.003\,\mathrm{M}_\odot$), showing that to the accuracy at which the test can be performed, general relativity provides a consistent description of this system.

For PSR B2127+11C in the globular cluster M15 Deich & Kulkarni (1996) measured the orbital precession, the Einstein delay and the orbital decay rate, from which they derived $m_\mathrm{p} = 1.350 \pm 0.040\ \mathrm{M}_\odot$, and $m_\mathrm{c} = 1.363 \pm 0.040\ \mathrm{M}_\odot$.

Table 1: Neutron star masses from relativistic effects on binary pulsar timing

Name	Method[a]	$m_{\rm p}$ ($M_\odot$)	$m_{\rm c}$ ($M_\odot$)	Ref.
J1518+4904	R,$\dot\omega$	1.54 ± 0.22	1.09 ± 0.19	[1]
B1534+12	R,$\dot\omega$,E	1.32 ± 0.03	1.36 ± 0.03	[2]
B1802–07	R,$\dot\omega$	$1.4\,(+0.4, -0.3)$	$0.33\,(+0.13, -0.10)$	[3]
B1855+09	R,S	$1.27\,(+0.23, -0.15)$	$0.233\,(+0.026, -0.017)$	[4]
B1913+16	R,$\dot\omega$,E,G,S	1.442 ± 0.003	1.386 ± 0.003	[5]
B2127+11C	R,$\dot\omega$,E,G	1.350 ± 0.040	1.363 ± 0.060	[6]
B2303+46	R,$\dot\omega$	1.16 ± 0.28	1.37 ± 0.24	[3]

[a] R = Roemer delay; $\dot\omega$ = periastron advance; E = Einstein delay; G = orbital decay by gravitational radiation; S = Shapiro delay. References: [1] Nice et al. (1996); [2] Wolszczan (1991); [3] Thorsett et al. (1993); [4] Ryba & Taylor (1991); [5] Taylor & Weisberg (1989); [6] Deich & Kulkarni (1996).

Wolszczan (1991) measured the periastron advance and the Einstein delay for the double neutron star system PSR B1534+12; adding the constraint $\sin i \leq 1$ he derived $m_{\rm p} = 1.32 \pm 0.03$ $M_\odot$, and $m_{\rm c} = 1.36 \pm 0.03$ $M_\odot$. Arzoumanian (1995) detected also orbital decay and Shapiro delay, and derived substantially improved masses for PSR B1534+12.

For PSR B1855+09, which is seen almost edge-on, Ryba & Taylor (1991) could measure the Shapiro delay, which directly gives the orbital inclination and the companion mass ($m_{\rm c} = 0.233^{+0.026}_{-0.017}\,M_\odot$); together with the mass function this gives $m_{\rm p} = 1.27^{+0.23}_{-0.15}\,M_\odot$. The companion is a low-mass white dwarf; theoretical estimates for its mass (based on evolutionary scenarios for the formation of systems like PSR B1855+09) agree with the measured value (Joss et al. 1987; Savonije 1987).

Thorsett et al. (1993) measured the rate of periastron advance for PSR B1802–07 and PSR B2303+46, from which they derive total system masses M of $1.7 \pm 0.4\,M_\odot$ and $2.53 \pm 0.08\,M_\odot$, respectively. Combining this with the observed mass functions, and assuming a probable inclination range, they derive $m_{\rm p} = 1.4^{+0.4}_{-0.3}\,M_\odot$, $m_{\rm c} = 0.33^{+0.13}_{-0.10}\,M_\odot$, for PSR B1802–07, and $m_{\rm p} = 1.16 \pm 0.28\,M_\odot$, $m_{\rm c} = 1.37 \pm 0.24\,M_\odot$, for PSR B2303+46. Improved values for the masses of these two systems have been derived by Arzoumanian (1995).

Nice et al. (1996) measured the rate of periastron advance and the mass function for PSR J1518+4904. Following the same argument as Thorsett et al. (1993) one finds for this system $m_{\rm p} = 1.54 \pm 0.22\,M_\odot$, $m_{\rm c} = 1.09 \pm 0.19\,M_\odot$.

