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Probing Field Mode Densities in a Multilayer Structure with a Quantum Well

Abstract

A calculation of the spontaneous emission rate from a quantum well in a multilayer structure is presented. The dependence of the spontaneous emission rate on the parameters of the structure, (i.e. thickness of layers, position of quantum well, dielectric constants) is shown to be smooth. In common dielectric waveguide structures, the guided modes and radiation modes have comparable contributions to the spontaneous emission rate and the ratio of these contributions increases with increasing dielectric contrast.

Recently the influence of the dielectric environment on the spontaneous emission rate was studied for an atom inside a dielectric three-layer structure [1, 2]. Within a quantum-electrodynamical approach we have theoretically investigated similar effects for a quantum well embedded in a multilayer structure. Using the Heisenberg equations of motion for the charge carriers, the spontaneous emission rates may be obtained without an explicit modal decomposition of the radiation field. Considering a two-band model with an electron in the upper band (1), we find the following expression for the spontaneous emission rate in terms of the charge current densities and the electromagnetic transverse Greens function [3] as a summation over empty valence band (2) states:

$$\gamma = \frac{2}{\hbar}\sum_{\mathbf{k}_2} \mathrm{Im}\left(\int\int d^3r d^3r' \mathbf{j}_{12}(\boldsymbol{\rho},\mathbf{k}_1,\mathbf{k}_2)\cdot \overset{\leftrightarrow\perp}{G}(\boldsymbol{\rho},\boldsymbol{\rho}',z_0,\omega)\cdot \mathbf{j}_{21}(\boldsymbol{\rho}',\mathbf{k}_1,\mathbf{k}_2)\right). \tag{1}$$

Using a parabolic two-band model, we found the current density matrix element [3]:

$$\mathbf{j}_{12}(\mathbf{r},\mathbf{k}_1,\mathbf{k}_2) = \frac{i\hbar e}{mV}\mathbf{K}\exp[i(\mathbf{k}_2-\mathbf{k}_1)\cdot\boldsymbol{\rho}]. \tag{2}$$

where $\mathbf{K} =< 1|\nabla|2 >$. The multilayer structure consists of n layers of dielectric material, each with a constant index of refraction illustrated in Fig.1. The index steps are in the z-direction, there is no variation in dielectric constant in the x- and y-direction. A quantum well is assumed to be embedded in the j-th layer with permittivity ε_j. The Greens tensor relating an observation point $\mathbf{r} = (\boldsymbol{\rho}, z)$ and a source point $\mathbf{r}_0 = (\boldsymbol{\rho}_0, z_0)$ was derived by

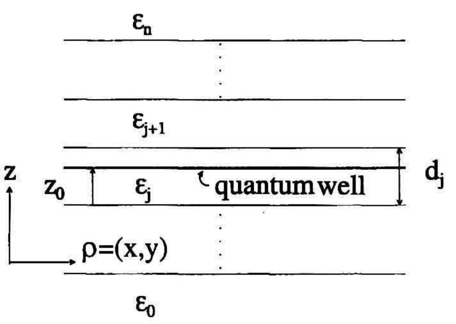

Figure 1: Multilayer structure with a quantum well embedded in one of its layers. Each layer has a constant index of refraction. The index of the quantum well, which is very thin, is set equal to ε_j of the cladding layer (with thickness d_j). The position of the quantum well is given by z_0.

Tomaš and reads for $\mathbf{r}$ and $\mathbf{r}_0$ both in the j-th layer as [4]:

$$
\begin{aligned}
\overleftrightarrow{G}_j^{(j)}(\mathbf{r},\mathbf{r}_0,\omega) &= \sum_{q=E,H} \frac{i\mu_0}{4\pi^2} \int \frac{d^2k}{2\beta_j} \zeta^q \frac{\exp(i\beta_j d_j)}{D_j^q} \left[\boldsymbol{\varepsilon}_j^{q>}(\mathbf{k},\omega;z)\boldsymbol{\varepsilon}_j^{q<}(-\mathbf{k},\omega;z_0)\theta(z-z_0) \right. \\
& \left. + \boldsymbol{\varepsilon}_j^{q<}(\mathbf{k},\omega;z_0)\boldsymbol{\varepsilon}_j^{q>}(-\mathbf{k},\omega;z_0)\theta(z-z_0) \right] \exp[i\mathbf{k}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_0)] e^{i\beta_j(z-z_0)} \\
& \qquad 0 < z,\ z_0 < d_j
\end{aligned} \tag{3}
$$

where the index q indicates the polarization of the field, which is either E-polarization or H-polarization, $\mathbf{k} = (k_x, k_y)$ is the conserved component of the wave vector parallel to the system interfaces, $\beta_j = \sqrt{\varepsilon_j \frac{\omega^2}{c^2} - k^2}$ is the local z-component of the wave vector, $\zeta^E = -1$, $\zeta^H = 1$ and

$$
D_j^q = 1 - r_{j-}^q r_{j+}^q e^{2i\beta_j d_j} \tag{4}
$$

with r_{j+}^q (r_{j-}^q) being the reflection coefficients of the upper (lower) interface. These coefficients obey the usual recurrence relations [4]

