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Atom Optics and the Discreteness of Photons

Abstract

The deflection of atoms in a quantized light field brings out the granular structure of the photon field.

1 The Marriage of Atom Optics and Cavity QED

In the year 1933 P.L. Kapitza and P.A.M. Dirac calculated the deflection of an electron from a standing light wave [1]. They found a cross section too small to be measured experimentally in the near future:

> "We see, therefore, that the experiment could scarcely be made with ordinary continuous sources of light, and it seems to us that the only possibility would be to produce the illumination by using an intense spark discharge instead of a mercury arc."

Indeed, only recently experiments with ultra short pulses from high power lasers have provided the first indirect evidence of this phenomenon [2].

The reason for the smallness of this effect lies in the absence of the internal structure of the electron and, in particular, in the absence of a resonant enhancement. In contrast, atoms have internal structures and we can have a resonance between the frequencies of an atomic transition and of the light field. (See, for example, [3, 4]). This has led to the pioneering experiments by Moskowitz *et al.* [5] observing the deflection of atoms due to a laser field. Here, the center of mass motion of the atom is treated quantum mechanically and the atom displays wave features. This has suggested the name *atom optics* for this field. In the meantime interferometers, mirrors and cavities for atoms have been realized experimentally. (For a review we refer to [6, 7, 8, 9, 10]. This field has blossomed over the last years and many interesting branches of atom optics have developed. Space does not allow us to elaborate more on the physics of Bose-Einstein condensates or atom optics as a testing ground of quantum chaos [11] nor can we but mention atom optics as a tool for nanotechnology [12].

Another exciting field of quantum optics is the area of cavity QED. The development of high-Q cavities for microwave fields in combination with Rydberg atoms has lead to unique light sources such as the micromaser. Moreover, the use of gyroscope mirrors has resulted in high-Q cavities in the optical regime.

It is therefore an interesting enterprise to combine both fields and consider atom optics in quantized light fields [13, 14]. In particular, we can consider the deflection of atoms from a quantized light field [15, 16]. This phenomenon allows us to measure the photon statistics and the discreteness of the electromagnetic field [17, 18, 19, 20].

A hundred years ago M. Planck proposed the quantization of the energy of a material harmonic oscillator in order to derive the correct black body radiation law [21]. In the context of the micromaser Rempe *et al.* [22] showed by measuring quantum revivals [23] that the maser field consists of discrete photons. Recently, Brune *et al.* [24] has used the time evolution of an atom in a high-Q cavity to prove the discreteness of the photon field. Moreover, Varcoe *et al.* [25] have realized for the first time in a controlled way Planck oscillators, that is electromagnetic fields of one or two photons. Despite this impressive progress, it is still interesting to see the granular structure of photons in a direct way. The deflection of atoms is such a method.

In this context it is interesting to note that the Paris group has made a similar suggestion using the whispering gallery mode of a microcavity [26]. However, no experiment has been made so far. In contrast, K.A.H. van Leeuwen reports in these proceedings the first experiments on the way towards the deflection of atoms from a quantum field (see also [27]).

Our paper is organized as follows. We first briefly summarize our model. We then derive the generalized Rabi equations for the probability amplitudes describing the state vector of the total system consisting of the center of mass motion, internal states of the atom and the states of the electromagnetic field. We reduce the problem to a one-dimensional scattering problem and solve the resulting equations in the Raman-Nath approximation. This allows us to obtain analytical expressions for the momentum distribution of the scattered atoms.

We consider various scattering situations: In the joint measurement scheme we make use of the entanglement between the center of mass motion and the cavity field. Here, we only retain those atoms that have not changed the phase of the field. This allows us a perfect read out of the photon statistics of the cavity field without extracting the field. (For a deflection experiment, making use of the entanglement between the center of mass motion and internal states see [28].

In the averaged momentum distribution we ignore the information contained in the field since we do not measure the field. Therefore the momentum distribution does not contain the full information about the initial photon statistics of the field. However, there are still some imprints of the field statistics left. In both cases we consider the so-called Kapitza-Dirac regime in which the initial wave packet is broad compared to the period of the standing wave.

The other extreme is the so-called Stern-Gerlach regime. Here the width of the wave packet is small compared to the period of the grating provided by the light field. The name originates from the analogy to the deflection of atoms in an inhomogeneous magnetic field [29].

In order to focus on the main ideas we do not present detailed derivations but still give enough steps to follow the calculations. For more information we refer to the literature.

