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Quantum Optics in Photonic Band Structures

Abstract

We address several generic quantum optical processes that undergo basic modifications in photonic crystals: (a) spontaneous formation of atomic coherence; (b) two-photon binding and entanglement; (c) self-induced transparency and gap solitons.

1 Introduction

Dielectric structures whose refractive index is periodically modulated on a submicron scale, known as photonic crystals (PCs) [1] are attracting considerable interest at present. Optical processes involving many atoms or excitons in such (PCs) undergo *basic modifications* as compared to the corresponding processes in open space or in bulk media. Here we survey our recent results on these modifications, which we attribute to three fundamental properties of PCs: (a) *Modified density of modes (DOM)*, which is characterized in PCs, by strong suppression of the background DOM within photonic band gaps (PBGs), by *sharp band-edge cutoffs* and *intra-gap* narrow lines associated with *defects* [1]. Many of the results detailed below (Sec. 2,3) stem from the failure of perturbation theory and the onset of strong field-atom (-exciton) coupling near sharp band edges or narrow lines in various PCs [2]. (b) *Band-edge and Bragg reflections*, which cause *spatial interference* effects in pulse propagation through the structure (Sec. 4). (c)*Photon effective masses*, which are associated with band-dispersion effects in PCs, and allow (*i*) photon "binding" to one or many atoms (Sec. 3), or (*ii*) Kerr-nonlinear inter-photon binding corresponding to quantum-soliton states [3].

2 Spontaneously formed coherence in PCs

A fundamental process relying on mode-density spectra in such structures is spontaneously induced coherence, due to strong coupling effects. We have developed an *exact* (non-perturbative) theory of near-resonant interaction with quantized fields having arbitrary DOM spectra [2]. This theory implies that an excited two-level system, whose resonance lies near a band-edge or narrow line in the DOM of the vacuum field would *spontaneously* evolve into a superposition of single-photon dressed states. This is a generalization of vacuum Rabi splitting, which occurs when the atom is near-resonant with a narrow mode. For a finite-width band, the states are pushed away from the edge, the ones ending up in the allowed band decay, whereas the ones pushed into the gap do not. Remarkably, there *can be multiple stable dressed states* in a *coherent superposition*, as many as there are gaps in the DOM.

In Fig.1 we show the splitting of an excited state $|e\rangle$ into superposed stable states, oscillating at band-gap frequencies ω_j, each consisting of superposed zero-photon $|e,0\rangle$ and

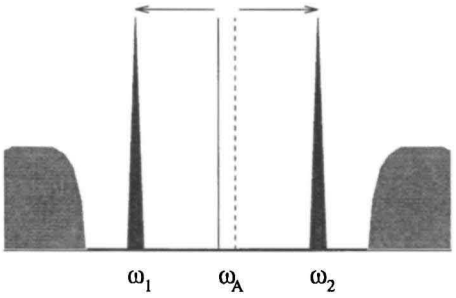

Figure 1: Splitting of an atomic excited state (dashed) into superposed stable dressed states in gaps (narrow lines) and decaying states (broad line shapes).

one-photon $|g, 1_\omega\rangle$ states, weighted by the field-dipole coupling $\kappa(\omega)$ and DOM $\rho(\omega)$

$$|e\rangle \to |\Psi_{1+2}\rangle = \sum_{j=1,2} c_j \left[|e, \{0\}\rangle - \int \frac{\kappa\rho(\omega)}{\omega - \omega_j} |g, 1_\omega\rangle \, \mathrm{d}\omega\right] \exp\left(i\omega_j t\right) \tag{1}$$

The amplitudes of these states, c_j, and their eigenfrequencies ω_j are *controllable* by the atomic transition detuning from cutoff and by the band DOM $\rho(\omega)$. This *spontaneous coherence control* opens interesting perspectives for lasing without inversion (LWI) in PCs containing 3-level atoms which have one resonant transition near a band edge.

3 Two-photon bound states by resonant interactions with atoms in PCs

The one-dimensional interaction of light with a system of atoms has been shown, by Bethe's ansatz, to produce multi-photon correlated states (referred to as string states) [4]. However, the energy of the two-photon correlated state in the string model is *exactly the same* as that of the unbound state. This makes any direct observation of the string two-photon state highly difficult.

