F. Jahnke, M. Kira and S.W. Koch

Quantum Theory of the Semiconductor Luminescence

Abstract

A fully quantum mechanical theory for the interaction of light and electron-hole excitations in semiconductor quantum-well systems is developed. The resulting many-body hierarchy for the correlation equations is truncated using a dynamical decoupling scheme leading to the semiconductor luminescence equations. Numerical results are presented for the photoluminescence of incoherently excited quantum wells.

1 Introduction

Recombination of electron-hole pairs and luminescence are fundamental processes in semiconductors. It is known from atomic systems that luminescence is modified when various atoms are optically coupled [1] or when atoms are positioned in a high-quality cavity [2, 3]. An analogous situation in semiconductors can be realized e.g. with an array of quantum wells (QW) [4] or for QWs in a semiconductor microcavity [5]. Like in atomic systems, strong coupling effects [5, 6] and suppressed or enhanced spontaneous emission [7, 8, 9, 10] due to the high quality optical resonances have been observed. For low excitations, the semiconductor material shows excitonic resonances below the fundamental absorption edge. In a microcavity, such a resonance is strongly coupled to the cavity mode leading to the double peaked normal-mode coupling spectrum which has been observed, e.g., in transmission and reflection [5] as well as in photoluminescence [6].

The transmission spectrum can experimentally be determined using pump-probe techniques. Such experiments can be explained in great detail [11, 12] on the basis of a classical description of the light field [13, 14]. The theory/experiment agreement might suggest that a quantum treatment of light only leads to minor corrections. However, this is usually true only as long as the classical fields exceed the vacuum fluctuations. Therefore, photoluminescence in such a correlated system is an important phenomenon which cannot be explained semiclassically.

Without external driving field the polarization and the coherent microcavity field $\langle E \rangle$ typically decays on a ps time scale after the excitation pulse. However, in many cases a substantial number of incoherent electrons and holes remain excited in the system. The system can then reach its ground state via non-radiative electron-hole recombination and through spontaneous emission leading to photoluminescence, which cannot be explained by the classical properties of the light.

The quantum mechanical analysis of the interacting photon-semiconductor system poses a considerable challenge to current theories. In the classical description of light, the major difficulties arise from the consistent inclusion of carrier-carrier Coulomb interaction effects.

In this paper, we review a general theory for the semiconductor luminescence of electron-hole pairs where many-body effects are included. Staring point is a quantum mechanical treatment of the interacting carrier-photon system in the electron-hole picture. The operator equations presented provide a general starting point for investigating quantum properties of light in semiconductor systems. The approach is not only valid for stationary emission under equilibrium conditions but also for the temporal emission dynamics under nonequilibrium conditions resulting from the interplay of incoherent and coherent fields, e.g., luminescence in the presence or after excitation by external laser fields [15, 16].

The emission properties critically depend on the excitation conditions of the system. Even though incoherent excitonic populations are not included at the present level of the theory, excitonic effects enter through the Coulomb interaction between the carriers [17]. For the description of incoherent photoluminescence, we develop the "semiconductor luminescence equations" which are based on a generalization of the Hartree-Fock decoupling scheme. In some respect, these equations are the analog to the "Maxwell-Semiconductor Bloch equations" describing the coherent excitation dynamics. In their most elementary form the semiconductor Bloch equations are based on the Hartree-Fock decoupling; the addition of many-body-correlations is subject of intense current research, see [13] and references therein. On the other hand, carrier-correlations and non-Markovian effects are already partially included in the presented semiconductor luminescence equations.

2 Equations of Motion for Photons and Carriers

For a classical field, the light-matter coupling is described by the dipole interaction Hamiltonian proportional to the scalar product of the field and the carrier polarization. When the treatment includes a quantum field, the specific form of the interaction Hamiltonian cannot be extracted trivially from the semiclassical Hamiltonian [18]. In [16], the quantized interaction Hamiltonian is derived in order to correctly include the quantum aspects of the light and the carrier systems.