2 Formulation of the Problem

Throughout this article we consider the scattering situation shown in Fig. 1. An atomic wave of a two-level atom with dipole moment $\wp$ propagates through a resonator and interacts res-

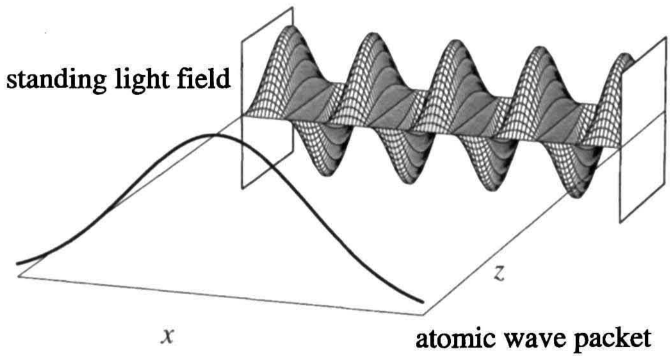

Figure 1: An atomic wave propagates through a cavity and interacts resonantly with a single mode of the standing light field. Here we consider the case where the atomic wave packet covers many wavelengths of the light field. The atomic wave packet is a plane wave in z-direction. Therefore we only show a cut through the probability distribution for one value of z.

onantly with a single mode of the radiation field. In the interaction picture the Hamiltonian for this process takes the form

$$\hat{H} = \frac{\hat{\mathbf{p}^2}}{2M} + \wp \cdot \mathbf{u}(\hat{\mathbf{p}})\, \mathcal{E}_0 \left(\hat{\sigma}\mathbf{a}^\dagger + \hat{\sigma}^\dagger \mathbf{a}\right). \tag{1}$$

Here $\hat{\mathbf{p}^2}/(2M)$ describes the kinetic energy of the center of mass motion, $\mathbf{u}(\hat{\mathbf{p}})$ and $\mathcal{E}_0$ are the mode function at the position $\mathbf{r}$ of the atom and the vacuum electric field of this mode, respectively.

Since we are now treating the center of mass motion quantum mechanically the position $\mathbf{r}$ and the momentum $\mathbf{p}$ are conjugate operators and obey the commutation relation

$$[r_l, p_m] = i\hbar\delta_{lm}.$$

Moreover, $\mathbf{a}$ and $\mathbf{a}^\dagger$ denote the annihilation and creation operators of the field and the Pauli spin matrices $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ destroy and create an atom in the excited state.

The dynamics of the state vector

$$|\Phi(t)\rangle = \sum_{m=0}^{\infty} \int d^3r' \, [\Phi_{a,m-1}(\mathbf{r}', t)|a, m-1\rangle + \Phi_{b,m}(\mathbf{r}', t)|b, m\rangle] \, |\mathbf{r}'\rangle, \tag{2}$$

describing the combined system of center of mass motion, internal states of the atom and the states of the electromagnetic field follows from the Schrödinger equation

$$i\hbar \frac{\partial|\Phi\rangle}{\partial t} = \hat{H}|\Phi\rangle. \tag{3}$$

Here $\Phi_{a,m-1}(\mathbf{r}, t)$ or $\Phi_{b,m}(\mathbf{r}, t)$ denote the probability amplitudes to find the atom at time t and position $\mathbf{r}$ and in the internal state $|a\rangle$ with $m-1$ photons in the field or in the internal state $|b\rangle$ with m photons in the field, respectively.

When we substitute this ansatz into the Schrödinger equation (3) we find with the Hamiltonian (1) the generalized Rabi equations

$$i\hbar \frac{\partial \Phi_{a,n-1}(\mathbf{r},t)}{\partial t} = \frac{\hat{\mathbf{p}}^2}{2M} \Phi_{a,n-1}(\mathbf{r},t) + \wp \cdot \mathbf{u}(\mathbf{r})\, \mathcal{E}_0 \sqrt{n}\, \Phi_{b,n}(\mathbf{r},t) \tag{4}$$

and

$$i\hbar \frac{\partial \Phi_{b,n}(\mathbf{r},t)}{\partial t} = \frac{\hat{\mathbf{p}}^2}{2M} \Phi_{b,n}(\mathbf{r},t) + \wp \cdot \mathbf{u}(\mathbf{r})\, \mathcal{E}_0 \sqrt{n}\, \Phi_{a,n-1}(\mathbf{r},t). \tag{5}$$

We can decouple these two equations by introducing the linear combinations

$$\Phi_n^{(\pm)} \equiv \Phi_{b,n} \pm \Phi_{a,n-1}. \tag{6}$$

In this case the generalized Rabi equations read

$$i\hbar \frac{\partial \Phi_n^{(\pm)}(\mathbf{r},t)}{\partial t} = \left[\frac{\hat{\mathbf{p}}^2}{2M} + U_n^{(\pm)}(\mathbf{r}) \right] \Phi_n^{(\pm)}(\mathbf{r},t) \tag{7}$$

where we have introduced the potentials

$$U_n^{(\pm)}(\mathbf{r}) \equiv \pm \wp \cdot \mathbf{u}(\mathbf{r})\, \mathcal{E}_0 \sqrt{n}. \tag{8}$$

Hence the probability amplitudes $\Phi_n^{(\pm)}$ satisfy a Schrödinger equation corresponding to a particle of mass M moving in a potential $U_n^{(\pm)}$. This potential is formed by the scalar product of the dipole moment $\wp$ and the mode function $\mathbf{u}(\mathbf{r})$. Moreover, it scales with the vacuum electric field strength $\mathcal{E}_0$ and the square root of the photon number. Since the nodes of the potential are independent of the photon number and the amplitude of the modulation in space is proportional to $\sqrt{n}$, the potentials $U_n^{(\pm)}$ get steeper as n increases. The potential $U_n^{(-)}$ is just the negative of $U_n^{(+)}$.

