

# ON THE ANALYSIS OF OCULAR MOVEMENTS

BY

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(WITH 4 PLATES AND 4 TEXTFIGURES)

VERHANDELINGEN DER KONINKLIJKE AKADEMIE  
VAN WETENSCHAPPEN TE AMSTERDAM

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In my former paper <sup>1)</sup> I pointed out that the method of investigation which it described should show us the place occupied by the eye in a number of successive positions. Formulae were derived, which enable us to compute the location of the instantaneous centres of rotation. For convenience these formulae were derived on the assumption that for each position a line, fixed in relation to the eye, and one fixed point on the line, could be located. For a fixed line the visual line <sup>2)</sup> was taken, i.e., the straight line which in the normal eye joins the point of fixation with the middle of the fovea centralis, the place of highest visual acuity <sup>3)</sup>; for fixed point, provisionally, the intersection of this line and the anterior surface of the cornea was taken. These assumptions were made in order to fix the notions. Only those movements are considered in which all points describe paths parallel to a horizontal plane, the "primary position" being the starting point of the movements to the right and to the left side. However, for deriving the curve of the instantaneous centres of rotation, it is not necessary to determine at every turn the *location* of the fixed line: the knowledge of its *direction* is sufficient for the purpose. On the other hand, the fixed point does not need to be situated on the considered line; it is only necessary that this point be one with the eye, kinematically speaking; the location of this point must be known, of course.

If one derives the curve of the centres of rotation from such data, one knows its situation with respect to the line described by the fixed point. However, the location of the successive positions of the eye is not yet known. In order to consider these positions too, a knowledge of the situation of the fixed point in relation to the eye is necessary. It is indeed desirable

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<sup>1)</sup> Communicated at the meeting of February 25th, 1928; Proceedings, Volume 31, N<sup>o</sup>. 3.

<sup>2)</sup> When writing the former paper, I did not know the English edition of HELMHOLTZ's Treatise, edited by JAMES P. C. SOUTHALL, published by the Optical Society of America 1924/1925. Here, the „Gesichtslinie" is called the "visual axis"; the "Visierlinie" the "line of sight"; the "Blicklinie" the "line of fixation" (Vol. I, p. 315). The visual line, defined above, is identical with the *one* line of sight possible of which GULLSTRAND speaks, in accordance with the fact that the act of sighting requires central visual acuity (Vol. I, p. 124, note). Upon this line GULLSTRAND remarks that since its position can be accurately found, whereas that of the visual axis cannot, it is better generally to reckon only with the former (Handbuch I Bd., S. 271; Am. Transl., Vol. I, p. 315).

<sup>3)</sup> Considering the uncertainty about the extent of area over which the elements may be supposed to be equivalent for fixation, it might be safer to use the notion of the line of sight for our purposes. The actual accuracy with which a line of sight may be located in the eye depends upon the accuracy with which the eye may reproduce a certain position in sighting. We shall see (p. 21) that the exactness of sighting which could be reached in the experiments was about one half of a minute. As the act of sighting is certainly different from the mere act of fixing a point, which involves probably less elements, this number does not give an direct information about the question of the central area.

to separate this investigation from the former ; the realisation of a fixed point and the determination of its successive positions is easier, and can be performed more exactly, than the precise determination of the situation of the point in relation to the eye. The accuracy of investigating the curve of instantaneous centres of rotation is therefore greater than the precision with which we can establish the corresponding positions of the eye. In the method of investigation these two determinations are already separated.

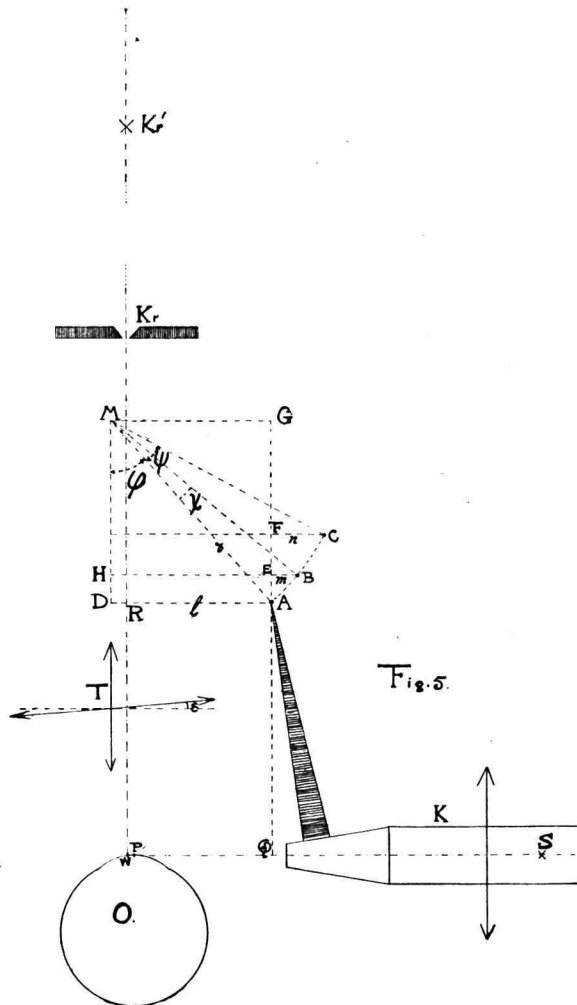
In the present paper, I shall first explain the mathematics of the method, and then their experimental realisation.

#### A. MATHEMATICAL METHODS.

The mathematical treatment will be preceded by an outline of the experimental methods.

The direction of the visual line is fixed by means of an instrument for sighting. This is built upon the alidade of a theodolite which can give the required rotations. The theodolite itself is mounted upon a cross-support. The screws of the latter enable us to shift the combination of theodolite and sighting-instrument in two mutually perpendicular directions. On the sighting-instrument is fitted a firm protuberance, the end of which bears another smaller cross-support. On the latter is mounted a little telescope (fig. 5, *K*), the optical axis of which is perpendicular to the visual line of the eye. This cross-support is adjusted so that the direction of one of its screws is parallel to the visual line. Thus by means of this screw the telescope *K* can be shifted in such a direction. The other screw of the small cross-support serves to focus the telescope *K* on the anterior surface of the subject's cornea. The telescope magnifies about 50 times. For each series of observations with a particular subject, the telescope is focussed once at the beginning, after which it is moved only in the direction parallel to the visual line. Experiments are made in the following manner : first the sighting-instrument is rotated at each turn over a certain angle, by means of the theodolite ; then the whole apparatus, which is mounted on the bigger cross-support, is shifted by means its screws, in such a manner, that first the subject, whose head is fixed, can sight again, and secondly, the observer can see the anterior surface of the subject's cornea in the telescope *K*. This telescope possesses a reticle in the plane where the image is formed by the objective. Now, by means of the screw of the smaller support the telescope is always shifted into such a position that in the image the vertical thread of the reticle is tangent to the anterior surface of the cornea. As it would be practically impossible to reach this position with the bigger screws alone, this separate adjusting process is necessary. A vertical spider-thread is firmly mounted on the telescope *K*. In fig. 5 the point *A* represents the projection of this spider-thread on the horizontal plane. It is obvious that this spider-thread forms one kinematic whole with the eye, if the telescope is always adjusted in the manner just described. *A* is the fixed point specified in the kinematic considerations.

Readings on the theodolite give the alterations in the direction of the visual line. It is assumed that in the zero-position the direction of the visual



line is the same as the direction of movement of one of the screws of the bigger cross-support. This direction is perpendicular to the plane of the face of the subject. Now, if we know at every turn the distance of  $A$  to the centre of the theodolite, and all rotations of the latter and all shiftings of the screws are read, noting meanwhile the directions and the order of succession in which they occur, we have all the data required for the construction of the path which  $A$  describes, and of the curve of instantaneous centres of rotation, which corresponds to it.

The data thus obtained contain many errors, proceeding partly from the imperfections of the apparatus, partly from the inaccuracy of the observations of the subject in sighting, and of the investigator in adjusting the telescope  $K$ .

Measurements of the position of  $A$  in relation to the centre-point of the theodolite succeed the investigations with the subject. In these investigations the telescope  $K$  is first focussed on the anterior surface of the subject's cornea. As stated above, the screw which enables this focussing is not adjusted further during the course of the experiment; the other smaller screw, however, was moved continually in order to adjust the telescope. Now, after each series of investigations, this screw is replaced in exactly the same position that it had at the beginning of the experiment. With it in this zero-position the location of  $A$  is determined. In fig. 5  $O$  represents the eye of the subject;  $Kr$  is the cross-shaped aperture of the sighting-instrument which corresponds to the cross  $Kr'$ , about 16 cm. behind it; the subject has to see  $Kr'$  in the middle of  $Kr$ .