In principle, we could use the vector potential operator to describe the quantum properties of light, e.g., within a Green's function method [19, 20]. However, in this paper we choose an alternative method where the quantum aspects of light are derived directly from the nonlocal boson operators $b^\dagger_{\mathbf{q}}$ and $b_{\mathbf{q}}$ describing the creation and anihilation of photons in the mode $\mathbf{q}$. This scheme directly leads us to a generalization of the semiclassical semiconductor Bloch equations [21] for the quantum case.

The dynamics of the photon operator $b_{\mathbf{q}}$ is obtained from the Heisenberg equation of motion $i\hbar\partial b_{\mathbf{q}}/\partial t = [b_{\mathbf{q}}, H]$ with the Hamiltonian of the interacting system [16],

$$i\hbar\frac{\partial}{\partial t}b^\dagger_{q_z,\mathbf{q}_{||}} = -\hbar\omega_{|\mathbf{q}|}\, b^\dagger_{q_z,\mathbf{q}_{||}} + i\mathcal{E}_{\mathbf{q}}\tilde{u}_{\mathbf{q}}\hat{P}^\dagger_{QW}(\mathbf{q}_{||}), \quad (1)$$

where $\mathcal{E}_{\mathbf{q}}$ and $\tilde{u}_{\mathbf{q}}$ determine the vacuum field amplitude and mode strength at the QW, respectively. The evolution of the photon operators is coupled to the QW polarization operator

$$\hat{P}^{\dagger}_{QW}(\mathbf{q}_{||}) = \sum_{\mathbf{k}} \Big[d^{\star}_{cv}(\mathbf{q}_{||}) v^{\dagger}_{\mathbf{k}+\mathbf{q}_v} c_{\mathbf{k}-\mathbf{q}_c} + d_{cv}(\mathbf{q}_{||}) c^{\dagger}_{\mathbf{k}+\mathbf{q}_c} v_{\mathbf{k}-\mathbf{q}_v} \Big] \quad (2)$$

with the dipole matrix element $d_{cv}(\mathbf{q}_{||})$. In a Bloch basis for a two-band semiconductor, the interacting carrier system is described by creation ($c^{\dagger}_{\mathbf{k}}$, $v^{\dagger}_{\mathbf{k}}$) and annihilation ($c_{\mathbf{k}}$, $v_{\mathbf{k}}$) operators for conduction and valence band electrons, respectively. According to Eq. (2), optical processes can be described with the microscopic polarization operators

$$\hat{P}_{\mathbf{k}}(\mathbf{q}_{||}) = v^{\dagger}_{\mathbf{k}-\mathbf{q}_v} c_{\mathbf{k}+\mathbf{q}_c}; \;\; \hat{P}^{\dagger}_{\mathbf{k}}(\mathbf{q}_{||}) = c^{\dagger}_{\mathbf{k}+\mathbf{q}_c} v_{\mathbf{k}-\mathbf{q}_v}, \quad (3)$$

where $\mathbf{q}_c + \mathbf{q}_v = \mathbf{q}_{||}$. The operator $\hat{P}^{\dagger}_{\mathbf{k}}(\mathbf{q}_{||})$ simultaneously creates an electron in the conduction band and destroys an electron in the valence band, i.e., it creates an electron-hole pair. The center of mass of this electron-hole pair moves with the momentum $\hbar\mathbf{q}_{||}$. In principle, the ratio of $\mathbf{q}_c$ and $\mathbf{q}_v$ can be chosen arbitrarily, here we use the center of mass coordinates

$$\mathbf{q}_c = \frac{m_e}{m_e + m_h}\mathbf{q}_{||}; \;\; \mathbf{q}_v = \frac{m_h}{m_e + m_h}\mathbf{q}_{||}. \quad (4)$$