Non-relativistic mass determinations

Van Kerkwijk et al. (1996) combined the pulse delay curves of the millisecond pulsar PSR J1012+5307 with the radial-velocity curve of its white-dwarf companion. This led to an accurately determined mass ratio $m_p/m_c = 13.3 \pm 0.7$. For a given white-dwarf composition the surface gravity acceleration, as inferred from model atmosphere fits to the profiles of the Balmer absorption lines in the white-dwarf spectrum, uniquely determine the white-dwarf mass, for which Van Kerkwijk et al. derive a value of $0.16 \pm 0.02\,(1\sigma)\,\mathrm{M}_\odot$. Combining this with the mass ratio and the mass function they find that the pulsar mass is in the range 1.5 to 3.2 $\mathrm{M}_\odot$ (95% confidence).

IV Mass determinations for neutron stars and black holes in X-ray binaries

Mass function

In determining the mass of an X-ray source, using Newtonian effects only, the fundamental quantity is the mass function $f_{\mathrm{opt}}(M)$ which is determined from the orbital period, P_{orb}, and the amplitude, K_{opt}, of the radial-velocity variations of the mass donor by

$$f_{\mathrm{opt}}(M) \equiv M_{\mathrm{X}}^3 \sin^3 i/(M_{\mathrm{X}} + M_2)^2 = \frac{K_{\mathrm{opt}}^3\, P_{\mathrm{orb}}}{2\pi \mathrm{G}}. \tag{7}$$

The corresponding quantity $f_{\mathrm{X}}(M)$ can be determined for binary X-ray pulsars:

$$f_{\mathrm{X}}(M) \equiv M_2^3 \sin^3 i/(M_{\mathrm{X}} + M_2)^2 = \frac{4\pi^2\, (a_{\mathrm{X}} \sin i)^3}{\mathrm{G} P_{\mathrm{orb}}^2}. \tag{8}$$

(The connection to observational parameters is written differently, since in the case of X-ray pulsars the observed quantities are usually pulse arrival times, whereas from optical spectra one measures radial velocities.)

Incomplete but occasionally extremely useful information may be obtained from a measurement of only one mass function, since it gives a lower limit to the mass of the companion of the star whose orbital motion is measured. As emphasized by McClintock & Remillard (1986) this is of great importance in distinguishing black holes from neutron stars in X-ray binaries.

If both mass functions can be measured, their ratio immediately gives the mass ratio $q \equiv M_{\mathrm{X}}/M_2 = f_{\mathrm{opt}}(M)/f_{\mathrm{X}}(M)$, and both masses are then determined separately, up to a factor $\sin^3 i$.

$$M_X \sin^3 i = f_{\rm opt}(M)\,(1+q^{-1})^2 \tag{9}$$

$$M_{\rm opt} \sin^3 i = f_X(M)\,(1+q)^2 \tag{10}$$

Inclination angle

To complete the mass determination one needs the orbital inclination i, for whose determination several methods are available, at least in principle: (i) X-ray eclipse durations, (ii) optical light curves, and (iii) polarization variations.

For a spherical companion star with radius R and a circular orbit (separation a, period P) the duration of the eclipse of a point-like X-ray source is related to i by the expression:

$$(R/a)^2 = \cos^2 i + \sin^2 i \sin^2 \theta_{\rm e}. \tag{11}$$

Here $\theta_{\rm e} = 2\pi t_{\rm e}/P$, with $t_{\rm e}$ half the duration of the eclipse. If the relative size of the companion is known, i is a function of $\theta_{\rm e}$ only. Direct estimates of R from the spectrum and luminosity class are not accurate enough to be useful. In general, the companion will not be spherical, due to the gravitational perturbation of the compact star. The relative size of the primary can be expressed as a function of the mass ratio q and a dimensionless potential parameter Ω, which is a measure of the extent to which the companion fills its Roche lobe. An observed eclipse duration then determines a relation between q, Ω, and i. The eclipse duration may be affected by absorbing effects on the X-rays by a stellar wind from the companion (see, e.g., Woo et al. 1995).