The initial condition for $|\Phi\rangle$ at time $t = 0$, that is, before the interaction, is a direct product

$$|\Phi(t=0)\rangle \;=\; |b\rangle \otimes \sum_{n=0}^{\infty} \psi_n |n\rangle \otimes \int d^3r'\, \mathcal{F}(\mathbf{r}')|\mathbf{r}'\rangle$$

of the atomic state which we take to be the ground state, the field state in a superposition of photon number states with probability amplitudes ψ_n and the initial distribution $\mathcal{F}$ of the atom in space.

When we compare this initial condition with the ansatz Eq. (2) we find

$$\Phi_n^{(\pm)}(\mathbf{r},0) = \mathcal{F}(\mathbf{r})\, \psi_n. \tag{9}$$

Hence the dynamics of the state vector $|\Phi\rangle$ of the combined system follows from the dynamics of the wave functions $\Phi_n^{(\pm)}$ moving in the potentials $U_n^{(\pm)}$ subject to the initial condition Eq. (9). Note that since in general the potential is three-dimensional and can be rather complicated it is a non-trivial task to solve the Schrödinger equation (7) for $\Phi_n^{(\pm)}$.

3 Reduction to One-Dimensional Scattering

We now consider a situation in which the atomic beam propagates initially orthogonal to the wave vector of the field in the cavity. In the remainder of this article we call this the x-direction, or the transverse direction. The motion along the z-axis, that is the longitudinal motion, we treat classically since we assume that the initial kinetic energy $Mv_z^2/2$ in z-direction is much larger than the change of the longitudinal momentum due to the interaction with the light field.

In this case the equations reduce to a one-dimensional Schrödinger equation of the form

$$i\hbar \frac{\partial \Psi_n^{(\pm)}(x,t)}{\partial t} = \left[\frac{\hat{p}_x^2}{2M} \pm \wp \mathcal{E}_0 \sqrt{n} \sin(\pi v_z t/L_z) \sin(kx)\right] \Psi_n^{(\pm)}(x,t). \tag{10}$$

Here we have also assumed for the sake of simplicity the specific mode function

$$\mathbf{u}(\mathbf{r}) = \mathbf{e}_y \sin(kx) \sin(\pi z/L_z)$$

of a box-shaped resonator of length L_z in the z-direction. Moreover, k denotes the wave number along the x-axis and $\wp \equiv \boldsymbol{\wp} \cdot \mathbf{e}_y$.

We have therefore reduced the three-dimensional scattering problem to the problem of solving a one-dimensional time dependent Schrödinger equation. We emphasize, however, that even this problem is non-trivial, since due to the motion of the atom through the resonator—the interaction switches on and switches off via the mode function $\sin(\pi v_z t/L_z)$—the potential is explicitly time dependent. Moreover, the potential in x-direction is periodic and can allow for rather complicated solutions.

4 State Vector in Raman-Nath Approximation

In the present discussion we confine ourselves to an approximate but analytical analysis of Eq. (10). For this purpose we recall that the atomic beam enters the resonator orthogonal to the wave vector of the electromagnetic field. Therefore its classical kinetic energy along the standing wave initially vanishes. Consequently the kinetic energy gained by the atom is due to the interaction with the light field. When the displacement caused by the electromagnetic field is smaller than its wavelength we can neglect the transverse kinetic energy term.

In this Raman-Nath approximation we can solve the Schrödinger equation (10) in an exact way and find

$$\begin{aligned} |\Psi(t)\rangle &= \sum_{n=0}^{\infty} \psi_n \int dx'\, f(x') \left\{\cos[\kappa\sqrt{n}\,\sin(kx)]|b,n\rangle \right. \\ &\qquad \left. -i \sin[\kappa\sqrt{n}\,\sin(kx)]|a,n-1\rangle\right\} |x'\rangle. \end{aligned} \tag{11}$$

Here we have introduced the dimensionless interaction parameter

$$\kappa \equiv \frac{2}{\pi} \frac{\wp \mathcal{E}_0}{\hbar} \tau.$$

We note that the position dependent interaction of the atom with the quantized light field has created a strong entanglement between the transverse motion, the field and the energy levels of the atom.