The Heisenberg equation of motion for $P_{\mathbf{k}}(\mathbf{q}_{||})$ in the fully quantized case differs from the semiclassical calculation [21] only in those terms that involve commutators with the light-matter interaction Hamiltonian. We obtain

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\hat{P}_{\mathbf{k}}(\mathbf{q}_{||}) &= \left(\epsilon^{c}_{\mathbf{k}+\mathbf{q}_c} - \epsilon^{v}_{\mathbf{k}-\mathbf{q}_v}\right)\hat{P}_{\mathbf{k}}(\mathbf{q}_{||}) \\ &+ d_{cv}(\mathbf{q}_{||})\left[c^{\dagger}_{\mathbf{k}+\mathbf{q}_c}\hat{E}(\mathbf{q}_{||})c_{\mathbf{k}+\mathbf{q}_c} - v^{\dagger}_{\mathbf{k}-\mathbf{q}_v}\hat{E}(\mathbf{q}_{||})v_{\mathbf{k}-\mathbf{q}_v}\right] \\ &+ \sum_{\mathbf{q}'_{||}\neq\mathbf{q}_{||}}\left\{\left[\hat{P}_{\mathbf{k}}(\mathbf{q}_{||}), \hat{P}^{\dagger}_{QW}(\mathbf{q}'_{||})\right]\hat{E}(\mathbf{q}'_{||})\right\}_N + \sum_{\mathbf{k}',\mathbf{k}''} V_{\mathbf{k}'-\mathbf{k}} \\ &\times \Big[v^{\dagger}_{\mathbf{k}-\mathbf{q}_v}\left(c^{\dagger}_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}}c_{\mathbf{k}''} + v^{\dagger}_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}}v_{\mathbf{k}''}\right)c_{\mathbf{k}'+\mathbf{q}_c} \\ &- v^{\dagger}_{\mathbf{k}'-\mathbf{q}_v}\left(c^{\dagger}_{\mathbf{k}''}c_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}} + v^{\dagger}_{\mathbf{k}''}v_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}}\right)c_{\mathbf{k}+\mathbf{q}_c}\Big] \end{aligned} \quad (5)$$

where $\epsilon^{c(v)}_{\mathbf{k}}$ determines the free carrier energies,

$$\epsilon^{c}_{\mathbf{k}+\mathbf{q}_c} - \epsilon^{v}_{\mathbf{k}-\mathbf{q}_v} = E_g + \frac{\hbar^2\mathbf{k}^2}{2\mu} + \frac{\hbar^2\mathbf{q}^2_{||}}{2M}, \quad (6)$$

using $M = m_e + m_h$ and $\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_h}$. In Eq. (5), $V_{\mathbf{k}}$ is the QW matrix element of the Coulomb potential, and $\{...\}_N$ denotes normal ordering. The operator for the electric field in the dipole picture [18] is given by [16]

$$\epsilon_0\hat{E}(\mathbf{q}_{||}) = \hat{D}(\mathbf{q}_{||}) - \frac{\tilde{g}}{n^2 S}\hat{P}_{QW}(\mathbf{q}_{||}), \quad (7)$$

with the mode expansion of the displacement operator,

$$\frac{1}{\epsilon_0}\hat{D}(\mathbf{q}_{||}) = \frac{1}{\sqrt{S}}\sum_{q} i\mathcal{E}_{\mathbf{q}}\left[\tilde{u}_{q_z,\mathbf{q}_{||}} b_{q_z,\mathbf{q}_{||}} - \tilde{u}^{\star}_{q_z,-\mathbf{q}_{||}} b^{\dagger}_{q_z,-\mathbf{q}_{||}}\right]. \tag{8}$$