We study here a principally new mechanism, whereby *group-velocity dispersion* in a 1D PC where light interacts with a collection of two-level atoms may lead to the production of a bound two-photon state with *lowered energy*, which makes its creation more favorable. The construction of two-photon solutions is then possible. Group-velocity dispersion may be obtained in a PC *outside* the band gap. This dispersion allows us to control the characteristics of the diphoton bound state.

We shall consider a one-dimensional structure, in which all the relevant mode frequencies ω are not too far from the atomic resonance Ω, i.e.

$$|\Omega - \omega| \ll \Omega \tag{2}$$

Introducing the notation $\lambda = k - k_0$, we can write the dispersion relation as

$$\omega(k) = \Omega + v_{gr}\lambda + \zeta\lambda^2 \tag{3}$$

where $v_{gr} = \partial\omega(k)/\partial k|_{k=k_0}$ is the group velocity (we use units $\hbar = c = 1$) and the last term corresponds to the inverse effective "mass" of the photon, $\zeta = 1/2\ \partial^2\omega(k)/\partial k^2|_{k=k_0}$ being the group velocity dispersion.

In what follows we shall use the field operator $\epsilon^+(x)$, which is the Fourier transform from k to x of the photon creation operator $a^+(\lambda)$

$$\epsilon(x) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} a(\lambda) e^{i\lambda x} \tag{4}$$

One-photon eigenstates of this system can be obtained in the Wigner-Weisskopf form [5]

$$\begin{aligned} |\lambda\rangle &= \int_{-\infty}^{\infty} dx\ \hat{\Psi}^\dagger(\lambda, x)|0\rangle \\ &= \int_{-\infty}^{\infty} dx\ e^{i\lambda x} f(\lambda, x)\epsilon^+(x)|0\rangle \\ &+ \sum_j g_j(\lambda) S_j^+|0\rangle \end{aligned} \tag{5}$$

where $S_j^{\pm}$ are the pseudospin operators of the j-th two-level atom, which is located at the point x_j and the field amplitude envelope $f(\lambda, x)$ and atomic excitation $g_j(\lambda)$ are obtained from the Schrödinger equation $\mathcal{H}|\lambda\rangle = \omega(\lambda)|\lambda\rangle$.

We search for a two-photon solution in the Bethe-ansatz form

$$\begin{aligned} |\lambda_1, \lambda_2\rangle = \int\int_{-\infty}^{\infty} & dx_1\, dx_2\ \mathcal{A}_{\lambda_1,\lambda_2}(x_1 - x_2) \\ & \times \hat{\Psi}^\dagger(\lambda_1, x_1)\hat{\Psi}^\dagger(\lambda_2, x_2) \otimes |0\rangle \end{aligned} \tag{6}$$

Here the Bethe factor $\mathcal{A}_{\lambda_1,\lambda_2}(x_1 - x_2)$ reflects the appearance of photon-photon correlations.

The two-photon bound state "diphoton" corresponds to complex λ's

$$\begin{aligned} \lambda_1 &= \Lambda + i\gamma \\ \lambda_2 &= \Lambda - i\gamma \end{aligned} \tag{7}$$

with positive γ (the inverse width of the correlation length of the diphoton state).

We have the following equation for the inverse correlation length of the diphoton

$$\begin{aligned} -4\zeta^2\gamma^3 + [2(v_{gr} + 2\zeta\Lambda)^2 \\ +4\zeta(v_{gr}\Lambda + \zeta\Lambda^2)]\gamma = \eta \end{aligned} \tag{8}$$

To first order in the velocity dispersion ζ the inverse width of the diphoton may be estimated as

$$\gamma \approx \frac{\eta}{2v_{gr}^2 + 12v_{gr}\zeta\Lambda} \tag{9}$$

and is of the order of the inverse life time of the excited atomic level.

The binding energy of the diphoton state is

$$\mathcal{E} = -2\zeta\gamma^2 \tag{10}$$

Thus for positive group-velocity dispersion ζ, the energy of the diphoton state is lower than the energy of the unbound state. We have considered the case when corrections due to group-velocity dispersion are small $\zeta\Lambda \ll v_{gr}$. In this case the width of the diphoton state may be estimated to be close to the natural line-width of the resonant atoms, $\gamma \sim \eta \sim 10^9$ s^{-1}, and the group-velocity dispersion coefficient for the photonic band gap structure may be estimated to be inversely–proportional to the band-width of the forbidden zone $\Delta\omega \sim 10^{12}$ s^{-1}. In this case the binding energy of the diphoton would be of the order of 10^6 s^{-1}.