Electron scattering of originally unpolarized light in close-binary stars may yield a net polarization which varies with orbital phase, due to the deformation from spherical symmetry of the system (e.g., deformation of the companion star, presence of an accretion disk). Under rather general conditions the fundamental and first harmonic (in orbital frequency) of the variations of the Stokes parameters Q and U (see Tinbergen 1996) describe ellipses in the (Q, U) plane, whose eccentricity e is related to the orbital inclination by $e = \sin i$ for the fundamental, and $e = \sin^2 i/(1+\cos^2 i)$ for the first harmonic (Brown et al. 1978; Rudy & Kemp 1978 ; Milgrom 1979). Polarization variations may therefore provide a measurement of i. The method has been applied to several HMXB, e.g., Cyg X-1 (Dolan & Tapia 1989); for references to early work, see Van Paradijs (1983).

Many HMXB with an evolved companion show moderate (up to $\sim 10\,\%$) optical brightness variations, with two approximately equal maxima, and two different minima, which occur at the quadratures and conjunctions, respectively. These so-called "ellipsoidal" light curves are caused by the rotational and tidal distortion of the companion star, and a non-uniform surface brightness distribution ("gravity darkening"). The double-waved shape of the light curve reflects the pear-like shape of the companion: near conjunctions the projected stellar disk is smallest, near quadratures largest. For assumed co-rotation of the companion star, its distortion is determined by the mass ratio q, and by the dimensionless potential parameter Ω (see above); Roche lobe filling of the companion corresponds to a (q dependent) critical value of Ω. The distortion, and therefore the shape and amplitude of the ellipsoidal light curve, are determined by q and Ω, and furthermore by i. Thus, in principle, the optical light curve can provide a relation between q, Ω and i. Together with other constraints on these parameters (eclipse duration, q from two observed mass functions) this may lead to a solution for the masses of both components of the binary (see Van Paradijs 1983, for extensive references to early work on light curve modeling of HMXB).

The analysis by Tjemkes et al. (1986) showed that for well-studied HMXB with evolved companions the optical light curves may be reproduced if the best-known system parameters are used. Conversely, however, X-ray heating of the companion, the presence of an accretion disk, and intrinsic variability of the companion have a significant effect on the light curve, to the extent that it is difficult to derive significant constraints on these systems from an analysis of their light curves. For Be/X-ray binaries, irregular brightness variations related to equatorial mass shedding in general dominate any orbital brightness variations.

Ellipsoidal light curves have been observed for several soft X-ray transients in quiescence, whose optical light is then dominated by the companion star. Since these companions fill their Roche lobes, the potential parameter Ω is determined by q, and therefore the light curve depends on q and i only. If needed, the generally small contribution from the quiescent accretion disk may be estimated from a study of the broad-band energy distribution, and corrected for. The contribution of the disk diminishes with increasing wavelength, and recent orbital light curve studies are therefore preferentially carried out in the infra-red (Casares et al. 1993; Beekman et al. 1996; Shahbaz et al. 1997). The ellipsoidal light curves of SXT in quiescence have played an important role in the mass estimates of black holes. However, one should not ignore Haswell's (1995) emphasis that also for quiescent SXT the ellipsoidal light curves may be affected by systematic effects, possibly caused by variable contributions from an accretion disk, analogous to the superhumps in the SU UMa subgroup of the cataclysmic variables.