In order to discuss the momentum transfer from the electromagnetic field to the atom we express the state vector

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} \psi_n \int dp' \left[c_n(p')|b,n\rangle - is_n(p')|a,n-1\rangle\right] |p'\rangle \tag{12}$$

in the momentum representation where

$$c_n(p) \equiv \frac{1}{\sqrt{2\pi\hbar}} \int dx\, f(x) \cos\left[\kappa\sqrt{n}\sin(kx)\right] e^{-ipx/\hbar} \tag{13}$$

and

$$s_n(p) \equiv \frac{1}{\sqrt{2\pi\hbar}} \int dx\, f(x) \sin\left[\kappa\sqrt{n}\sin(kx)\right] e^{-ipx/\hbar}. \tag{14}$$

The state vector of the combined system allows us to answer questions concerning the momentum distribution of the scattered atoms, especially when we consider joint measurements between the transverse motion and the quantum field in the cavity.

5 Deflection of Atoms

In the present section we consider the deflection of the atom in the Raman-Nath approximation. In this regime the field does not displace the atom significantly but still changes the momentum.

One of the initial conditions in the scattering process is the transverse position amplitude $f(x)$ of the atoms. According to Eqs. (13) and (14) the probability amplitudes c_n and s_n for finding the momentum p are Fourier transforms of the product of the initial position amplitude $f(x)$ and trigonometric functions of the mode function $\sin(kx)$ of the electromagnetic field. We can therefore distinguish two characteristic cases for these Fourier integrals: *(i)* In the Kapitza-Dirac regime the initial position distribution $|f(x)|^2$ of the atoms is broad compared to the period of the standing wave, or *(ii)* in the Stern-Gerlach regime the distribution is narrow.

Throughout the paper we focus on the Kapitza-Dirac regime. However, we emphasize that the case of the Kapitza-Dirac scattering with a mask reduces in the limit of a single slit to the Stern-Gerlach regime.

5.1 *Measurement Schemes*

The three degrees of freedom of this quantum system, the center of mass motion, the field and the internal degrees of freedom are entangled. We can therefore make joint measurements of these variables. In principle we can use all three of them. However, in the present discussion we confine ourselves to the motion and the field only.

Joint Measurements

In this situation the atom traverses the cavity prepared in a given field state $|\psi_{\text{field}}\rangle$, interacts with it and as a consequence gets deflected. After the atom has left the cavity we observe the field and measure the momentum. We reprepare the complete atom-field system and repeat the experiment.

Quantum mechanics predicts the conditional probability distribution

$$W(p, |\tilde{\psi}_{\text{field}}\rangle) \equiv \frac{1}{W(|\tilde{\psi}_{\text{field}}\rangle)} \sum_{j=a,b} \left| \langle j|\langle p|\langle \tilde{\psi}_{\text{field}}|\Psi\rangle \right|^2$$

to find the momentum p given that the field is in the reference state

$$|\tilde{\psi}_{\text{field}}\rangle \equiv \sum_{n=0}^{\infty} \tilde{\psi}_n |n\rangle \tag{15}$$

and the probability that the vector $|\Psi\rangle$ contains this reference state reads

$$W(|\tilde{\psi}_{\text{field}}\rangle) \equiv \frac{1}{N} = \int_{\infty}^{\infty} p \sum_{j=a,b} \left| \langle j|\langle p|\langle \tilde{\psi}_{\text{field}}|\Psi\rangle \right|^2 .$$

Since we do not make a measurement of the internal states $|j\rangle = |a\rangle \text{or} |b\rangle$ we take the trace over them.

When we substitute the photon number representation Eq. (15) of the reference state into the above expression for W we find

$$W(p, |\tilde{\psi}_{\text{field}}\rangle) = N \sum_{j=a,b} \left| \sum_{n=0}^{\infty} \tilde{\psi}_n^* \langle j|\langle p|\langle n|\Psi\rangle \right|^2 , \tag{16}$$

and consequently the probability distribution originates from the coherent sum, that is the interference of many probability amplitudes.

We now make use of the explicit expression Eq. (12) for the state vector $|\Psi\rangle$ and the probability distribution reads

$$W(p, |\tilde{\psi}_{\text{field}}\rangle) = N \left\{ \left| \sum_{n=0}^{\infty} \tilde{\psi}_n^* \psi_n c_n(p) \right|^2 + \left| \sum_{n=0}^{\infty} \tilde{\psi}_n^* \psi_n s_n(p) \right|^2 \right\} . \tag{17}$$

Indeed, the two internal levels contribute in an incoherent way. In contrast, the field states represented by the probability amplitudes $\tilde{\psi}_n^*$ and ψ_n enter in a coherent way.