The  $X$ -axis of the system of coordinates to which observations and results are referred, is taken in the direction of the antero-posterior screw of the larger support, perpendicular to the plane of the subject's face. The direction towards the subject is positive; the  $Y$ -axis, perpendicular to it, is positive toward the right; all these specifications agree with those of the former paper.

In fig. 5,  $M$  is the centre of rotation of the theodolite.  $MD$  is drawn parallel to the  $X$ -axis, and  $AD$  normal to it. For the rest, the location of the  $X$ -axis has no importance in the following considerations. Let us call the angle  $DMA : \varphi$ ,  $DA : l$ ,  $MA : r$ . These magnitudes should be determined. Now, let the theodolite be rotated over a known angle  $\chi$ ; then  $A$  moves to  $B$ . The line  $AG$  is drawn parallel to  $DM$ , and  $BEH$  perpendicular to it;  $MG$  is parallel to  $DA$ . It is obvious that  $EB$  is the displacement of  $A$  in the direction of the  $Y$ -axis. Now we can again rotate the theodolite. Let us call the whole angle  $AMC$  over which it has rotated  $\psi$ , as  $A$  has moved to  $C$ . The total displacement of  $A$  in the direction of the  $Y$ -axis is now  $FC$ .

Designating  $EB$  and  $FC$  respectively  $m$  and  $n$ , we have

$$l = r \sin \varphi$$

$$l + m = r \sin (\varphi + \chi)$$

$$l + n = r \sin (\varphi + \psi)$$

from which equations we can derive the following formulae:

$$\tan \varphi = \frac{m \sin \psi - n \sin \chi}{m (1 - \cos \psi) - n (1 - \cos \chi)} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

$$r = \frac{m}{\sin (\varphi + \chi) - \sin \varphi} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (8)$$

It will be described below (page 16, sub 6) how the displacements in the direction of the  $Y$ -axis are measured. It would be possible, of course,

to compute  $\varphi$  and  $r$  from  $m$  and the displacement  $AE$  in the direction of the  $X$ -axis; but it is easier to realise experimentally the method just described.

Now, in the experiments with the subject, the shiftings of  $M$  are obtained by moving the screws of the bigger cross-support. As we have indicated above, the  $X$ -axis is taken in the direction of the antero-posterior screw of this support. Here we have to deal with one of the imperfections of the apparatus: the direction of the transverse screw is not exactly perpendicular to the direction of the antero-posterior one. This error must be measured; the deviation  $\epsilon$ , of the transverse screw from the positive  $Y$ -axis, appeared to be in the direction of the negative  $X$ -axis (p. 14). The angle  $\epsilon$  is only  $5'$ ; still it is necessary to introduce it into the computation-formulae. In fig. 5 the directions of the screws are drawn near  $T$ . It is obvious that, in order to know the real shiftings along the  $X$ -axis, it is necessary to subtract from the antero-posterior screw-readings the product of the readings on the transverse screw multiplied by  $\sin \epsilon$ ; the shiftings along the  $Y$ -axis are found by multiplying the screw-readings by  $\cos \epsilon$ . We may now leave for a space the errors of the screws; these will be dealt with in the second part of the paper.

The point  $A$  follows all *shiftings* of  $M$ ; naturally the latter does not change its location in rotations of the theodolite. But then the whole rectangle  $ADMG$  moves over the angle which is read on the theodolite. We can compute the displacements of  $A$ , if we also know for every turn the radius  $r$  of the arc through which  $A$  moves, as well as the angle which  $MA$  makes with the  $X$ -axis. We will call this angle  $\varphi$ , indicating with  $\varphi_0, \varphi_1, \varphi_2 \dots$  etc. the values of  $\varphi$  preceding respectively the first, second, third ... etc. rotations. So  $\varphi_0$  is the angle  $\varphi$  of the equation (7). We call the angles over which the theodolite at each turn is rotated  $\delta_0, \delta_1, \delta_2 \dots$  etc., and the whole angle with respect to the zero position after the first, second, third ... rotations  $\alpha_1, \alpha_2, \alpha_3 \dots$  etc. Fig. 6 gives a scheme of the displacement of  $A$  during several rotations and shiftings. The intersection of the  $X$ - and  $Y$ -axes is made to fall on  $A_0$ : the zero-position of  $A$ . In this zero-position the sighting-instrument is adjusted so as to give to the visual line the direction of the antero-posterior screw, i.e., of the  $X$ -axis. Now  $A$  may be displaced along the track drawn from  $A_0$  via  $A_0', A_0'', A_0'''$  to  $A_1$ .  $A_1$  indicates the location of  $A$ , if the above mentioned conditions are satisfied (p. 5). We can now set up formulae which describe the manner in which the coordinates of  $A$  change; the successive rotations of the theodolite, and shiftings of the three screws used, are supposed to be known exactly. The angle is  $\delta_0$ ; the successive shiftings by means of the  $X$ -axis-screw are represented by  $\bar{x}_k$ :  $\bar{x}_0, \bar{x}_1 \dots$  etc.; the shiftings produced by the other screw of the bigger support are  $\bar{y}_0, \bar{y}_1, \dots$  etc., and we call the shiftings parallel to the visual line which are produced by one of the screws of the

small cross-support  $\bar{x}_0, \bar{x}_1 \dots$  etc., Here  $\bar{x}_k$  is positive in the direction toward the subject.

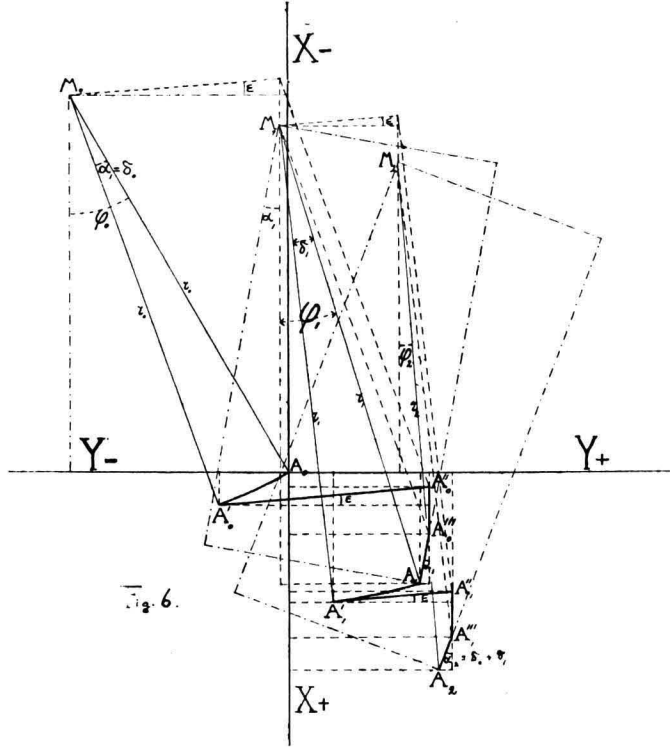


Fig. 6 shows clearly that the displacements of  $A$  in the direction of the  $+X$ -axis and the  $+Y$ -axis in consequence of the rotation are respectively :

$$r_0 \cos (\varphi_0 - \delta_0) - r_0 \cos \varphi_0 \quad \text{and} \quad - \{ r_0 \sin \varphi_0 - r_0 \sin (\varphi_0 - \delta_0) \}$$

The movement  $\bar{y}_0$ , of the transverse screw, gives the displacements :

$$- \bar{y}_0 \sin \varepsilon \quad \text{and} \quad \bar{y}_0 \cos \varepsilon$$

The antero-posterior screw gives the shiftings :

$$\bar{x}_0 \quad \text{and} \quad 0$$

The screw movement  $\bar{x}_0$  gives the displacements :

$$\bar{x}_0 \cos \alpha_1 \quad \text{and} \quad - \bar{x}_0 \sin \alpha_1$$

So the total displacements in the directions of the  $+X$ -axis and  $+Y$ -axis (respectively  $b_1'$  and  $c_1'$ ) are :

$$b_1' = r_0 \{ \cos (\varphi_0 - \delta_0) - \cos \varphi_0 \} - \bar{y}_0 \sin \varepsilon + \bar{x}_0 + \bar{x}_0 \cos \alpha_1 \quad . \quad (9)$$

$$\text{and} \quad c_1' = r_0 \{ \sin (\varphi_0 - \delta_0) - \sin \varphi_0 \} + \bar{y}_0 \cos \varepsilon - \bar{x}_0 \sin \alpha_1 \quad . \quad (10)$$