Additional Constraints

In addition to the above standard ingredients of a mass determination for X-ray binaries two additional pieces of information may be used. The first applies when the source distance is known. Since the apparent magnitude of the companion star (corrected for interstellar extinction) and its spectral type together determine its angular radius, for these sources one can estimate the radius, R_2, of the companion star, and this constrains the relation between M_X and M_2. This was used by Gies & Bolton

Table 2: Neutron Star Masses: X-ray Binaries and Binary Radio Pulsars

Name	i (°)	M_X ($M_\odot$)	M_{comp} ($M_\odot$)	Ref.
HMXB				
Vela X-1	> 74	1.88(+0.69,–0.47)	23.5(+2.2,–1.5)	[1]
4U 1538–52	68(+9,–8)	1.06(+0.41,–0.34)	16.4(+5.2,–4.0)	[1]
SMC X-1	70(+11,–7)	1.6 ± 0.1	17.2 ± 0.6	[2]
LMC X-4	65(+7,–6)	1.47(+0.44,–0.39)	15.8(+2.3,–2.0)	[1]
Cen X-3	> 66	1.09(+0.57,–0.52)	18.4(+4.0,–1.8)	[1]
Her X-1[a]	> 79	1.47(+0.23,–0.37)	2.32(+0.16,–0.29)	[1]
Her X-1	> 79	1.5 ± 0.3	2.3 ± 0.3	[3]
LMXB				
Cen X-4	30-37	1.1-1.9	< 0.2	[4,5]
4U 1626–67[a]	9-36	1.8(+2.8,–1.3)	< 0.5	[6]
Cyg X-2	< 73	$> 1.42(\pm 0.08)$	$> 0.47(\pm 0.03)$	[7]
LMBP				
J1012+5307		1.5-3.2	0.16 ± 0.02	[8]

[a] From Doppler shifted optical pulsations.
References: [1] Van Kerkwijk, Van Paradijs & Zuiderwijk (1995); [2] Reynolds et al. (1993); [3] Reynolds et al. (1997); [4] Chevalier et al. (1989) [5] McClintock & Remillard (1990); [6] Middleditch et al. (1981); [7] Casares, Charles & Kuulkers (1997); [8] Van Kerkwijk, Bergeron & Kulkarni (1996).

(1986) in their analysis of Cyg X-1. In case the companion star fills its Roche lobe, the constraint is simple: for such stars the density is determined by the orbital period alone (see, e.g., Frank et al. 1992), and M_2 follows immediately from R_2. In the case of the SXT Cen X-4 (whose compact star is a neutron star) this served to show that the companion mass is extremely low, less than $0.2\,M_\odot$ (Chevalier et al. 1989; McClintock & Remillard 1990).

If the orbital angular momentum is parallel to that of the companion star (and therefore of the matter flowing through the accretion disk), both K_{opt} and the observed rotational velocity $V_{rot} \sin i$ of the secondary have been decreased by the same projection factor $\sin i$. Therefore, their ratio is not affected. As shown by Gies & Bolton (1986) this constrains the relation between M_X and M_2. In case the companion star co-rotates at the same angular velocity as the orbit, this constraint is an estimate of the mass ratio. Using the expressions for the radius of the Roche lobe by Paczynski (1971), one obtains (Kuiper et al. 1988; Wade & Horne 1988):

$$\left(V_{rot} \sin i / K_{opt}\right) = 0.462\, q^{-1/3}\, (1+q)^{2/3}. \tag{12}$$

Table 3: Black-Hole Binary Candidates from Radial-Velocity Measurements

Name	Nova	$P_{\rm orb}$ (days)	$f(M)$ ($M_\odot$)	i (°)	M_X ($M_\odot$)	Ref.
HMXB						
Cyg X-1	No	5.6	0.25	28–38	16 ± 5	[1-3]
LMC X-3	No	1.7	2.3	64–70	3.5–10	[4,5]
LMC X-1	No	4.2	0.144	40–63	4–10	[6]
LMXB						
A 0620–00	Yes	0.32	2.70	66.5–73.5	3.3–4.2	[7,8]
				31–54	14 ± 7	[9]
GS 2023+338	Yes	6.47	6.08	52–60	10–15	[10-12]
GS/GRS 1124–68	Yes	0.43	3.1	55–65	4.5–7.5	[13]
GRO J0422+32	Yes	0.21	1.2	45–51	3.2–3.9	[14,15]
				13–31	> 9	[16]
GRO J1655–40	Yes	2.60	3.16	63–71	7 ± 0.7	[17-19]
GS 2000+25	Yes	0.35	5.0	43–85	4.8–14.4	[20-23]
H 1705–25	Yes	0.52	4.0	60–80	3.5–8.5	[24]