Averaged Measurements

We now consider a completely different experiment. The atom traverses the cavity and we only measure the momentum of the atom. We therefore ignore the change of the field due to the atom. In this case we have to take the trace over the cavity state. When we use photon number states to perform this trace, the resulting probability reads

$$W(p) = \sum_{j=a,b} \sum_{n=0}^{\infty} |\langle j|\langle p|\langle n|\Psi\rangle|^2 .$$

In contrast to Eq. (16) here we first square and then take the sum. The resulting probability distribution therefore originates from an incoherent sum, that is a sum of probabilities.

When we substitute the explicit representation Eq. (12) of the state vector into the above expression for the averaged momentum distribution we arrive at

$$W(p) = \sum_{n=0}^{\infty} |\psi_n|^2 \left[|c_n(p)|^2 + |s_n(p)|^2\right] . \tag{18}$$

Indeed, here we only sum probabilities.

5.2 *Kapitza-Dirac Regime*

We now consider the case where the initial atomic position distribution of width L reaches over N periods λ of the standing wave. For the sake of simplicity we assume it to be constant.

Momentum Quantization

In this case we can evaluate the amplitudes c_n and s_n explicitly and find

$$c_n(p) = \frac{1}{\sqrt{\hbar k}}\, \delta_N^{(1/2)}\left[\frac{p}{\hbar k}\right] \frac{1}{2} \left[1 + (-1)^{p/(\hbar k)}\right] J_{p/(\hbar k)}(\kappa\sqrt{n}) \tag{19}$$

and analogously

$$s_n(p) = \frac{1}{\sqrt{\hbar k}}\, \delta_N^{(1/2)}\left[\frac{p}{\hbar k}\right] \frac{1}{2i} \left[1 - (-1)^{p/(\hbar k)}\right] J_{p/(\hbar k)}(\kappa\sqrt{n}). \tag{20}$$

We notice that the function

$$\delta_N^{(1/2)}(\xi) \equiv \frac{1}{\sqrt{N}} \sum_{\nu=0}^{N-1} \exp(-2\pi i \xi \nu)$$

is periodic and has maxima at integer values of ξ. Indeed, at these positions the phase factors are integer multiples of 2π and each term in the sum is unity giving the value $\sqrt{N}$ for the function $\delta_N^{(1/2)}$. Hence, as $N \to \infty$ the maxima of $\delta_N^{(1/2)}$ approach infinity. For non integer values ξ the individual terms cancel each other.

This behavior suggests that $\delta_N^{(1/2)}$ acts as a comb of δ-functions at integer values of ξ. However, we can show that only the square of $\delta_N^{(1/2)}$ displays this behavior. Since we are interested in momentum distributions and hence probabilities the function $\delta_N^{(1/2)}$ only appears as a square. The argument of $\delta_N^{(1/2)}$ is $p/(\hbar k)$. Consequently the momentum of the atom can take on only multiple integers of the momentum $\hbar k$.

We therefore find a quantization of the atomic momentum in multiples of the photon momentum. However, the association with the momentum of the light field is slightly misleading. This quantization does not arise from the quantization of the radiation field. It rather emerges from the periodicity of the potential, namely the mode function of the electromagnetic field.

Moreover, we recognize from Eqs. (19) and (20) that the probability amplitude $c_n(p)$ is only nonzero for even integer multiples of $\hbar k$. In contrast, $s_n(p)$ only takes on nonzero values for odd integer multiples of $\hbar k$. We recall that $c_n(p)$ and $s_n(p)$ are associated with the atom leaving the cavity in the ground or excited state, respectively when it has entered the cavity in the ground state. Therefore, in order to leave it in the ground state it has to undergo an even number of Rabi cycles and thus exchanges an even number of photon momenta. Likewise, an atom leaving in the excited state needs an odd number of momenta exchange in order to make the transition from its initial ground state. This is just another manifestation of the entanglement of the field variables with the momentum of the atom.

Momentum Distribution

We are now in a position to derive explicit expressions for the momentum distributions discussed in the preceding section. We start our analysis with the averaged momentum distribution, Eq. (18).

We can combine the contributions from the atoms leaving the cavity in the ground state or in the excited state coresponding to the probabilities $|c_n(p)|^2$ and $|s_n(p)|^2$ when we note that due to the special form Eqs. (19) and (20) of c_n and s_n the first sum only contains the even multiples of $\hbar k$ whereas the second contribution only contains the odd multiples. However, in both cases the probability is given by the square of the Bessel function. Hence, we arrive at

$$W(p) \;=\; \sum_{\wp=-\infty}^{\infty} \delta(p-\wp\hbar k)W_\wp \tag{21}$$

where we have introduced the dimensionless and discrete momentum distribution

$$W_\wp\left[|\psi_{\text{field}}\rangle\right] \equiv \sum_{n=0}^{\infty} W_n\left[|\psi_{\text{field}}\rangle\right] J_\wp^2(\kappa\sqrt{n}). \tag{22}$$