4 Self-induced transparency in photonic band structures: gap solitons near absorption resonances

Pulse propagation in a non-uniform resonant medium, e.g., a periodic array of resonant films, can destroy self-induced transparency (SIT) [6], because the pulse area is then split between the forward and backward (reflected) coupled waves, and is no longer conserved [7]. Should we then anticipate severely hampered transmission through a medium whose resonance lies in a reflective spectral domain (photonic band gap) of a PC (a Bragg reflector)? We have shown analytically and numerically [8, 9, 10] that it is possible for the pulse to overcome the band-gap reflection and produce SIT in a near-resonant medium embedded in a Bragg reflector. The predicted SIT propagation is a *principally new type of a soliton*, which does not obey any of the familiar soliton equations, such as the non-linear Schrödinger equation (NLSE) or the sine-Gordon equation.

Qualitatively, the SIT soliton in a PC may be understood as the addition of a near-resonant non-linear refractive index to the modulated index of refraction of the Bragg structure. When this addition compensates the linear modulation, then there is no band gap and soliton propagation is possible (Fig.2).

The proposed mechanism of gap solitons is revealed in a periodic array of thin layers of *resonant two-level systems* (TLS) separated by half-wavelength nonabsorbing dielectric layers, i.e., a *resonantly absorbing Bragg reflector* (RABR). Such a RABR has been shown by us to have, for *any* Bragg reflectivity, a vast family of stable solitons, both standing and moving [9, 10]. As opposed to the 2π-solitons arising in self induced transparency, i.e., resonant field – TLS interaction in a uniform medium, gap solitons in a RABR can have an *arbitrary* pulse area. The main innovation of our findings is that they demonstrate an unexpected property of a RABR with active layers. The RABR with thin active layers provides, to the best of our knowledge, the first example of a nonlinear optical medium in which *stable* bright and dark solitons exist for the *same values* of the model's parameters (at different frequencies).

The periodic grating gives rise to band gaps in the system's linear spectrum, i.e., the medium is totally reflective for waves whose frequency is inside the gaps. The central frequency of the fundamental gap is $\omega_c = k_c c/n_0$, c being the vacuum speed of light, and the gap edges are located at the frequencies $\omega_{1,2} = \omega_c\,(1 \pm a_1/4)$, where a_1 is the grating modulation

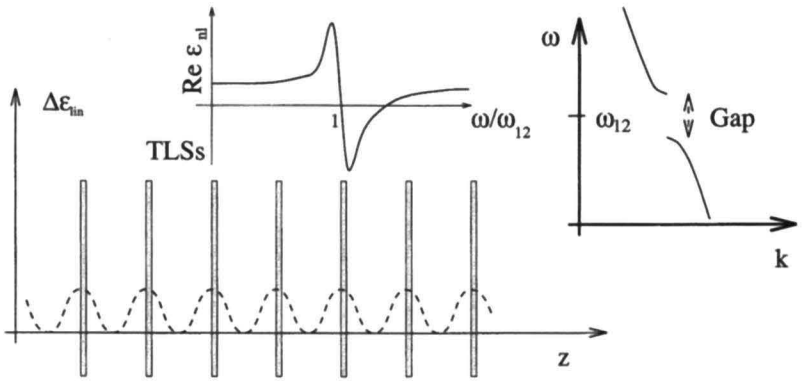

Figure 2: The first-harmonic modulation $\Delta\epsilon_1 \cos 2kz$ of the linear refractive index (dashed curve) in a structure of periodically alternating layers. This modulation can be canceled by the near-resonant nonlinear response Re $\epsilon_{\rm nl}$ (inset), if it has the opposite sign to $\Delta\epsilon_1$ at the TLS positions.

depth. We further assume that *very thin* TLS layers (much thinner than $1/k_c$) whose resonance frequency ω_0 is close to the gap center ω_c, are placed at the maxima of the modulated refraction index. In other words, the thin active layers are placed at the points $z_{\rm layer}$ such that $\cos(k_c z_{\rm layer}) = \pm 1$. Quantum wells embedded in Bragg mirrors are adequately described as TLS layers.