[1] Webster & Murdin (1972); [2] Bolton (1972); [3] Gies & Bolton (1986) [4] Cowley et al. (1983); [5] Kuiper et al. (1988) [6] Hutchings et al. (1987); [7] McClintock & Remillard (1986); [8] Marsh et al. (1994); [9] Shahbaz et al. (1994a); [10] Casares et al. (1992); [11] Casares & Charles (1994); [12] Shahbaz et al. (1994b); [13] Remillard, McClintock & Bailyn (1992); [14] Filippenko, Matheson & Ho (1995); [15] Casares et. al. (1995a); [16] Beekman et al. 1997; [17] Bailyn et al. (1995); [18] Van der Hooft et al. (1998); [19] Orosz & Bailyn (1997); [20] Casares, Charles & Marsh (1995b); [21] Filippenko, Matheson & Barth (1995); [22] Beekman et al. (1996); [23] Callanan et al. (1996); [24] Remillard et al. (1996).

The cancellation of the $\sin i$ factor has also been applied to the rotational velocity at some radial distance in the accretion disk, as inferred from emission line profiles. Warner (1976) applied this to cataclysmic variables, and Johnston et al. (1990) to the SXT A0620–00. Since relatively little is known about the emission line structure of accretion disks, the interpretation of the q values inferred from this method is somewhat uncertain.

Optical pulsations

Optical pulsations have been detected from Her X-1 and 4U 1626–67. These pulsations arise from the reprocessing of pulsed X rays in the companion star and the accretion disk, and can be used to study the orbital parameters of these systems.

A detailed analysis of the optical pulsations of Her X-1 was made by Middleditch & Nelson (1976). The orbital motion of the neutron star is known from the Doppler shifts of the X-ray pulse arrival times. Optical pulsations in phase with the X-ray pulsations are present, but also optical pulsations with a slightly different frequency. The former arise from reprocessing in the accretion disk, the latter from the surface of the companion; their frequency difference is just the orbital frequency (beat frequency relation). The pronounced variation with orbital phase of the amplitude of the optical pulsations indicates that the companion is non-spherical and fills its Roche lobe. By assuming that across the surface of the Roche lobe filling companion the X-ray reprocessing time is constant Middleditch & Nelson used the Doppler shift information of these optical pulsations to estimate the mass ratio, which together with the eclipse duration and the X-ray mass function leads to $M_X = 1.30 \pm 0.14\,M_\odot$ and $M_{opt} = 2.18 \pm 0.11\,M_\odot$. These values are consistent with those obtained from the radial-velocity curve of the late A-type companion star (Reynolds et al. 1997).

Doppler shifts of the 7.7 s X-ray pulsations of 4U 1626–67 have not been detected so far: $a_X \sin i < 10$ ms (Levine et al. 1988). Optical pulsations, arising from reprocessing of X rays in the disk and in the companion star were detected by Middleditch et al. (1981; see also Chakrabarty 1998). From a detailed modeling of these pulsations, similar to that for Her X-1, they derived $M_X = 1.8^{+2.9}_{-1.3}\,M_\odot$ and $M_2 < 0.5\,M_\odot$.

V Summary of mass determinations of neutron stars and black holes

The results of the mass estimates, described in §§III and IV, are summarized in Table 1 (relativistic effects in binary radio pulsars), Table 2 (neutron stars in X-ray binaries, and non-relativistic orbital analyses of binary radio pulsars), and in Table 3 (black holes in X-ray binaries). From these tables one may draw the conclusion that with few exceptions neutron star masses are consistent with a relatively narrow mass range near the "canonical" value of $1.4\,M_\odot$. Vela X-1 and Cyg X-2 present interesting examples of substantially higher neutron star masses, but the possible presence of systematic effects on these mass determinations has to be investigated before the consequence, i.e., that very soft equations of state are excluded, can be accepted.

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