We note that this averaged momentum distribution $W_\wp$ involves only the photon statistics $W_n \equiv |\psi_n|^2$ of the cavity field. In particular, it does not bring in the probability amplitudes ψ_n. In Figs. 2, 3 and 4 we depict the averaged momentum distributions for a number state $|n\rangle$, a coherent state $|\alpha\rangle$ and a highly squeezed state $|\psi_{sq}\rangle$ in the cavity. All three momentum distributions are different. For the number state we find oscillations and a dominant maximum at $\wp = \kappa\sqrt{\bar{n}}$. The oscillations are very reminiscent of the Franck-Condon oscillations in molecules. Indeed, we can show, that both oscillations have a common origin: Interference in phase space. For the coherent and the squeezed state the oscillations for small momenta have been averaged out but the dominant maximum at $p = \kappa\sqrt{\bar{n}}$ remains. Moreover, for the case of the squeezed state we note, that the oscillatory photon statistics manifests itself in the decay of the right side of the maximum.

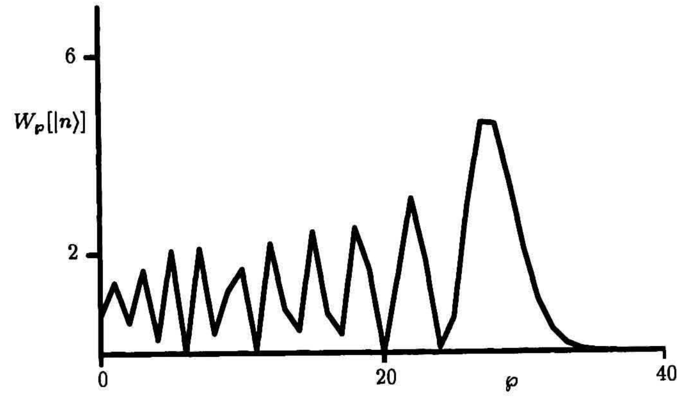

Figure 2: Momentum distribution of atoms scattered off a single mode of a cavity field in a number state $n = \bar{n} = 9$ photons for an interaction parameter $\kappa = 10$. The distribution shows a dominant peak at $\wp = \kappa\sqrt{\bar{n}} = 30$ and a strong decay for momenta larger than this critical value. For $\wp$ smaller than $\kappa\sqrt{\bar{n}}$ the distribution is oscillatory. These oscillations result from quantum interference of translational motion. The envelope follows the classical cross section.

Joint Measurements

We now turn to the case of a joint measurement between the momentum of the atom and the field. When we substitute the explicit expressions Eqs. (19) and (20) for s_n and c_n into the expression for the joint momentum distribution Eq. (17) we arrive at

$$W\left[p, |\tilde{\psi}_{\text{field}}\rangle\right] = \sum_{\wp=-\infty}^{\infty} \delta(p - \wp\hbar k) W_{\wp}(|\tilde{\psi}_{\text{field}}\rangle, |\psi_{\text{field}}\rangle) \tag{23}$$

where we have introduced the dimensionless and discrete momentum distribution

$$W_{\wp}\left[|\tilde{\psi}_{\text{field}}\rangle, |\psi_{\text{field}}\rangle\right] \equiv \frac{1}{W(|\tilde{\psi}_{\text{field}}\rangle)} \left|\sum_{n=0}^{\infty} \tilde{\psi}_n^* \psi_n J_{\wp}(\kappa\sqrt{n})\right|^2 \tag{24}$$

and

$$W(|\tilde{\psi}_{\text{field}}\rangle) = \sum_{\wp=-\infty}^{\infty} \left|\sum_{n=0}^{\infty} \tilde{\psi}_n^* \psi_n J_{\wp}(\kappa\sqrt{n})\right|^2$$

denotes the probability to find the reference field state $|\tilde{\psi}_{\text{field}}\rangle$ after the interaction.

Nowhere clearer than in the comparison between the averaged and the joint momentum distributions $W_{\wp}\left[|\psi_{\text{field}}\rangle\right]$ and $W_{\wp}\left[|\tilde{\psi}_{\text{field}}\rangle, |\psi_{\text{field}}\rangle\right]$ do we recognize the power of entanglement: In the averaged distribution we sum the squares of Bessel functions. In the joint distribution we first sum the Bessel functions and then square the result. Since the Bessel functions oscillate between positive and negative values cancellations can occur in the summation over Bessel functions. No such cancellation arises in the averaged distribution.