$$
r_{j,\pm}^q = \frac{1}{D_{j\pm1}^q} \left[\tilde{r}_{j,j\pm1}^q + r_{j\pm1,\pm}^q \exp(2i\beta_{j\pm1} d_{j\pm1}) \right] \tag{5}
$$

with $\tilde{r}_{j,j\pm1}^q$ the reflection coefficients for a single interface $j/k(=j\pm1)$ reduce to

$$
\tilde{r}_{jk}^H = \frac{\varepsilon_k\beta_j - \varepsilon_j\beta_k}{\varepsilon_k\beta_j + \varepsilon_j\beta_k}; \quad \tilde{r}_{jk}^E = \frac{\beta_j - \beta_k}{\beta_j + \beta_k}. \tag{6}
$$

The vectors $\boldsymbol{\varepsilon}_{qj}^<$ and $\boldsymbol{\varepsilon}_{qj}^>$ describe the z-dependence of the electric field in the cavity of a $q = H$ polarized or a $q = E$ polarized plane wave of unit strength incident on the system from its upper (downward) and lower (upward) side, respectively. They are defined by

$$
\begin{aligned}
\boldsymbol{\varepsilon}^<(\mathbf{k},\omega;z) &= \mathbf{e}_j^{q-}(\mathbf{k})e^{-i\beta_j z} + r_{j-}^q \mathbf{e}_j^{q+}(\mathbf{k})e^{i\beta_j(d_j-z)} \\
\boldsymbol{\varepsilon}^>(\mathbf{k},\omega;z) &= \mathbf{e}_j^{q+}(\mathbf{k})e^{-i\beta_j(d_j-z)} + r_{j+}^q \mathbf{e}_j^{q-}(\mathbf{k})e^{i\beta_j z} \\
\mathbf{e}_j^{H\mp} &= \frac{c}{\sqrt{\varepsilon_j}\omega}(\pm\beta_j\hat{\mathbf{k}} + k\hat{\mathbf{z}}), \quad \mathbf{e}_j^{E\mp} = \hat{\mathbf{k}} \times \hat{\mathbf{z}}
\end{aligned} \tag{7}
$$

where $\mathbf{e}_j^{q\mp}$ are the orthonormal polarization vectors associated with the downward and upward propagating waves in the cavity respectively; $\hat{\mathbf{k}}$ and $\hat{\mathbf{z}}$ are unit vectors.

We assume that the quantum well is just a thin sheet at the $z = z_0$ plane. The transverse Greens tensor $\overset{\leftrightarrow(j)\perp}{G_j}(\boldsymbol{\rho}, \boldsymbol{\rho}', z_0, \omega)$ at the z_0-plane is obtained by subtracting the longitudinal part. The result is:

$$\overset{\leftrightarrow(j)\perp}{G_j}(\boldsymbol{\rho}, \boldsymbol{\rho}', z_0, \omega) = \frac{i\mu_0}{4\pi^2} \sum_{q=p,s} \int \frac{d^2k}{2\beta_j} \zeta_q \frac{e^{i\beta_j d_j}}{D_{qj}} \tilde{\boldsymbol{\varepsilon}}_{qj}^{>}(\mathbf{k}, \omega, z_0) \tilde{\boldsymbol{\varepsilon}}_{qj}^{<}(-\mathbf{k}, \omega, z_0) \exp[i\mathbf{k} \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}')] \quad (8)$$

with

$$\begin{aligned}
\tilde{\boldsymbol{\varepsilon}}_j^{E>}(\mathbf{k}, \omega, z_0) &= \left(\hat{\mathbf{k}} \times \hat{\mathbf{z}}\right)\left(e^{-i\beta_j(d_j - z_0)} + r_{j+}^E e^{i\beta_j(d_j - z_0)}\right) \\
\tilde{\boldsymbol{\varepsilon}}_j^{E<}(\mathbf{k}, \omega, z_0) &= \left(\hat{\mathbf{k}} \times \hat{\mathbf{z}}\right)\left(e^{-i\beta_j z_0} + r_{j-}^E e^{i\beta_j z_0}\right) \\
\tilde{\boldsymbol{\varepsilon}}_j^{H>}(\mathbf{k}, \omega, z_0) &= \hat{\mathbf{z}}\left(e^{-i\beta_j(d_j - z_0)} + r_{j+}^H e^{i\beta_j(d_j - z_0)}\right) \\
\tilde{\boldsymbol{\varepsilon}}_j^{H<}(\mathbf{k}, \omega, z_0) &= \hat{\mathbf{z}}\left(e^{-i\beta_j z_0} + r_{j-}^H e^{i\beta_j z_0}\right).
\end{aligned} \quad (9)$$