The QW confinement wavefunctions $\xi(z)$ enter via

$$\tilde{g} = \int dz\, |\xi(z)|^4. \tag{9}$$

For the carrier occupation number operators $\hat{n}^c_{\mathbf{k}} = c^{\dagger}_{\mathbf{k}} c_{\mathbf{k}}$ and $\hat{n}^v_{\mathbf{k}} = v^{\dagger}_{\mathbf{k}} v_{\mathbf{k}}$ as we obtain the equations of motion

$$i\hbar\frac{\partial}{\partial t}\hat{n}^c_{\mathbf{k}} = \left[\sum_{\mathbf{q}_{||}} d^{\star}_{cv}(\mathbf{q}_{||}) v^{\dagger}_{\mathbf{k}+\mathbf{q}_{||}} \hat{E}(\mathbf{q}_{||}) c_{\mathbf{k}} - \sum_{\mathbf{k}',\mathbf{k}''} V_{\mathbf{k}'-\mathbf{k}} c^{\dagger}_{\mathbf{k}'}\left(c^{\dagger}_{\mathbf{k}''} c_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}} + v^{\dagger}_{\mathbf{k}''} v_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}}\right) c_{\mathbf{k}}\right] - \text{h.c.}, \tag{10}$$

$$i\hbar\frac{\partial}{\partial t}\hat{n}^v_{\mathbf{k}} = \left[-\sum_{\mathbf{q}_{||}} d^{\star}_{cv}(\mathbf{q}_{||}) v^{\dagger}_{\mathbf{k}} \hat{E}(\mathbf{q}_{||}) c_{\mathbf{k}-\mathbf{q}_{||}} + \sum_{\mathbf{k}',\mathbf{k}''} V_{\mathbf{k}'-\mathbf{k}} v^{\dagger}_{\mathbf{k}}\left(c^{\dagger}_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}} c_{\mathbf{k}''} + v^{\dagger}_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}} v_{\mathbf{k}''}\right) v_{\mathbf{k}'}\right] - \text{h.c.} \tag{11}$$

In Eqs. (5), (10), and (11), the ordering of $\hat{E}$ and carrier operators is crucial since $\hat{E}$ contains both field and particle operators. Together with the operator equations (1), these equations serve as a general starting point for our investigations of quantum correlations.

The equations of motion for $\hat{P}_{\mathbf{k}}(\mathbf{q}_{||})$ and $\hat{n}^{c,v}_{\mathbf{k}}$ contain four-particle operator combinations as a consequence of the Coulomb interaction and the dipole self-energy. As usual, this leads to an infinite hierarchy of equations since expectation values consisting of n particle operators are always coupled to higher order terms having $n+2$ particle operators. In practice, this hierarchy has to be truncated using a suitable decoupling. In the semiclassical regime, the simplest decoupling scheme is the dynamic Hartree-Fock approximation,

$$\langle a^{\dagger}_1 a^{\dagger}_2 a_3 a_4\rangle|_{HF} = \langle a^{\dagger}_1 a_4\rangle\langle a^{\dagger}_2 a_3\rangle - \langle a^{\dagger}_1 a_3\rangle\langle a^{\dagger}_2 a_4\rangle, \tag{12}$$

combined with the random phase approximation,

$$\langle c^{\dagger}_{\mathbf{k}} c_{\mathbf{k}'}\rangle = \delta_{\mathbf{k},\mathbf{k}'}\langle\hat{n}^c_{\mathbf{k}}\rangle; \qquad \langle v^{\dagger}_{\mathbf{k}} v_{\mathbf{k}'}\rangle = \delta_{\mathbf{k},\mathbf{k}'}\langle\hat{n}^v_{\mathbf{k}}\rangle. \tag{13}$$

This approach can be extended for the fully quantized system where it is important to retain also field-particle correlations in terms like $\langle a_1^\dagger a_2^\dagger \hat{O}_F a_3 a_4 \rangle$ with a single photon operator $\hat{O}_F$. The corresponding truncation [16, 17] permutes $\hat{O}_F$ between all possible Hartree-Fock terms of the carriers. For the incoherent excitations studied here, the intraband transitions vanish such that we find additional factorizations

$$\begin{aligned} \langle c_{\mathbf{k}}^\dagger c_{\mathbf{k}'} \hat{O}_F \rangle &= \delta_{\mathbf{k},\mathbf{k}'} \langle \hat{n}_{\mathbf{k}}^c \rangle \langle \hat{O}_F \rangle; \\ \langle v_{\mathbf{k}}^\dagger v_{\mathbf{k}'} \hat{O}_F \rangle &= \delta_{\mathbf{k},\mathbf{k}'} \langle \hat{n}_{\mathbf{k}}^v \rangle \langle \hat{O}_F \rangle. \end{aligned} \tag{14}$$