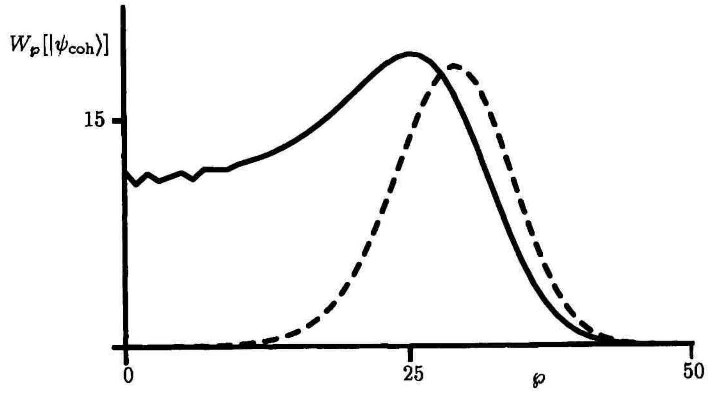

Figure 3: Influence of the photon distribution of a coherent state of average number of photons $\bar{n} = 9$ on the momentum distribution. The Poissonian photon distribution (dashed curve) creates a smooth momentum distribution. The maximum of W_n governs the maximum of $W_{\wp}$. The right edge of W_n controls the right edge of $W_{\wp}$.

Bessel functions enjoy a dominant maximum when the index is equal to the argument. When we assume that the product $\tilde{\psi}_n^* \psi_n$ is slowly varying on the scale of the oscillations in the Bessel function the main contribution to the sum arises for $\wp = \kappa\sqrt{n}$, that is $n = (\wp/\kappa)^2$. This yields the approximate expression

$$W_{\wp}\left[|\tilde{\psi}_{\text{field}}\rangle, |\psi_{\text{field}}\rangle\right] \cong \mathcal{N} W_{n=(\wp/\kappa)^2}$$

where $\mathcal{N}$ denotes a normalization constant.

This expression clearly shows that in this case the joint momentum distribution follows precisely the photon statistics of the field state in the cavity. This is very different from the case of the averaged momentum distribution where we have to average the photon statistics with respect to the square of the Bessel function.

We illustrate this for the example of a highly squeezed state as the initial state in the cavity and a phase state

$$|\tilde{\psi}_{\text{field}}\rangle \equiv |\varphi = 0\rangle \equiv \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} |n\rangle$$

as a reference state. We note, that this reference state satisfies the requirement that the product $\tilde{\psi}_n^* w_n$ is slowly varying since $\tilde{\psi}_n^* = const.$ Indeed, we find that the momentum distribution follows precisely the oscillatory photon statistics.

In this context it is interesting to understand why there is such a close connection between the momentum and photon distributions. We note, that this is not true if the reference state is a single photon number state or, likewise, if the initial field state is a photon number state. In both cases the summation over the photon number states reduces to a single term and the cancellation due to the oscillatory behavior of the Bessel function does not occur. We

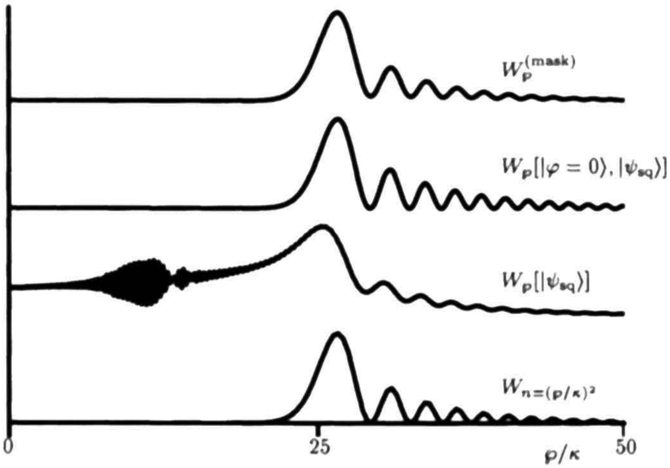

Figure 4: The photon statistics of a squeezed, displaced state of squeezing parameter $s = 50$ and displacement parameter $\alpha = 10$ (lower curve) and its read out via the momentum distribution of deflected atoms. The curve $W_\wp([|\varphi = 0\rangle, |\psi_{sq}\rangle]$ corresponds to a joint measurement of the atomic momentum and the field phase whereas the distribution $W_\wp[|\psi_{sq}\rangle]$ ignores the field phase. The top curve $W_\wp^{(\text{mask})}$ gives the momentum distribution of atoms filtered by a mask of slit width $d = \lambda/10$ placed at the nodes of the standing wave. The joint measurement strategy gives an adequate readout while ignoring the field phase results in a less effective readout as well as in additional rapid oscillations. We note that there is a modulation on the left side of the first maximum of $W_\wp[|\varphi = 0\rangle, |\psi_{sq}\rangle]$ and the period of the oscillations is slightly different from W_n. Here we have chosen $\kappa = 110$.

obviously need an initial and a reference field state that have broad photon distributions. In the case of the squeezed state and the phase state this condition is satisfied.