We remember that  $\varphi_0$  and  $r_0$  were found by means of the equations (7) and (8). As  $A$  is not a point of the eye itself, the displacements are called  $b_1'$  and  $c_1'$  in order to distinguish them from the values  $b_k$  and  $c_k$ , mentioned in the former paper. The whole curve of  $A$  is, of course, a *transformed* curve of the cornea, and also therefore of the whole eye. After the analogous movement of  $A$  from  $A_1$ , via  $A_1'$ ,  $A_1''$ ,  $A_1'''$  towards  $A_2$ , the coordinates are :

$$\begin{aligned} b_2' &= b_1' + r_1 \{ \cos (\varphi_1 - \delta_1) - \cos \varphi_1 \} - \bar{y}_1 \sin \varepsilon + \bar{x}_1 + \bar{x}_1 \cos a_2, \\ c_2' &= c_1' + r_1 \{ \sin (\varphi_1 - \delta_1) - \sin \varphi_1 \} + \bar{y}_1 \cos \varepsilon - \bar{x}_1 \sin a_2. \end{aligned}$$

The value of  $\varphi_1$  is easily computed, as

$$\tan (\varphi_1 + a_1) = \frac{r_0 \sin \varphi_0}{r_0 \cos \varphi_0 + \bar{x}_0}.$$

Besides  $\sin (\varphi_1 + a_1) = \frac{r_0 \sin \varphi_0}{r_1}$ , and so

$$r_1 = \frac{r_0 \sin \varphi_0}{\sin (\varphi_1 + a_1)}.$$

The general formulae for the computations of  $\varphi_n$ ,  $r_n$ , and the coordinates  $b_{n+1}'$  and  $c_{n+1}'$  of the point  $A_{n+1}$  are given in the following equations :

$$\tan (\varphi_n + a_n) = \frac{r_0 \sin \varphi_0}{r_0 \cos \varphi_0 + \sum_0^{n-1} \bar{x}_k} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$r_n = \frac{r_0 \sin \varphi_0}{\sin (\varphi_n + a_n)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$b_{n+1}' = b_n' + r_n \{ \cos (\varphi_n - \delta_n) - \cos \varphi_n \} - \bar{y}_n \sin \varepsilon + \bar{x}_n + \bar{x}_n \cos a_{n+1} \quad (13)$$

$$c_{n+1}' = c_n' + r_n \{ \sin (\varphi_n - \delta_n) - \sin \varphi_n \} + \bar{y}_n \cos \varepsilon - \bar{x}_n \sin a_{n+1} \quad . \quad . \quad (14)$$

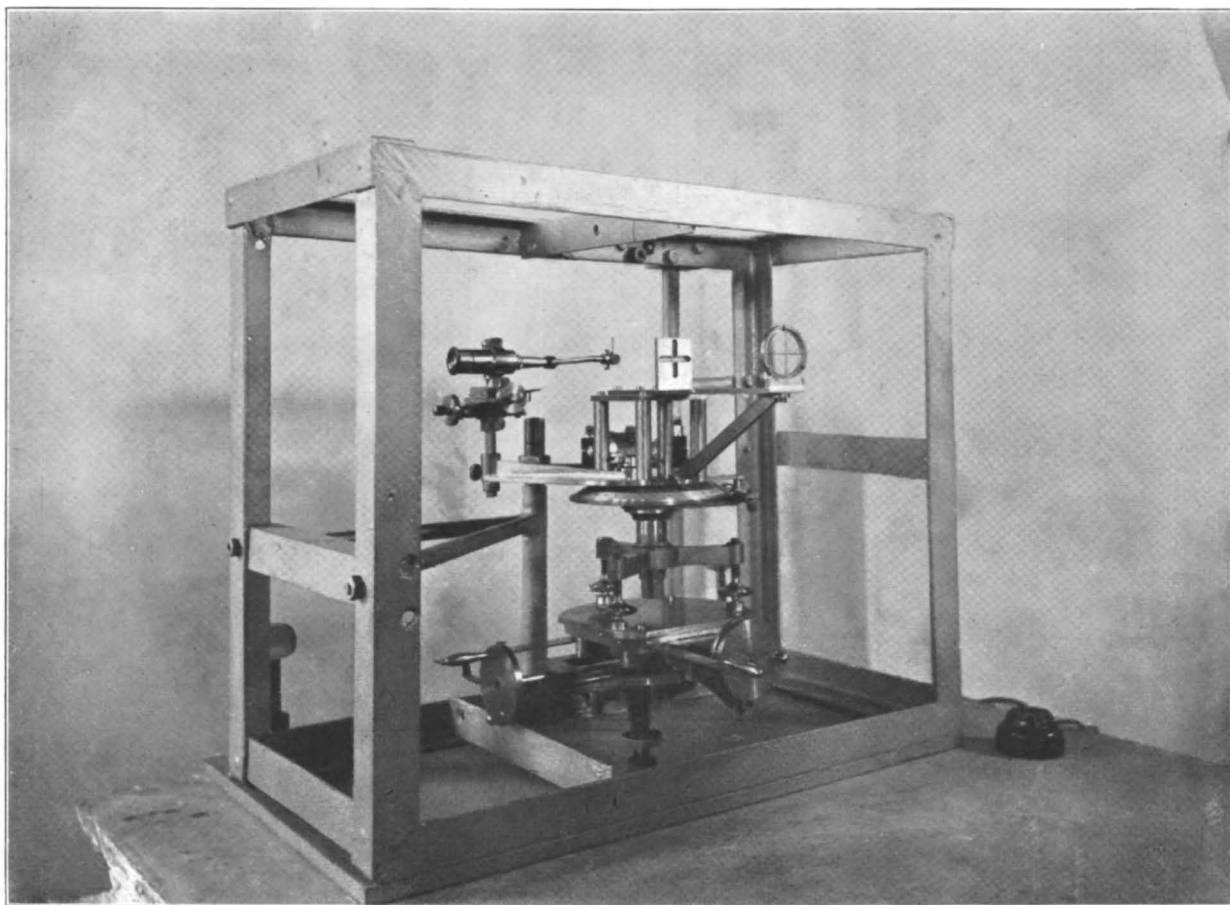
So the construction of the transformed curve of the eye  $A_0 A_1 A_2 \dots A_{n+1}$  is performed by introducing in the computation of each pair of coordinates the values of the former pair.

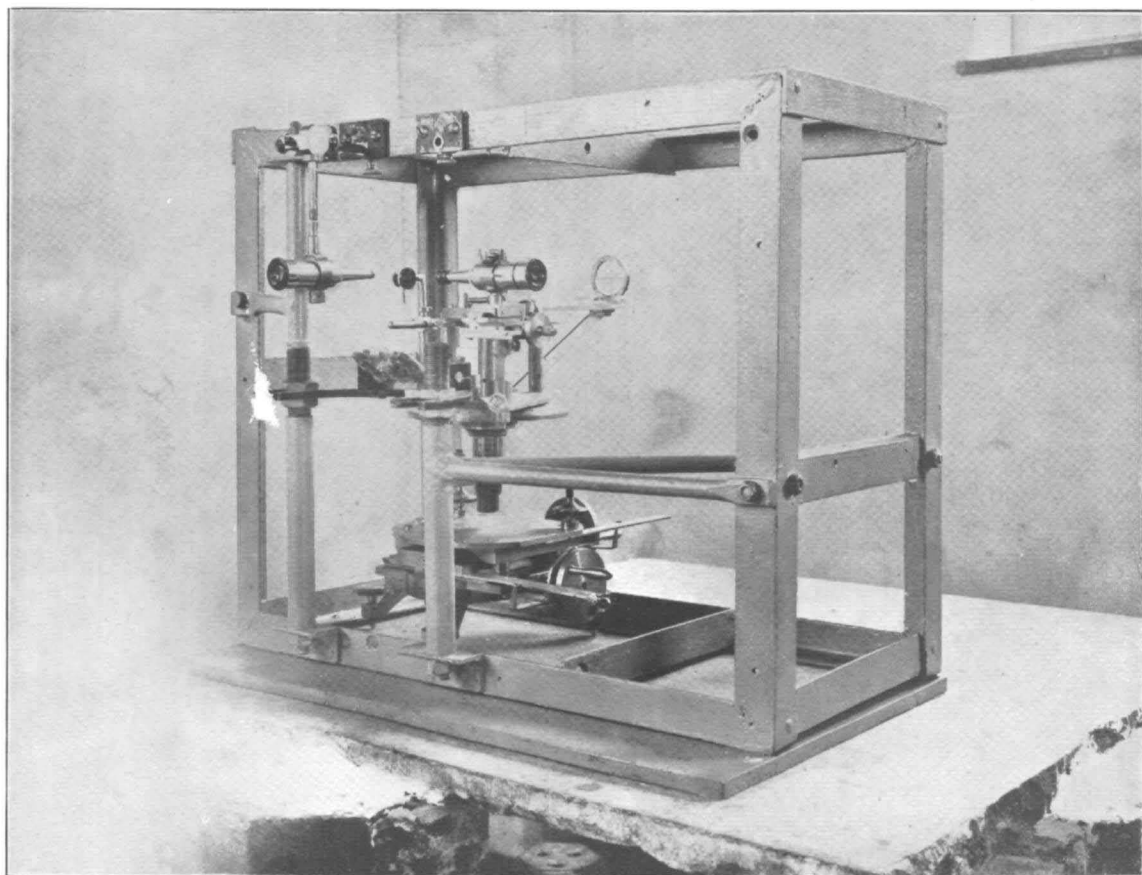
## B. EXPERIMENTAL METHODS.