The electric field $E(z,t)$ can be decomposed into cosine and sine spatial components, having the dimensionless slowly varying amplitudes Σ_+ and Σ_-, respectively,

$$\begin{aligned} E(z,t) &= \hbar(\mu\tau_0)^{-1}\left(\mathrm{Re}\,\left[\Sigma_+(z,t)e^{-i\omega_c t}\right]\cos k_c z\right. \\ &- \left.\mathrm{Im}\,\left[\Sigma_-(z,t)e^{-i\omega_c t}\right]\sin k_c z\right), \end{aligned} \tag{11}$$

where μ is the transition dipole moment of the TLS, and the characteristic absorption time of the field by the TLS is $\tau_0 = n_0\mu^{-1}\sqrt{\hbar/2\pi\omega_c\varrho_0}$, with ϱ_0 being the TLS density (averaged over z).

The equations for the field envelope in the symmetric mode Σ_+ and the polarization envelope P form a *closed system*,

$$\begin{aligned} \frac{\partial^2\Sigma_+}{\partial\tau^2} - \frac{\partial^2\Sigma_+}{\partial\zeta^2} &= -\eta^2\Sigma_+ + 2i(\eta-\delta)P \\ &- 2\sqrt{1-|P|^2}\,\Sigma_+, \end{aligned} \tag{12}$$

$$\frac{\partial P}{\partial\tau} = -\,i\delta P\; -\sqrt{1-|P|^2}\,\Sigma_+, \tag{13}$$

where dimensionless time τ, coordinate ζ, and detuning δ are defined as follows:

$$\tau \equiv t/\tau_0,\ \zeta \equiv (n_0/c\tau_0)\,x,\ \delta \equiv (\omega_0-\omega_c)\tau_0. \tag{14}$$

The dimensionless modulation strength η is the ratio of the TLS absorption distance to the Bragg reflection distance, which can be expressed as $\eta = a_1\omega_c\tau_0/4$. The envelope of the antisymmetric field component Σ_- is driven by $\partial P/\partial\zeta$.

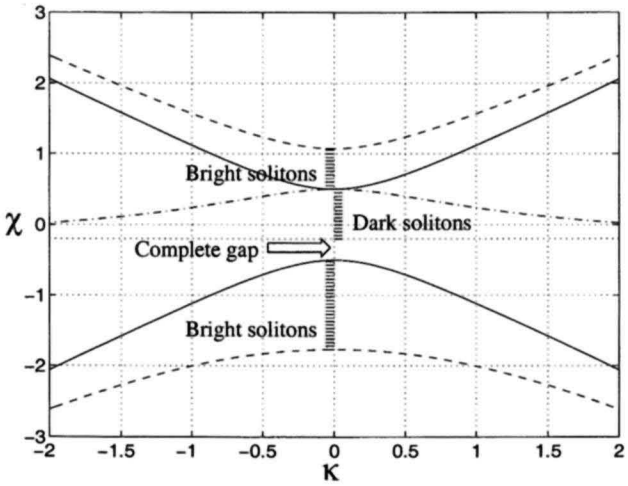

Figure 3: The dispersion curves (dimensionless frequency χ versus dimensionless wavevector k) at $\eta = 0.5$ and $\delta = -0.2$. The solid lines show the dispersion branches corresponding to the "bare" (noninteracting) grating, while the dashed and dash-dotted lines stand for the dispersion branches of the grating "dressed" by the active medium. The frequency bands that support the standing dark and bright solitons are shaded. The arrow indicates a complete gap, where no field propagation takes place.

The spectrum produced by the linearized version of Eqs. (12) and (13) is obtained on assuming the TLS population to be uninverted; we then arrive at the dispersion relation for the wavenumber κ and frequency χ. Different branches of the dispersion relation are shown in Fig. 3. The roots $\chi = \pm\sqrt{\kappa^2 + \eta^2}$ (corresponding to the solid lines in Fig. 3) originate from the driven equation for Σ_- and represent the dispersion relation of the Bragg reflector with the gap $|\chi| < \eta$, that does not feel the interaction with the active layers. The important roots are described by the dashed and dash-dotted lines; they will be shown to correspond to bright or dark solitons in the indicated (shaded) bands.