The spontaneous emission coefficient then reduces to

$$\gamma = \gamma^E + \gamma^H, \quad (10)$$

where γ^E is associated with the E-polarized waves and γ^H is associated with the H-polarized waves and

$$\begin{aligned}
\gamma^E &= \frac{\mu_0 \hbar e^2}{4\pi m^2} \mathrm{Re}\left(\int_0^\infty dk \frac{k}{\beta_j} \frac{K_\perp^2}{D_j^E} (1 + r_{j+}^E e^{2i\beta_j(d_j - z_0)})(1 + r_{j-}^E e^{2i\beta_j z_0})\right), \\
\gamma^H &= \frac{\mu_0 \hbar e^2}{4\pi m^2} \mathrm{Re}\left(\int_0^\infty dk \frac{k}{\beta_j} \frac{2K_z^2}{D_j^H} (1 + r_{j+}^H e^{2i\beta_j(d_j - z_0)})(1 + r_{j-}^H e^{2i\beta_j z_0})\right).
\end{aligned} \quad (11)$$

According to eq. (11), the spontaneous emission rate depends on the reflectivity coefficients of upper and lower layer stacks, the thickness of the j-th layer and the position of the quantum well in this layer. The E-polarized emission is only related to the current matrix element parallel to the quantum well, the H-polarized emission to that perpendicular to the quantum well.

Typical results are shown in Fig. 2. The values for the index of refraction used are taken from Saleh and Teich [5]. In Fig. 2(a) and 2(b) cusps are seen in the separate contributions of guided and radiation modes. These cusps can be attributed to the birth of new guided modes [3], causing the suddenly higher contribution from the guided modes. Just before each cusp, the precursor of the new guided mode is composed of radiation modes. That is why the contribution from the radiation modes drops in the complimentary fashion at the birth of the new guided modes, such that the total spontaneous emission rate shows no effect.

The results presented here show that changing the thickness of the cladding layer and the position of the quantum well in the cladding layer can modify the spontaneous emission

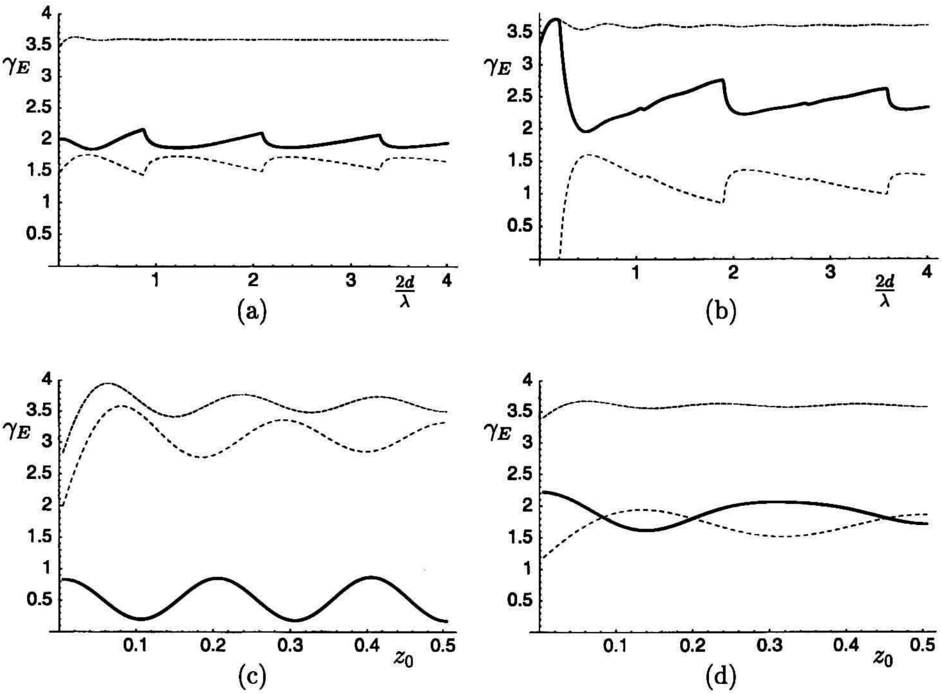

Figure 2: Spontaneous emission rate into E-polarized modes versus thickness d of the middle layer (figures (a) and (b)) and versus position z_0 of the quantum well in this layer at a fixed thickness of 0.75λ (figures (c) and (d)). The contribution of the radiation modes (solid line) and the guided modes (long-dashed line) is summed to give the total spontaneous emission rate (short-dashed line). Configuration from top to bottom layer, figure (a): five layer, AlAs, $Ga_{0.5}Al_{0.5}As$ (thickness 0.5λ), GaAs, $Ga_{0.5}Al_{0.5}As$ (thickness 0.5λ) , AlAs; figure (b): three-layer, AlAs, GaAs, $Ga_{0.5}Al_{0.5}As$; figure (c): three-layer, AlO, GaAs, AlO; figure (d): three-layer, AlAs, GaAs, AlAs; refractive indices: AlAs 3.2, $Ga_{0.5}Al_{0.5}As$ 3.4, GaAs 3.6, AlO 1.76. Plots for the H-polarized modes are very similar.

rate of the quantum well significantly. This can be used to control the spontaneous emission lifetime of the quantum well. Moreover the guided modes do play an important role in the spontaneous emission rate. One can see from Fig. 2(c) and 2(d) that, as long as one stays away from the interfaces, the spontaneous emission rate is not very sensitive to the position of the quantum well.

Acknowledgements

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