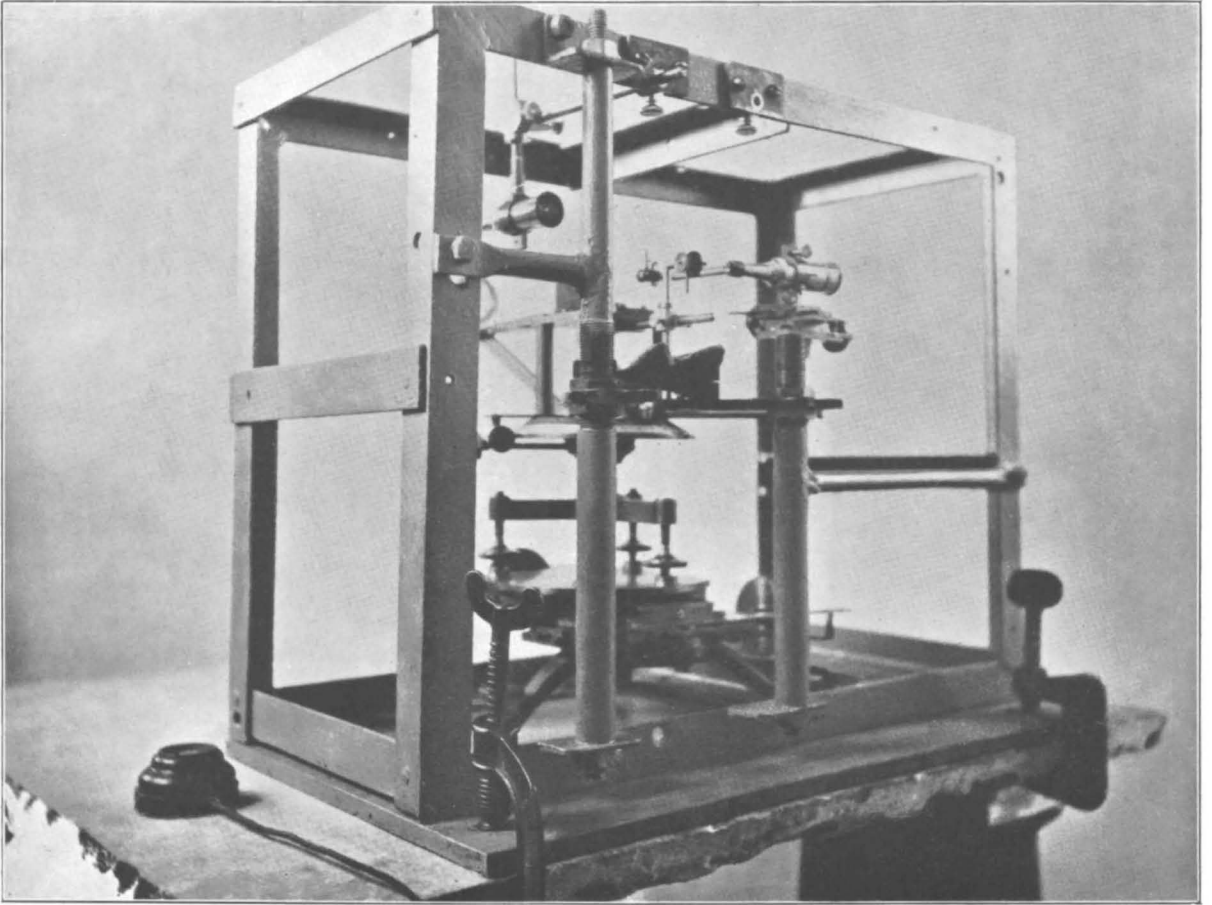
The apparatus with which the investigations are carried on is shown in Plates I and II.

The screws of the larger support permit about ten centimeters-movement. They are made of steel; the other parts of the support are made of brass. The support stands on three set-screws. A horizontal brass plate is fitted upon the support at the height of about 12 cm. above its bearing surface. On the upper side of the plate three hollowed little blocks are attached, in which

rest the set-screws of the theodolite. The speed of the screws (motion per turn) is one millimeter. Their heads are divided in hundredths. Of course these graduations cannot be used without further corrections. I will deal with these later. The theodolite, which rests on the blocks of the brass plate, is about 13 centimeters high. By means of its own set-screws it can be adjusted to a horizontal position, independent of the larger support. The precision of adjustment of the theodolite is in minutes. The sighting-apparatus and the protuberance which bears the small cross-support with the little telescope *K* (p. 4) are mounted upon the upper side of the rotating part of the theodolite; the diameter of its disc is about 13 cm. Four vertical little brass pillars are screwed upon the upper side of the disc. Two of them are 8.7 cm., the other two 7.2 cm. high; the latter are shorter, because they rest upon the protuberance which their screws pass in order to enter the disc of the theodolite. In this way the base of the protuberance is fixed upon the disc. The protuberance is a triangular plate of brass provided with a vertical rib. The sides of the triangle are about 17 and 11 cm., the base 9 cm. Near the outer point of the triangle a little bar is mounted, on which the small cross-support is fixed. The bar can be moved in a vertical direction. The screws of the cross-support permit a movement of about 3.5 cm., their speed is 0.5 mm. The heads of these screws are graduated into fifty divisions. I will deal with the control of their errors below. The direction of the upper screw is adjusted so as to be parallel to the visual line (p. 4) i.e. it is adjusted so as to be parallel to the antero-posterior larger screw when the apparatus is in its zero-position (p. 16); in order to make this possible the cross-support is so mounted that it may be revolved upon a cone at the upper end of the small vertical bar. Upon the cross-support is also fixed a small cone on which the telescope *K*, with which the observer can focus the cornea of the subject, can be revolved. A light and strong metallic little tube is fitted upon the telescope *K*; this tube supports at its other end the vertical spider-thread, the projection upon the horizontal plane of which represents the point *A*. This spider-thread may be moved in two vertical planes which are perpendicular one to the other; therefore it is possible to adjust it in an exactly vertical position (for the execution see p. 17). The pillars mentioned above support a horizontal brass plate which is screwed upon their upper ends. A frame which continues in a direction away from the subject is fitted upon the plate. This frame supports the two vertical sighting-crosses *Kr* and *Kr'*, already mentioned. The cross *Kr'* fixed upon the frame at its outer end, is photographed on a little glass plate. The lines are sharp, the cross being the reduced image of a carefully made ink-drawing. The breadth of the lines is about 0.3 mm.; the length of the arms of the cross is 2 cm. The glass plate may be revolved about a horizontal axis and shifted in the vertical plane upon a little circular frame, in order to adjust the cross. The other cross, *Kr*, of the sighting-apparatus is fixed upon the frame nearer to the subject, at a distance of 16 cm. from the cross *Kr'*. A vertical brass plate







(3 mm. thick) is mounted on the frame. This plate contains the cross-shaped opening *Kr*, the length of whose arms is 1.5 cm., their breadth about 0.35 mm. The plate is at a distance of about 15 cm. from the eye; the distance changes somewhat with the positions of the larger screws. The spider-thread *A* is half-way between the telescope *K* and the plate with *Kr*. This is necessary for the measurement of the position of *A*, with which we will deal below.

The whole apparatus is placed in a cage, built of T-shaped iron bars. The breadth of the cage is 62 cm., its depth 30 cm., and its height 51 cm. The cage is fixed upon a heavy iron plate; the set-screws of the large cross-support rest upon this plate. It was not necessary to make little holes in the plate for the set-screws, since the apparatus is heavy enough to prevent changes in its position during the experiments; indeed, the forces which are exerted upon the apparatus are small.