Stationary solutions for the symmetric-mode field envelope Σ_+ and polarization envelope P are sought in the form $\Sigma_+ = e^{-i\chi\tau}\mathcal{S}(\zeta)$ and $P = i\, e^{-i\chi\tau}\, \mathcal{P}(\zeta)$ with real $\mathcal{P}$ and $\mathcal{S}$. Bright solitons can be shown to appear in two frequency bands χ; the lower band being $\chi_1 < \chi < \min\{\chi_2, -\eta, \delta\}$, and the upper band $\max\{\chi_1, \eta, \delta\} < \chi < \chi_2$, where the boundary frequencies $\chi_{1,2}$ are given by $\chi_{1,2} \equiv (1/2)\left[\delta - \eta \mp \sqrt{(\eta + \delta)^2 + 8}\right]$. The lower band exists for all values $\eta > 0$ and δ, while the upper one only exists for $\delta > \eta - 1/\eta$, which follows from the requirement $\chi_2 > \eta$. An example of bright solitons is depicted in Fig. 4. Note that, depending on the parameters η, δ and χ, the main part of the soliton's energy can be carried either by the Σ_+ or the Σ_- mode.

Dark solitons (DS's) are obtained similarly to the bright ones. The condition for their existence determines the following frequency interval χ (η is defined to be positive): $\max\{\delta, -\eta\} < \chi < \min\{\chi_2, \eta\}$, and $\delta < \eta$. The DS frequency range is marked by shading (to the right

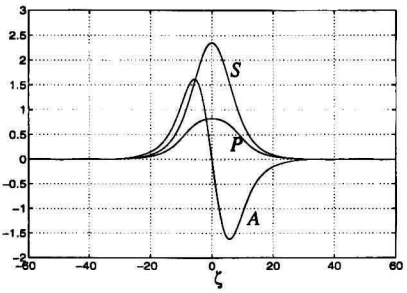

Figure 4: A typical example of a bright soliton. The variables $\mathcal{S}$, $\mathcal{P}$, and $\mathcal{A}$ are plotted as a function of ζ for the parameters $\eta = 0.2$, $\delta = -2$, and $\chi = 0.4$.

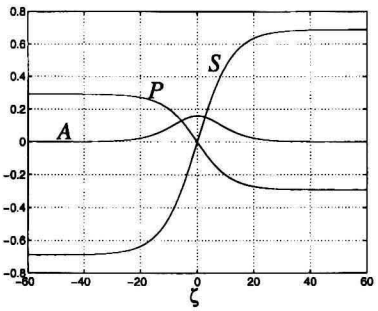

Figure 5: A typical example of a dark soliton, presented in terms of the variables $\mathcal{S}$, $\mathcal{P}$ and $\mathcal{A}$. The parameters are $\eta = 0.6$, $\delta = -2$, and $\chi = 0.25$.

from zero) in Fig. 3. An example of a DS amplitude in the Σ_+ mode, together with the corresponding quantities $\mathcal{P}$ and $\mathcal{A}$, are plotted in Fig. 5. The DS frequency band *always coexists* with one or two bands supporting the bright solitons. Quite naturally, the bright and dark solitons cannot have the same frequency.

Let us now discuss the experimental conditions for the realization of the solitons. Excitons in periodic quantum wells can, under certain conditions (such as low densities [11]) be described as effective two-level systems (TLS's). We consider their surface density to be $\approx 10^{10} - 10^{11}$ cm^{-2}. Structures occupying a region of approximately 100 absorption lengths would require a device of the total width of approximately 1 mm to 1 cm, which corresponds to $\approx 10^3$ to 10^4 unit cells. The modulation of the refraction index can be as high as $a_1 \approx 0.3$, so that the parameter η can vary from 0 to 10^2. The intensities of the applied laser field corresponding to $\Sigma_\pm \approx 1$ are then of the order $10^6 - 10^7$ W/cm^2. The dephasing time is currently $1/\Gamma_2 \approx 10^{-13}$ s. Decreasing the dephasing rate appears to be the main experimental challenge.

5 Conclusions

Strong field-atom coupling near band-edge cutoff or a narrow defect line in a PC has been shown to allow unprecedented control of the following properties: (a) *Single-atom* spontaneously-induced *coherence*, which can give rise to lasing without inversion (LWI). (b) Photon binding, resulting in stable propagation in resonant media has been connected with photon effective masses. (c) Gap solitons have revealed their capacity for "filtering" undesired pulse shapes and creating self-induced "cavities".

The above novel features offer the first glimpses into the remarkable possibilities offered by field confinement in PCs for the design and control of coherent/cooperative processes.

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