3 Semiconductor Luminescence Equations

Quantum corrections to the semiclassical limit can be determined by studying terms like $\Delta\langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle = \langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle - \langle b_{\mathbf{q}}^\dagger\rangle\langle \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ and $\Delta\langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle = \langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle - \langle b_{q_z,\mathbf{q}_{||}}^\dagger\rangle\langle b_{q_z',\mathbf{q}_{||}}\rangle$, where the classical factorization is subtracted from the full term. The significance of such corrections increases as the coherent terms $\langle b_{q_z,\mathbf{q}_{||}}\rangle$ and $\langle \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ become smaller.

In the following, we focus on the theoretical analysis of incoherent photoluminescence where carriers are nonresonantly generated in the QW by stationary or pulsed optical excitation high above the semiconductor band-edge. Since there is no coherent field or intraband polarization generated in the vicinity of the exciton resonances we can use

$$\langle \hat{P}_{\mathbf{k}}(\mathbf{q}_{||}, t_0 = 0)\rangle = 0, \qquad \langle b_{\mathbf{q}}(t_0 = 0)\rangle = 0. \tag{15}$$

Starting from these initial values, our equations show that for $t > t_0$

$$\begin{aligned} &\langle \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle = \langle b_{\mathbf{q}}\rangle = \langle \hat{n}_{\mathbf{k}}^{c,v} b_{\mathbf{q}}\rangle \\ &= \langle b_{\mathbf{q}} \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle = \langle b_{\mathbf{q}} b_{\mathbf{q}'}\rangle = 0. \end{aligned} \tag{16}$$

Under these incoherent conditions the quantum correlations are obtained directly from the full terms, i.e. $\Delta\langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle = \langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ and $\Delta\langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle = \langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle$. Furthermore, the only non-zero quantities are $f_{\mathbf{k}}^{e,h}$, $\langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$, and $\langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle$. The equation of motion for $\langle b_q^\dagger b_{q'}\rangle$ is obtained from Eq. (1) without the need of a factorization approximation

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle &= \hbar\left(\omega_{\mathbf{q}'} - \omega_{\mathbf{q}}\right)\langle b_{q_z,\mathbf{q}_{||}}^\dagger b_{q_z',\mathbf{q}_{||}}\rangle \\ &+ i\mathcal{E}_{\mathbf{q}}\tilde{u}_{\mathbf{q}}\langle b_{\mathbf{q}'} \hat{P}_{QW}^\dagger(\mathbf{q}_{||})\rangle + i\mathcal{E}_{\mathbf{q}'}\tilde{u}_{\mathbf{q}'}^*\langle b_{\mathbf{q}}^\dagger \hat{P}_{QW}(\mathbf{q}_{||})\rangle. \end{aligned} \tag{17}$$

Thus, the photon number expectation values are coupled to field-matter correlations of the type $\langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$. The equations of motion for $\langle b_{\mathbf{q}}^\dagger \hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ and $f_{\mathbf{k}}^{e,h}$ can be derived from the quantum operator equations (1) and (5)-(11) combined with the dynamic decoupling and the initial condition (16),