The invariable fixation of the subject with regard to the apparatus is secured by means of merely biting into a mass of wax upon a biting-plate (this plate is better shown in Plate II than in Plate I). Indeed, if a biting-plate is well constructed, fixation is excellent and no other support is necessary; during the course of the experiments this could be easily checked by means of the telescope *K*. The outline of the nicked iron plate is congruent to the curve of the jaws. As the plate has the form of a wide U, it does not accommodate the incisors only, but the molar-teeth as well. I consider this shape, which is different from the ordinary "Beiszbrettchen", essential for good fixation. Besides, the plate may be revolved round a transverse horizontal axis; this is necessary, because the jaws ascend in the antero-posterior direction. Only with this kind of adjustment is it possible to find a position in which one can easily look straight forward with the teeth set in the wax plate. For this purpose the biting-plate has two cylinder-shaped protuberances; each of them may be revolved between a pair of excavated iron blocks. Once the subject has chosen the suitable position, the blocks are screwed one upon the other, immovably.

In order to adjust the biting-plate approximately in the vertical direction, it may be moved up and down by being fixed upon two solid vertical bars with screw threads. Each pair of the excavated iron blocks is mounted upon a horizontal piece of iron, which has more or less the shape of a half horseshoe. Its straight end, which is turned towards the subject, has a 5 cm. deep indentation. The bars fit into these indentations; one solid nut under the horseshoe-shaped piece, and one nut above may fix it in every position desired. The length of the screw-threads is 7 cm.; the distance between them is 18 cm. The bars are firmly fixed upon the cage. In order to prevent the interference of any of them with the movement of the protuberance bearing the telescope *K*, it was necessary to shorten it, (Plates I and II). This bar has a special horizontal support. All parts of this biting-apparatus are so strong, and it is so firmly fixed, that it is impossible for the subject to disturb it sensibly.

The more subtle adjustment of the sighting-instrument is secured by means of the set-screws of the large cross-support.

The iron plate upon which the cage is mounted, has a protuberance, which is fixed upon a stone plate. The latter is cemented on to three big vertical pillars, sufficiently free from vibration.

*Calibration of the apparatus.*

1. *Horizontal motion.*

All movements must be performed in a really *horizontal plane*. First a water-level, placed upon the frame (swallow-tail) in which each of the bigger screws is fitted, secures their horizontal movements. Then the rotation of the theodolite is made a horizontal one by means of its own set-screws. A water-level permanently fitted upon the upper surface of the alhidade of the theodolite controls its position. The protuberance on which the little cross-support is mounted, was constructed in such a manner that, if the plane of rotation of the theodolite is a horizontal one, the directions of the screw-movements of the little cross-support are horizontal too; at least when the direction of the upper screw is parallel to the visual line. Left to themselves, all these parts remain horizontal; the movements cause only very small changes; in any case the errors which may be made in this way, cannot influence the results sensibly. I will return to this subject in the next section.

The construction of the sighting-apparatus secures a sufficiently vertical position of both crosses on the horizontal theodolite.

2. *Errors of the Screws.*

The screws which I had constructed for this apparatus were no real measuring screws, though they were made as carefully as possible.

The control of the displacements in the horizontal plane is as follows. A little apparatus was constructed, which could be mounted upon the smaller cross-support instead of the telescope *K*. In this apparatus could be fixed a little invar-metal block (10 cm.; graduation in tenths of a millimeter) of the S. I. P. Genève. It was possible to adjust the block in a horizontal position. Then it could be revolved about its vertical axis, and the apparatus was constructed in such a manner that this position remained a horizontal one during rotation. Two long parallel lines are drawn all along the little block, and perpendicular to the lines of the graduations. These long lines were used in the control of the direction of the screw-movements. A little microscope supplied with a reticle was focussed on the surface of the block. Now suppose that one of the threads of the reticle covers exactly the image of one of the lines on the block. If the direction of the line is the same as the direction of movement of the screw which is being examined, the image of the line will continue to be covered by the reticle, when the screw is turned. In the examination of the screws of the large cross-support the theodolite was used to adjust the block in the desired position; in the examination of the small screws the rotation in the

apparatus just described was used. In the former adjustment the angle of deviation could be measured. It appeared that this adjustment was sensitive enough, as it was accurate within one minute. The deviation of the screw-movements in the horizontal plane from one part to another could also be determined by this method. These deviations cause linear displacements in a direction perpendicular to the screw-movement, and they may influence the results sensibly. It was not difficult to construct the screws in such a manner that the periodical displacements of this kind (at every turn of the screw) were reduced to values which might be neglected. But it was more difficult to abolish the more "systematic" deviations, caused by the fact that the axes of the screws were not exactly straight. As I purposed to make as small errors as possible in computing the hundredths of millimeters, the maximum displacement in the perpendicular sense must not be more than 0.01 mm. This gives some difficulty when the length of the screws is about 10 cm.

Naturally, the same deviations exist in the vertical direction ; we deal with their measurements below (p. 16).

Once the little block was adjusted to have its lines in the "average" direction of a screw, it was possible to calibrate the displacements in that direction. At every turn the screw was revolved so far, that a reticle-thread of the microscope was seen just in the middle between two stripes of the graduation ; this can be made exact to 0.001 mm. The numbers, read on the head of the screw, were noted ; these numbers express the observed hundredths and the estimated thousandths of a millimeter. Now, if the difference between the successive numbers is always about 10, at any rate, if the deviation from a difference of 10 is not more than 0.4, we may assume that the movement of the screw is sufficiently regular. This was the case, as the deviations generally happened to be 0.1 to 0.2, sometimes 0.3, hundredths of mm. ; a deviation of 0.4 was very rare. The numbers thus observed were gathered together in tables. Besides, along the screw a little scale with an index is mounted ; it was observed, and noted, to which millimeter graduations the numbers on the head of the screw corresponded. Such a table was made for each screw in both directions. Now, in order to reduce the experimental values, it is first established between which pairs of numbers of the table the observed numbers are situated ; the number of intervals, which indicates the tenths of millimeters, can then be counted ; in order to obtain the hundredths (and thousandths) of millimeters, I have simply added the differences between the observed numbers and the nearest intermediate numbers of the table. With regard to the dead speed of the screws it is necessary to compare only positions which were obtained with the same direction of revolution ; we can always realise this easily.

The variation of the room-temperature was not great enough to influence the results sensibly.

The results of the measurements of  $r$  and  $\varphi$ , in which the screws are also used and which require a greater precision than that which we adopted for

the experiments with the subject, have proved that, when the described precautions are observed, only the thousandths of a mm. are not quite exactly determinable, (p. 18).

### 3. *Determination of the angle $\epsilon$ .*

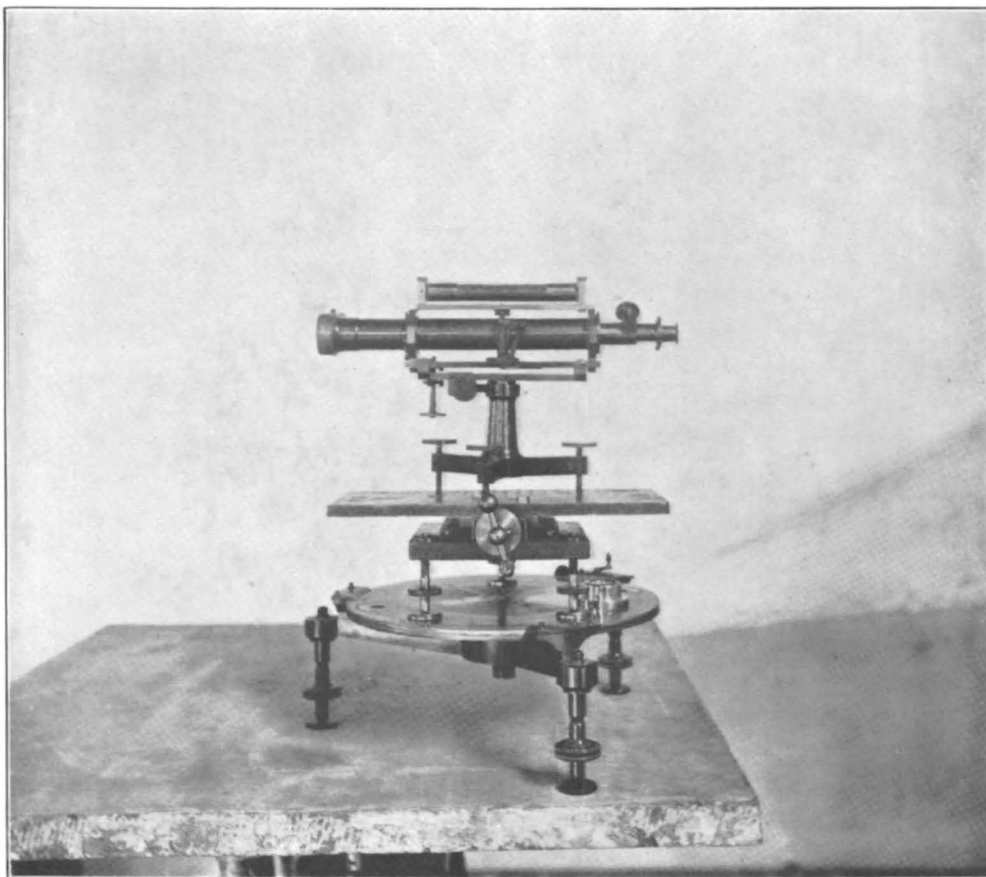
As we have defined on p. 7, this angle is the deviation from  $90^\circ$ , which is shown by the angle between the directions of the two larger screws. In determining this angle, the above-mentioned measuring-block was adjusted so as to have its long lines parallel to the direction of one of the screws. The rotations were made by means of the theodolite, and its position was noted. Then the theodolite was rotated so far, that the direction of the lines became parallel to the direction of the other screw. The angle between the two screw-directions can be read on the theodolite. As both adjustments are possible with an error smaller than  $1'$ , it could be expected that the results would diverge no more than about  $1'$ ; this was indeed the case. The value of  $\epsilon$  was settled to be  $5'$ ; and, as I have already said, it appeared that the deviation which the direction of the transverse screw makes with the positive  $Y$ -axis, takes place on the side of the negative  $X$ -axis, (the  $X$ -axis being taken in the direction of the antero-posterior screw).