There is a simple explanation for this phenomenon of exact read out of photon statistics from momentum statistics. Since we are performing joint measurements, we are selecting from our ensemble very specific atoms. The squeezed state we have choosen has a phase distribution that is centered around the origin. Likewise, the phase state corresponds to the phase $\varphi = 0$. Hence, the joint measurement selects atoms that have not changed the phase of the field. These are the atoms that have traversed the cavity at the nodes where the electric field vanishes. However, at the nodes the gradient of the field is nonzero. Consequently the atoms obtain a momentum. The steepness of the gradient depends on the photon number and therefore the momentum transfer depends on the photon number. Since photon numbers are discrete the momentum transfer is discrete. Moreover, the probability for a given deflection angle is determined by the probability to find the corresponding electric field gradient, that is to find the corresponding photon number. Hence, there is a one-to-one correspondence between the momentum distribution and the photon number distribution.

5.3 *Kapitza-Dirac Scattering with a Mask*

In the preceding section we have found that a joint measurement selecting only those atoms that pass the resonator at the nodes provides a perfect readout of the photon statistics. This suggests to replace the joint measurement strategy by a simple mask with narrow slits around the nodes of the field. These slits are then separated by half of the period λ of the standing wave and have to be narrower than $\lambda/2$. In the optical regime it is impossible to obtain such mechanical slits. However, it is possible to obtain such a grating by using an extra light field and absorption.

We therefore assume a DeBroglie wave

$$f(x) = \frac{1}{\sqrt{N}} \sum_{\nu=0}^{N-1} g(x - \nu\lambda/2), \tag{25}$$

that is a coherent superposition of N Gaussian wave packets $g(x)$ located at the nodes of the field and with a width $d \ll \lambda/2$. The case of a single narrow wave packet corresponds to the Stern-Gerlach regime.

We substitute this form $f(x)$ of the DeBroglie wave into the definition Eqs. (13) and (14) of $c_n(p)$ and $s_n(p)$ and perform the integration. Here we make use of the fact that the slits are much narrower than the period, that is $dk = 2\pi d/\lambda \ll 1$. This allows us to linearize the sine function and we can immediately perform the remaining Gauss integrals. We arrive at

$$c_n(p) = \delta_N^{(1/2)} \left[\frac{p}{2\hbar k}\right] \frac{1}{2} \left[\widetilde{g}(p - \kappa\sqrt{n}\hbar k) + \widetilde{g}(p + \kappa\sqrt{n}\hbar k)\right]$$

where $\widetilde{g}(p)$ is a Gaussian in momentum space of width $\Delta p \equiv \hbar/d$.

We first note that the period $\lambda/2$ of the grating instead of λ as in the first example has produced a discreteness of the scattered momenta of integer multiples of $2\hbar k$ rather than $\hbar k$. Moreover, we note that the initial momentum distribution $\widetilde{g}$ gets displaced to momenta $\pm\kappa\sqrt{n}\hbar k$. Hence, every number state in the cavity gives rise to a momentum transfer by $\pm\kappa\sqrt{n}\hbar k$. When the width Δp of the initial momentum distribution is smaller than the separation

$$\delta p \equiv \kappa(\sqrt{n+1} - \sqrt{n})\hbar k \cong \frac{\kappa}{2\sqrt{n}}\hbar k$$

of neighboring momentum peaks caused by neighboring number states the discreteness of the number states manifests itself in discrete peaks in the momentum distribution of the deflected atoms.

Similarly we find the probability amplitude

$$s_n(p) = \delta_N^{(1/2)} \left[\frac{p}{2\hbar k} - \frac{1}{2}\right] \frac{1}{2i} \left[\widetilde{g}(p - \kappa\sqrt{n}\hbar k) - \widetilde{g}(p + \kappa\sqrt{n}\hbar k)\right].$$

We note that the antisymmetry of the sine function together with the period $\lambda/2$ has created odd integer multiples of $\hbar k$.

When we substitute these expressions for c_n and s_n into the formula Eq. (18) for the averaged momentum distribution we arrive at

$$W(p) = \sum_{\wp=-\infty}^{\infty} \delta(p - \wp\hbar k) W_{\wp}^{(\text{mask})}$$

where

$$W_\wp^{(\mathrm{mask})}\left[|\psi_{\mathrm{field}}\rangle\right] \equiv \sum_{n=0}^{\infty} W_n\left[|\psi_{\mathrm{field}}\rangle\right] \frac{1}{2}\left[W(\wp-\kappa\sqrt{n})+W(\wp+\kappa\sqrt{n})\right].$$

Here $W(p) \equiv |\widetilde{g}(p)|^2$ is the initial momentum distribution.

According to this result the momentum is again quantized in units of $\hbar k$. This is a manifestation of the coherence of the initial atomic distribution Eq. (25) over many periods of the standing wave. Moreover, the distribution consists of symmetrically located copies of the initial momentum distribution $W(p)$. They are located at $\wp = \pm\sqrt{n}\kappa$. The envelope of these peaks is the photon statistics W_n of the cavity field.

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