$$
\begin{aligned}
& i\hbar\frac{\partial}{\partial t}\langle b^{\dagger}_{\mathbf{q}}\hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle \\
& = \left[\epsilon^{c}_{\mathbf{k}+\mathbf{q}_c} - \epsilon^{v}_{\mathbf{k}-\mathbf{q}_v} - \hbar\omega_{\mathbf{q}} - \Sigma(\mathbf{k},\mathbf{q}_{||})\right]\langle b^{\dagger}_{\mathbf{q}}\hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle \\
& - \left(1 - f^{e}_{\mathbf{k}+\mathbf{q}_c} - f^{h}_{\mathbf{k}-\mathbf{q}_v}\right)\Omega(\mathbf{k},\mathbf{q}) \\
& + f^{e}_{\mathbf{k}+\mathbf{q}_c} f^{h}_{\mathbf{k}-\mathbf{q}_v}\Omega_{SE}(\mathbf{k},\mathbf{q})
\end{aligned}
\tag{18}
$$

$$
\frac{\partial}{\partial t} f^{e}_{\mathbf{k}} = \frac{2}{\hbar}\sum_{q_z,\mathbf{q}_{||}} \mathrm{Im}\left[-i d^{\star}_{cv}(\mathbf{q}_{||})\mathcal{E}_{\mathbf{q}}\tilde{u}^{\star}_{\mathbf{q}}\langle b^{\dagger}_{q_z,\mathbf{q}_{||}}\hat{P}_{\mathbf{k}-\mathbf{q}_c}(\mathbf{q}_{||})\rangle\right], \tag{19}
$$

$$
\frac{\partial}{\partial t} f^{v}_{\mathbf{k}} = \frac{2}{\hbar}\sum_{q_z,\mathbf{q}_{||}} \mathrm{Im}\left[-i d^{\star}_{cv}(\mathbf{q}_{||})\mathcal{E}_{\mathbf{q}}\tilde{u}^{\star}_{\mathbf{q}}\langle b^{\dagger}_{q_z,\mathbf{q}_{||}}\hat{P}_{\mathbf{k}+\mathbf{q}_v}(\mathbf{q}_{||})\rangle\right]. \tag{20}
$$

Equations (17)-(20) give a closed set of *semiconductor luminescence equations* with the energy renormalization,

$$
\Sigma(\mathbf{k},\mathbf{q}_{||}) = \sum_{\mathbf{k}'} V_{\mathbf{k}'-\mathbf{k}}\left(f^{e}_{\mathbf{k}'+\mathbf{q}_c} + f^{h}_{\mathbf{k}'-\mathbf{q}_v}\right), \tag{21}
$$

and the renormalized stimulated contribution,

$$
\begin{aligned}
\Omega(\mathbf{k},\mathbf{q}) = {} & d_{cv}(\mathbf{q}_{||})\langle b^{\dagger}_{\mathbf{q}}\hat{E}(\mathbf{q}_{||})\rangle \\
& + \sum_{\mathbf{k}'} V_{\mathbf{k}'-\mathbf{k}}\langle b^{\dagger}_{\mathbf{q}}\hat{P}_{\mathbf{k}'}(\mathbf{q}_{||})\rangle.
\end{aligned}
\tag{22}
$$

In Eq. (18) the term proportional to $1 - f^{e}_{\mathbf{k}} - f^{h}_{\mathbf{k}}$ introduces either stimulated emission or absorption depending on the excitation conditions. The strength of the spontaneous emission,

$$
\Omega_{SE}(\mathbf{k},\mathbf{q}) = i\mathcal{E}_{\mathbf{q}}\tilde{u}_{\mathbf{q}} d_{cv}(\mathbf{q}_{||}), \tag{23}
$$

is determined by the dipole matrix element d_{cv} and the effective mode strength at the QW position $\tilde{u}_{\mathbf{q}}$.

