

# Spontaneous Emission Rate Alteration in Photonic Crystal Waveguides and Photonic Crystal Microcavities

## Abstract

A model for spontaneous emission rate alteration based on the Fermi Golden rule and the position-dependent photon density of states is outlined. Numerical results are given for the rate of spontaneous emission in a waveguide and a microcavity, where the design is based on introducing a defect in a two-dimensional photonic crystal.

Photonic crystals represent a new promising class of periodic dielectric structures for control of light-matter interaction [1]. A new class of waveguides and microresonators, which we shall refer to as photonic crystal waveguides and photonic crystal microcavities, may be designed by introducing a defect in a photonic crystal. In this paper the alteration of spontaneous emission in the region of a line defect and a point defect in a two-dimensional photonic crystal is considered. Recently, a laser design based on introducing a defect in a two-dimensional photonic crystal was experimentally demonstrated [2]. The model for spontaneous emission used in this paper is based on the Fermi Golden rule and the position-dependent photon density of states (PDOS). The position-dependent PDOS for two-dimensional photonic crystals with no defects introduced has previously been considered by Søndergaard and Busch [3, 4]. The structures considered in this paper are approximated by a periodic structure using a supercell approximation, and for periodic structures the complex modes of the electric field may, in accordance with Bloch's theorem, be written on the form

$$\mathbf{E}_{\mathbf{k},n}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\mathbf{U}_{\mathbf{k},n}(\mathbf{r}), \quad (1)$$

where  $\mathbf{k}$  is a wave vector,  $n$  is the band number, and  $\mathbf{U}_{\mathbf{k},n}(\mathbf{r})$  is a function with the same periodicity as the periodic structure. The corresponding angular frequency is denoted  $\omega_{\mathbf{k},n}$ . In terms of these modes the position-dependent PDOS is defined as

$$\mathcal{E}(\mathbf{r}, \omega) = \sum_{\mathbf{k},n} \delta(\omega - \omega_{\mathbf{k},n}) |\mathbf{E}_{\mathbf{k},n}(\mathbf{r})|^2, \quad (2)$$

where the energy of each mode within a period of the periodic structure is normalized to unity.

Enhancement and suppression of spontaneous emission is evaluated by comparing the PDOS (2) with the corresponding PDOS for a homogeneous dielectric with the same dielectric

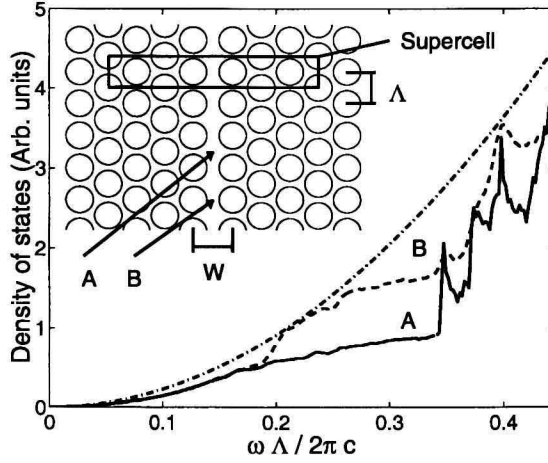


Figure 1: The position-dependent photon density of states is shown for two positions A and B (see inset) within a two-dimensional photonic crystal waveguide. For comparison the density of states is also shown for a homogeneous dielectric with the same dielectric constant as the background material.

constant as the background material. The electric field distributions (1) are calculated using fully vectorial plane wave expansion theory and a variational principle [5].

Spontaneous emission rate alteration for a photonic crystal waveguide is considered in Figure 1. The photonic crystal waveguide and the supercell used as an approximation are shown as an inset. The photonic crystal is characterized by circular air-holes arranged on a triangular lattice, and the waveguide may be thought of as a line defect in the crystal. The frequency is normalized using the center-to-center air-hole spacing  $\Lambda$ . The diameter of the air-holes is  $0.83\Lambda$ , and the width of the waveguide is given by  $W = 1.2\Lambda$ . The PDOS (2) is shown for the positions A and B within the waveguide. Also shown is the PDOS for a homogeneous dielectric with dielectric constant 13 (representative of GaAs at optical frequencies) corresponding to the photonic crystal background material. At both positions A and B the PDOS is below the parabolic curve for the homogeneous dielectric, and consequently the rate of spontaneous emission is reduced relative to the homogeneous dielectric.

Figure 2 shows the position-dependent PDOS for two positions A and B (see inset) in the region near a point-like defect in a two-dimensional photonic crystal. In this case the air-hole diameter is  $0.86\Lambda$ , and a defect has been introduced by reducing the diameter of a single air-hole to  $0.57\Lambda$ . As an inset the amplitude of the electric field squared is shown for a localized non-degenerate mode with frequency  $\omega\Lambda/2\pi c = 0.374$ . The amplitude of the electric field squared for this mode is strong at position A and close to zero at position B. The sum of the amplitudes of the electric field squared for two degenerate modes with frequency  $\omega\Lambda/2\pi c = 0.382$  is also shown as an inset. In this case the amplitude is strong at position B and not so strong at position A. Accordingly, for the frequency  $\omega\Lambda/2\pi c = 0.374$  a strong peak is seen in the PDOS at position A, whereas a strong peak is not seen at position B.

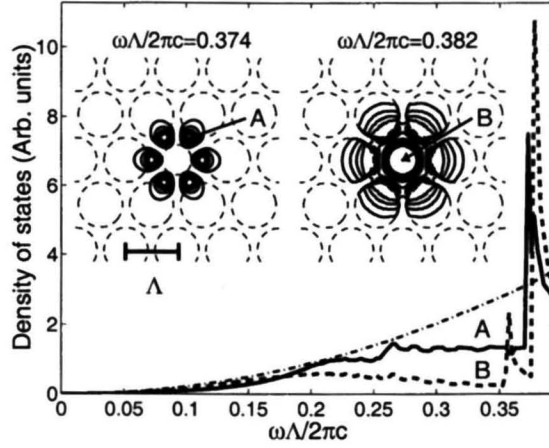


Figure 2: The position-dependent photon density of states is shown for two positions A and B (see inset) within a two-dimensional photonic crystal microcavity. For comparison the density of states is shown for a homogeneous dielectric with the same dielectric constant as the background material.

Similarly, the photon density of states is strong at position B and not so strong at position A for the frequency  $\omega\Lambda/2\pi c = 0.382$ . The PDOS for a homogeneous dielectric with dielectric constant 13 is also shown in Figure 2, and it is clear that the PDOS at position A for the frequency  $\omega\Lambda/2\pi c = 0.374$  is enhanced relative to the PDOS for the homogeneous dielectric, and consequently the rate of spontaneous emission is enhanced.

A similar conclusion may be drawn for the frequency  $\omega\Lambda/2\pi c = 0.382$  at position B. However, position B is at the center of an air-hole, whereas position A corresponds to a high-index material, where it is more natural to expect an emitter. Both for the case of the photonic crystal waveguide and the photonic crystal microcavity the PDOS is shown for two positions to illustrate that the PDOS depends strongly on position.

In conclusion, the numerical results show that the position-dependent photon density of states may depend strongly on the position within a photonic crystal waveguide and a photonic crystal microcavity. For the two positions considered within the waveguide spontaneous emission was not enhanced relative to the rate of spontaneous emission in a homogeneous dielectric with the same dielectric constant as the background material. However, for the case of a photonic crystal microcavity the photon density of states is strongly enhanced just above the cutoff frequency for a few localized modes, and in this case spontaneous emission may be enhanced.

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