Of course, for the determination of the corresponding angle in the smaller cross-support the theodolite could not be used, because its rotation moves the whole support. By means of the small revolving apparatus in which the measuring-block is fitted, this was first adjusted so as to be parallel to the direction of one of the small screws; then the block was made parallel to the direction of one of the larger screws, but this time by means of the theodolite, the revolving apparatus remaining stationary. Next the block was again revolved by means of the revolving apparatus to get it parallel to the direction of the other small screw. Finally it was made parallel again to the direction of the same large screw as before, by means of the theodolite. The angle between the directions of the two small screws could be read on the theodolite. Here, the block is adjusted four times, and the possibility of error is thus as great again as in the determination of  $\epsilon$ . However, the divergence of the values obtained was not sensibly larger. The value of the angle was settled to be  $2'$ , the deviation being in the same direction as in the case of  $\epsilon$ .

### 4. *Adjusting the theodolite into the "zero-position", in which the visual line is parallel to the direction of the antero-posterior screw ( $X$ -axis).*

A telescope (Plate III) is mounted at a distance of about 14 meters from the apparatus, on the side opposite the subject. The telescope is mounted upon a screw-support, by means of which it may be moved in a transverse direction. The screw-support, in its turn, is fitted upon a theodolite which permits a sensitive rotation. The whole was mounted upon a pillar, sufficiently vibration-free.

Now, it is possible to have a distinct image of both  $Kr$  and  $Kr'$  in the



telescope, which may be adjusted so as to have the image of  $Kr'$  just in the centre of the image of  $Kr$ . Moreover, if focussing took place with the antero-posterior screw of the apparatus in a middle position, it is not necessary to reset the telescope in order to have distinct images in the outer positions of the screw. Now the telescope is adjusted in such a manner that the two crosses appear in the centre,  $Kr'$  just in the middle of  $Kr$ ; then we may assume, that the direction of the visual line, which would be determined by the given position of the sighting-apparatus, is parallel to the optical axis of the telescope. Alterations of the direction of the sighting-apparatus, even smaller than a minute, give distinct changes of the image in the telescope, in which  $Kr'$  no more appears exactly in the centre of  $Kr$ . In the telescope there is a reticle in the plane of the image which is formed by the objective. Now, let us suppose, that the direction of the antero-posterior screw and that of the sighting-apparatus are divergent. Then it is obvious that the position of the images of the crosses, relative to the reticle, will change when the sighting-apparatus is moved by means of the antero-posterior screw. For this movement now causes a displacement of the sighting-apparatus in a perpendicular direction at the same time. If this displacement is not too extensive we shall continue to see the image of  $Kr'$  in the centre of  $Kr$  <sup>1)</sup>.

Only when the direction of the sighting-apparatus is the same as the direction of the antero-posterior screw, will the image of the crosses not change in relation to the reticle. The exactness of the adjustment depends first, on the antero-posterior distance of the two positions of the sighting-apparatus which are considered, and secondly on the accuracy of judging the transverse displacement. With regard to the errors of the screws just mentioned, a distance of about 9 cm. was available. The telescope was adjusted so that the vertical thread of the reticle lay just beside the image of the vertical thread of  $Kr'$ . A grain upon the reticle-thread was between them. It was not possible to arrange two spider-threads so close together in the reticle, that the thread of  $Kr'$  could be focussed in the centre of the space between them. The magnification of the ocular was too great for this. But the method just described gave quite satisfactory results. Indeed, it appeared that the sighting-apparatus could be adjusted with a precision of one minute. The error of this adjustment does not influence the accuracy of the transformed curve of the cornea; it only influences the accuracy of the location of this curve with regard to the visual line. As the mounting of the telescope was a permanent one, the position of the sighting-apparatus could be checked at every moment.

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<sup>1)</sup> This is only true, of course, if the antero-posterior screw moves in a straight direction. Dealing with the errors of the screws, we have pointed out, that this is not exactly the case. However, this possibility of error could be easily avoided; for the angular deviations from the average direction, and the subsequent perpendicular displacements, although very small, were noted in a range of positions all over the screw. In the adjustment described, positions were chosen in which the deviation was minimal.

The telescope was also used for the adjustment of the glass with the cross  $Kr'$ , in order to have the visual line in a horizontal plane. Indeed, in the horizontal telescope, the horizontal legs of the cross  $Kr'$  would then be seen in the centre of the corresponding legs of the cross  $Kr$ .

The deviations of the screw-movements in a vertical direction could also be checked by means of the telescope. They appeared to be small enough (a maximum of about  $1.5'$ ) to have no influence on the results.

5. *Adjusting the smaller cross-support in order to have the direction of one of its screws parallel to the antero-posterior larger screw when the theodolite is in the zero-position.*

The theodolite was adjusted, as just described, in the zero-position in which the visual line, which is fixed by the sighting-apparatus, is parallel to the direction of the antero-posterior screw.

The little measuring-block, already frequently mentioned, was first adjusted so as to be parallel to the screw of the smaller cross-support, (i. e. its upper screw), this adjustment being performed by means of the rotating apparatus upon which the block is mounted. As we have also mentioned, this cross-support may be revolved upon a cone at the upper end of the small vertical bar upon which it is fixed. Thus it is possible to rotate the support in such a manner that the direction of its upper screw is parallel to the direction of the antero-posterior screw of the larger support, the adjustment being controlled by means of the measuring block. As the fixation of the smaller support in this position changes the direction somewhat, it was checked after fixation. The error of the adjustment is within one minute.

#### 6. *Measurement of $r$ and $\varphi$ .*

In addition to the controls, described up to this point, adjustment of the apparatus includes the position of the telescope  $K$ . This will be dealt with in Section 7.

Once the apparatus was adjusted, measurements could be performed about the position of the vertical spider-thread in relation to the centre-point of the theodolite i.e.  $r$  and  $\varphi$ . As we have already said, these measurements were made after the investigations with the subject, the telescope  $K$  being replaced in the "zero-position" which it occupied at the beginning of the experiment (p. 6).

The data, necessary for the computations, are the displacements which the spider-thread  $A$  undergoes in the direction of the  $Y$ -axis, when the theodolite is rotated at least twice over a noted angle.

For these measurements another telescope,  $K'$ , exactly like  $K$ , was used. For this purpose it could be fitted upon the cage in which the apparatus is mounted. The fitting permitted a horizontal position and a rough adjustment in three perpendicular directions. Plate II shows the telescope  $K'$ , mounted upon the cage. The reticle of  $K'$  had two parallel spider-threads, one near the other. The optical axis of the telescope  $K'$  was