The term $\langle b^{\dagger}_{\mathbf{q}}\hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ gives the amplitude of a process where an electron hole pair, with center of mass momentum $\mathbf{q}_{||}$, recombines by emitting a photon with the same in-plane momentum. As long as there are carriers excited in the QW, this correlation starts to build up even if the field-particle and the field-field correlations are initially taken to be zero, because the term $f^{e}_{\mathbf{k}+\mathbf{q}_c} f^{h}_{\mathbf{k}-\mathbf{q}_v}\Omega_{SE}(\mathbf{k},\mathbf{q})$ entering Eq. (18) is nonzero. Thus, it provides a spontaneous emission source to the recombination process. According to the factor $f^{e}_{\mathbf{k}+\mathbf{q}_c} f^{h}_{\mathbf{k}-\mathbf{q}_v}$, the spontaneous recombination takes place only if an electron at $\mathbf{k}+\mathbf{q}_c$ and a hole at $\mathbf{k}-\mathbf{q}_v$ are present simultaneously. As the field correlations start to build up, the stimulated contribution $\Omega(\mathbf{k},\mathbf{q})$ can alter the photoluminescence spectrum. In other words, the observed photoluminescence is a result of the dynamic interplay of the field-field and field-particle correlations affected by the elementary processes of spontaneous emission and the stimulated contributions. These effects have been shown to explain nonlinear effects in semiconductor microcavity systems which previously had been incorrectly attributed to "boser" transitions [22].

Under coherent excitation conditions, quantum correlations of the type $\Delta\langle b^{\dagger}_{\mathbf{q}}\hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ contain contributions form the coherent dynamics well-known from the semiconductor Bloch equations and incoherent dynamics described by the semiconductor luminescence equations. The resulting interplay of coherent and incoherent dynamics is studied in [15, 16].

4 Quantum-Well Luminescence

To illustrate the theory we study in the following examples of the semiconductor photoluminescence when the carrier occupation functions can be approximated as Fermi-Dirac distributions with equal carrier density for electrons and holes. Such an approach is reasonably well justified for situations where the intraband carrier scattering time is much faster than the carrier recombination and generation times. An experimentally relevant example is the situation where near band-gap luminescence is measured after an excitation of the system into interband-absorption region. After the excited carriers are thermalized due to carrier-carrier and carrier-phonon interaction, there exists a temporal window of several tenth of picoseconds, where the carrier distribution is practically constant, provided that the recombination is weak.

For a description of the incoherent excitation regime, we start our calculation by setting all correlations initially to zero and evolve Eqs. (17)-(20) to steady state. The simplest way to include the effects of screening and dephasing is to phenomenologically replace the bare Coulomb potential by a screened one $V^S_{\mathbf{k}}$. Furthermore, one has to add a term $(\Delta E_g - i\gamma)\langle b^{\dagger}_{\mathbf{q}}\hat{P}_{\mathbf{k}}(\mathbf{q}_{||})\rangle$ in Eq. (18) where γ is the dephasing rate, and $\Delta E_g = \sum_{\mathbf{k}}(V^S_{\mathbf{k}} - V_{\mathbf{k}})$ is the Coulomb-hole gap shift. The microscopic treatment of interaction-induced dephasing and screening is further discussed in [16].

Under steady-state conditions, the measured luminescence spectrum is determined by the photon flux in a detector, i.e., the number of photons in the detector modes per time interval. The steady-state photon flux is given by

$$I_{PL}(\mathbf{q}) \propto \frac{\partial}{\partial t}\langle d^{\dagger}_{\mathbf{q}} d_{\mathbf{q}}\rangle. \tag{24}$$

For a similar definition, see [23]. If the light field is changing rapidly, a more general detector model has to be used [24].

For a QW embedded in a spatially homogeneous background, the free-field modes are plane waves. Using standard GaAs parameters, the resulting 3D exciton binding energy is $E_B = 4.2$ meV and the Bohr radius is $a_0 = 12.5$ nm. Then, assuming an 8 nm QW width, the quantum confined exciton has its 1s-resonance 2.45 E_B below the band gap energy E_g.

Figure 1 shows the computed photoluminescence in comparison with the absorption for different temperatures. The top row of figures displays the absorption spectra for low carrier densities and the middle row presents the corresponding photoluminescence spectra. We see that absorption and luminescence are peaked at the same exciton resonance energy, showing that even though our theory does not include population of incoherent excitonic states, the spectra are still peaked at the exciton resonance. This is a consequence of the well-known

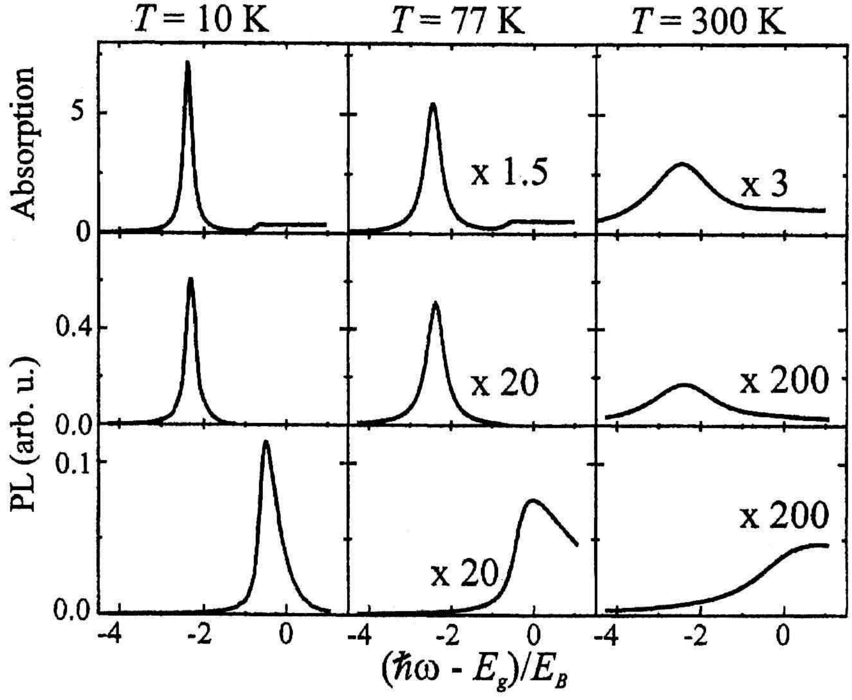

Figure 1: Quantum-well absorption and luminescence for three different carrier temperatures and carrier density 1×10^{10} cm^{-2}. The middle row shows the photoluminescence obtained from the full calculation while for the bottom row the Coulomb terms have been neglected. E_g is the unrenormalized band gap energy and E_B is the 3D exciton binding energy.

fact, that the strong interband Coulomb correlations in semiconductors lead to excitonic resonances in the interband polarization (in a semiclassical picture) and in the photon assisted polarization (in the full quantum theory). The bottom row of figures shows the artificial results obtained by shutting off the interband Coulomb term. Then the photoluminescence peak shifts to the band edge, as expected from free, i.e., non-interacting carrier theory. Further inversitgations [17] show, that for increased carrier density, where the exciton is gradually bleached and eventually gain occurs the QW luminescence stays peaked at the exciton resonance energy even when the absorption peak vanishes. For these elevated excitations, the band edge nonlinearities make the photoluminescence increasingly asymmetric.

In summary, this article presents a quantum theory of semiconductor light emission. Examples for the evaluation of this theory have been shown for the electron-hole regime, where incoherent excitonic populations can be ignored. The theory has also been evaluated in the low excitation purely excitonic regime and interesting results regarding exciton formation dynamics and related photoluminescence have been reported [25]. Work is in progress to include excitonic populations also in the electron-hole approach and to study the influence of structural sample disorder on the exciton formation and light emission dynamics.

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Author's address

F. Jahnke and S.W. Koch:
Department of Physics and Material Sciences Center,
Philipps-University,
Renthof 5, D-35032 Marburg,
Germany;

M. Kira:
Royal Institute of Technology,
Lindstedsvägen 24,
S-10044 Stockholm,
Sweden.