brought as nearly as possible into the direction of the  $X$ -axis. Then a plumb with a very thin metal thread was brought into the focus of the telescope. Movements of the plumb were abolished, without endangering its apparent vertical position, by supporting it very slightly in a vertical plane going through the axis of the telescope  $K'$ . By means of the image of the plumb, viewed centred between the two spider-threads, it could be adjusted in a vertical position. Then the plumb was removed. Nothing was changed at the telescope  $K'$ , in the further course of these measurements. Next, the spider-thread  $A$  was brought into the focus of the telescope  $K'$  by means of the larger screws of the apparatus. This thread in its turn could be adjusted into a vertical position by means of the now vertical threads of  $K'$ . This adjustment was performed in two vertical planes, one perpendicular to the other, the rotation being obtained by means of the theodolite. This was then put in its zero-position again. By means of the larger screws the now vertical thread  $A$  was placed into the focus in the telescope  $K'$  again. The antero-posterior screw was used for focussing as exactly as possible, and the transverse screw adjusted the image of the thread  $A$  centred exactly between the two reticle-threads. The position of the transverse screw was noted. The same adjustments were made after several noted rotations of the theodolite. Only two of them are necessary; for check, there were always four of them made. The notations of the positions of the theodolite and the transverse screw procure the data for  $\chi$ ,  $\psi$ ,  $m$  and  $n$ , from which  $r$ ,  $\varphi$ , and  $l$  may be computed. Focussing of the thread  $A$  has to be performed as exactly as possible, as it appeared that errors in the 0.001 mm. place assert themselves sensibly in the resultant values of  $r$  and  $\varphi$ . The screws were adjusted, of course, several times at every turn; the accuracy of focussing is great enough, so that the readings on the screw-head are reproducible to the 0.001 mm. place; however, as previously set forth, the 0.001 mm. place of the screw measurements is not quite exact. Therefore, I always rotated over four angles instead of two. It was not necessary to introduce the angle  $\epsilon$  in the computations, as  $\cos 5'$  differs too little from 1, to influence in these relatively small distances even the thousandth millimeter place. On the other hand, it is important that the optical axis of the telescope  $K'$  is as precisely as possible in the direction of the  $X$ -axis; an inexact antero-posterior adjustment of the thread  $A$ , when focussing, causes a positive or negative error of the transverse displacement. The magnitude of the error is the product of the error of focussing and the sin of the angle between the optical axis of  $K'$  and the  $X$ -axis. However, a simple computation will show, that the possibility of error is not very great: with an error of focussing of 0.1 mm., which is certainly improbable, the angle of deviation has to be 34—35 minutes to cause an error of 0.001 mm. in the transverse direction.

Several series of determinations have been performed in this way. The results have been treated in the following manner. In one of the series for

instance we obtained, when the whole angle over which the theodolite had been rotated was successively  $5^\circ$ ,  $10^\circ$ ,  $15^\circ$  and  $20^\circ$ , the following values for the correspondent displacements along the  $Y$ -axis : 3.248 mm. ; 6.292 mm. ; 9.100 mm. ; 11.649 mm. So we have (see fig. 5) :

$$\begin{aligned} r \sin(\varphi + 5^\circ) - r \sin \varphi &= 3.248 \\ r \sin(\varphi + 10^\circ) - r \sin \varphi &= 6.292 \\ r \sin(\varphi + 15^\circ) - r \sin \varphi &= 9.100 \\ r \sin(\varphi + 20^\circ) - r \sin \varphi &= 11.649. \end{aligned}$$

If we put  $r \sin \varphi = p$  and  $r \cos \varphi = q$  we get:

$$\begin{aligned} -2 \sin^2 2\frac{1}{2}^\circ \cdot p + \sin 5^\circ \cdot q &= 3.248 \\ -2 \sin^2 5^\circ \cdot p + \sin 10^\circ \cdot q &= 6.292 \\ -2 \sin^2 7\frac{1}{2}^\circ \cdot p + \sin 15^\circ \cdot q &= 9.100 \\ -2 \sin^2 10^\circ \cdot p + \sin 20^\circ \cdot q &= 11.649. \end{aligned}$$

The most probable values of  $p$  and  $q$  were computed by means of the method of least squares. In this case, we get :  $r \sin \varphi = 24.398$  and  $r \cos \varphi = 38.365$ , from which we can compute the values of  $r$  and  $\varphi$ . We get :  $\varphi = 32^\circ 27' 15''$  and  $r = 45.465^+$ .

We may check the experimental data with these values. The results are shown in this table :

Angle	Calculated	Measured
$5^\circ$	3.250	3.248
$10^\circ$	6.291	6.292
$15^\circ$	9.098	9.100
$20^\circ$	11.650	11.649

Other series gave similar results.

#### 7. *The position of the spider-thread A in relation to the eye.*

In the first part of this paper we explained the necessity of separating the determination of the successive positions of the point  $A$  from the investigation we shall now deal with. This separation is necessary, because of the difference in the obtainable precision of measurement.

Nevertheless the spiderthread is only an intermediary in the investigation of the motion of the eye itself, and so its location draws its interest chiefly from our knowledge of its position with relation to the eye.

As we have already said, the telescope  $K$  contains a reticle in the plane of the image, formed by the objective. The reticle may be shifted in this vertical plane, the movements being performed by means of four small screws which perforate the metal mantle of the telescope. The vertical thread of the reticle enables adjustment of the telescope with regard to the anterior surface of the subject's cornea. We had a reticle constructed which contains

a double set of spider-threads, vertical and horizontal. It could be firmly fitted upon the apparatus, and in this fixation the position of the vertical set of threads might be adjusted in two vertical planes, the one perpendicular to the other. Besides, the double reticle could be focussed in the telescope *K*, so that the spider-threads of the latter were just centred between the images of the corresponding sets of the former. For the further adjustment of the reticle of the telescope it was mounted so that it could be rotated around the mantle-axis. Now the position of the screws of the reticle was changed until a rotation of the telescope around its axis over  $180^\circ$  gave no change in the relative positions of the images of the single and of the double reticle, the threads of the former remaining centred between the threads of the latter. When this condition had been obtained, we could assume that the line which joined the point of intersection of the threads in *K* with that point of the object, of which the objective forms an image in the point of intersection, is parallel to the mantle-axis of *K*.

In its definitive fixation the telescope *K* could be revolved around a vertical axis. It was adjusted so as to have its mantle-axis parallel to the direction of the transverse screw of the smaller cross-support. This was performed in a manner similar to the adjustments of direction, already described. In the zero-position of the apparatus the above-mentioned line now makes an angle of  $2'$  with the positive *Y*-axis, as the transverse screw does so, the deviation taking place on the side of the negative *X*-axis. However, the measurement of location, now to be described, is not so exact that this deviation is of great influence.

Figure 5 shows the situation in relation to the right eye. In drawing this figure it is assumed, first, that the fore-most point of the cornea, *P*, on which the telescope *K* is focussed with the point of intersection, *S*, of its threads, lies in a horizontal plane with the visual line *Kr'*—*Kr*—*W*; secondly, that the line *PS* is perpendicular to the visual line; and thirdly, that the visual line passes through the anterior surface of the cornea on the nasal side of the foremost point. We already know that the second assumption does not hold. The correction here to be made, is, that the real foremost point *P* is situated about 0.005 mm. more to the right, and less than 0.001 mm. before the point drawn. In computing this I assumed the curvature to be spherical here <sup>1)</sup>, and the radius of the curvature to be about 8 mm.; the transverse displacement is  $8 \times \sin 2'$ .

The third assumption of course is not binding, if it follows from the data obtained that it does not hold in a certain subject.

As for the first assumption, in general, the point *P* and the visual line are not really in the same horizontal plane. By means of the vertical little bar, already mentioned, which can be moved in a vertical direction, the telescope *K* was adjusted so as to have the point of intersection of the threads in a horizontal plane with the visual line, i.e., the centres of the crosses *Kr*

<sup>1)</sup> See: HELMHOLTZ' Treatise (GULLSTRAND), Engl. translation, Vol. I, p. 314.

and  $Kr'$ . This adjustment was made by means of the telescope  $K'$  and the double reticle. The former was first focussed on the centre of the cross  $Kr$ ; then the double reticle was centred to  $K'$ ; in focussing  $K$  on the centre of the reticle the desired position was obtained. Now, in the inverse image of the subject's cornea its foremost point was situated higher than the point of intersection of the threads. So the point  $P$  lay below the point  $W$ . As, however, the distance in the vertical direction is less than 0.1 mm., the antero-posterior position is not influenced in the 0.001 mm. place.

The transverse distance  $PQ$ , and the antero-posterior distance  $QA$ , from  $A$  to the point  $P$ , which is focussed by the telescope  $K$ , are measured in the same way as  $m$  and  $n$  in the determination of  $r$  and  $\varphi$  by means of the larger screws of the apparatus. The double reticle is fixed upon the apparatus and adjusted so as to be in a verticle position in two places, one perpendicular to the other, by means of the vertical threads of the telescopes  $K$  and  $K'$ . The telescope  $K$  remains in a position in which the double reticle is focussed; the latter represents the point  $P$ . Of course, it is not possible, in general, to have the telescope  $K$  in exactly the same position as it occupied in the experiment. But as the positions of the smaller screws are noted, corrections may easily be made. Now the double reticle and the thread  $A$  are successively focussed by the telescope  $K'$  <sup>1)</sup>, the displacements being made and measured by means of the larger screws.  $PQ$  may be measured quite as exactly as  $m$  and  $n$ . The measurement of  $QA$ , however, is not so exact, as it is not possible to adjust the telescope so precisely in the antero-posterior direction; slight errors are made in the 0.01 mm. place.

The measurement of  $AR$  was performed in the same manner. As the antero-posterior screw is not long enough to cover the whole distance  $W-Kr$ , the measurement is made in another position of the telescope  $K'$ . It is not possible to focus  $Kr$  with the same exactness as a thread; slight errors are made in the hundredths of millimeters.

#### 8. *The errors of the observations and the judgment of the results.*

As we have already pointed out, in the experiments the sighting-instrument is rotated at each turn over a certain angle, by means of the theodolite; then the apparatus is adjusted in such a manner by means of the screws that, first, the subject sights again, and, secondly the observer sees the anterior surface of the subject's cornea in the telescope  $K$ , touching the vertical thread. So the results contain two kinds of errors.

First, in each chosen position the apparatus is supposed to meet the demands which we formulated in the first part of this paper. The real position, however, differs from the ideal one in which the situation of the spider-thread  $A$  in relation to the eye would always be exactly the same; on the one hand, the subject makes an error in sighting, and consequently

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<sup>1)</sup> The anterior vertical thread of the double reticle with regard to  $K'$  is focussed; the distance of the two threads might be measured by means of  $K$ .

the position of the visual line is not settled exactly ; on the other hand, the observer makes an error in adjusting the telescope, and so the spider-thread is not adjusted exactly. In order to settle a location, adjustments were always made several times, and the most probable value was chosen. It appeared that, in looking for the "sighting-position", the transverse screw was fixed within a range of about 0.04 mm. We can easily compute the exactness of sighting, which corresponds to this value, if we consider the distances of the crosses  $Kr$  and  $Kr'$  from the eye (p. 10). The angle is about one half of a minute, and I suppose that one cannot expect greater precision, if one takes into consideration the fact that utmost accuracy was not attempted, in order to avoid fatigue of the subject, and, more particularly, to obtain a relatively large number of points <sup>1)</sup>. The cornea of the subject gives, under suitable illumination, a distinct image in the telescope  $K$  ; under certain illuminations it is possible to distinguish between the anterior surface of the cornea and the thin fluid layer resting on it. This illumination, however, was not used, as it was not suitable for the adjustment with regard to the thread. The possible error of the adjustment is about 0.01 mm. ; it is principally due to the fact that the eye is not completely motionless during the entire time necessary for adjusting. I will not now discuss in detail the causes which may underlie the explanation of this fact, because I did not make special investigations along this line. It may be sufficient only to mention the forward movement of the eyeball as the slit between the eyelids grows wider, which was first described by J. J. MÜLLER <sup>2)</sup>, and the opposite movement during the shutting of the eyelids, first described by DONDERS <sup>3)</sup>. During the adjustment the eye was always normally open ; small changes in the position of the eyelids, however, might change somewhat the antero-posterior position of the eyeball. Although I did not observe a regular pulsation, there may have been some influence of heart-beat and respiration <sup>4)</sup>. We may further consider the continuous small movements which the eye makes during fixation, as described by several authors.

The sighting-apparatus is somewhat farther from the eye at the end of the experiment, than it was at the beginning. The difference is small, however, and it is not very likely that the corresponding difference in the state of accommodation, which amounts to about 0.2  $D$  (calculated for the cross  $Kr'$ , which is accommodated for), will influence the contraction of the external eye-muscles at all <sup>5)</sup>.

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<sup>1)</sup> The precision usually stated for sighting is not much greater. See for instance : *Handbuch der Vermessungskunde* von Weil. Dr. W. JORDAN, fortgesetzt von Weil. Dr. C. REINHERTZ, II Band, 7 Aufl., bearb. von Dr. O. EGGERT, Stuttgart 1908, S. 187.

<sup>2)</sup> J. J. MÜLLER, *Graefe's Archiv*, 14 Band, 3 Abt., 1868, S. 205.

<sup>3)</sup> F. C. DONDERS, *Graefe's Arch.*, 17 Band, 1 Abt., 1871, S. 99.

<sup>4)</sup> A. TUYL, *Graefe's Arch.*, 52 Band, 1901, S. 245 ff.,

<sup>5)</sup> About the influence of accommodation, see F. C. DONDERS l.c. p. 100, A. TUYL l.c. p. 252.

Secondly, the data obtained from the theodolite and from the screws contain errors. We have already dealt with the latter. They proved to be 0.002 mm. at most (p. 18). The theodolite gives the minutes exactly. The utmost error may be evaluated at 10".

The result of a series of observations is the corresponding series of points which represent the transformed curve of the eye, computed by means of the formulae (11), (12), (13) and (14). Assuming for the moment that each of these points is exact, and that, in going from one situation to the next one, the eye has rotated around one centre-point or axis, we can, for every turn, compute these corresponding centres of rotation by means of the formulae (1) and (2) which I have derived in my former paper. On the other hand, we may assume that perhaps none of the points are exact and that the real transformed curve of the eye would be the best smooth line, drawn through all those points. We might try to set up a formula for this smooth curve or for a part of it and then we should be able to compute the coordinates of corresponding centres of rotation by means of the formulae (3) and (4) of the former paper.

This is, in fact, the way in which such data are usually treated. However, this treatment is only likely to be correct, if it can be proved that the differences between the experimental data and the coordinates of the corresponding points of the smooth curve are not greater than can be explained by means of the errors which the experimentation could possibly involve. It seems to me, that this condition has not always been satisfied, or may even have been overlooked entirely.

The two kinds of errors mentioned — those which are included in the observations of subject and observer and those which are caused by the inaccuracy of the apparatus — influence the result in quite different ways.

Let us suppose for a moment that the apparatus is quite exact, so that we have only to deal with the first category of errors. At any turn the position of the apparatus is brought into accord with and controlled by the new position of the subject's eye. The utmost difference that is possible between the coordinates of a certain experimental point and the coordinates of the corresponding point of the smooth curve is the sum of the utmost possible errors in the observations of subject and observer. If the difference surpasses this sum, we must admit that the deviation of the point is partly, at least, "real" and that the smooth curve does not reproduce the real movement. Although in the formulae (13) and (14) the values of the coordinates of each point are computed by adding the coordinates of the former point to the recent shiftings, the errors in the latter do not influence the succeeding data, because each position is checked again on the eye. An erroneous position will only influence the shiftings necessary to attain the next position.

The other category of errors influence the result in a more complicated manner. In drawing a graph which represents the data, it is supposed that each shifting of *A* is exactly expressed in the changes of the coordinates,

so that each point corresponds exactly to a position of the apparatus. As this is not true, the erroneous notation of each point influences the notations of the following points. Therefore, the shape of the whole curve may change in a quite incalculable manner, and we are not sure that even the smooth curve whose points differ less from the corresponding data than the amount of the sum of the possible errors of the observations of subject and observer, reproduces the real movement. On the other hand, it is not likely that all errors will have the same direction, and so errors may be corrected to a certain extent by those of the next readings. In any case, it is necessary that the errors of the readings on the apparatus should be much smaller than those of the observations. Even then the evaluation of the real situation may be rather impossible, especially when the function of the smooth curve is a complicated one. If we change the situation of one point on the assumption that its coordinates involve a certain error, all the following points should be changed in accordance with this simple assumption, and it is likely that the smooth curve which we shall now have to draw, will not be the same as the former one ; moreover, as we assume that the readings of the theodolite were not exact, we have to compare the corrected experimental points with points of the smooth curve which correspond to different angles.

Fortunately, the function of the smooth curve which we can draw between the points of each series of our experimental data, proves to be a simple one, as this curve is a circle. Moreover, coordinates of the centre-point of the circle are the averages of the coordinates of the (almost) "instantaneous" centres of rotation which are calculated from the experimental data, from point to point, by means of the formulae (1) and (2). We shall now deal with these data, and we shall see that the circle may not describe the real motion, even if we assume, that the utmost errors were made in every case, which is very unlikely.

### *Results and Conclusions.*

Definitive experiments were made with one subject only, male, age 24. V. O. D. =  $\frac{6}{4} +$ ; V. O. S. =  $\frac{6}{4}$ ; V. O. D. S. =  $\frac{6}{4} +$ , with  $+ 0.5$  sph. =  $\frac{6}{4} +$ ; slight exophoria in seeing close up; orthophoria in the vertical direction. The right eye was used; it was otherwise normal. Several series of experiments were first made with the eye turning to the right; then they were repeated with the eye turning to the left. The apparatus does not occupy the same position in the cage in both directions. The range, both to the right and to the left, was extended to thirty degrees; the length of the screws was not sufficient to cover the whole range of 60 degrees. In order to permit good continuity between the series in both directions, the sighting-apparatus was adjusted so as to have the same direction at the zero-position for both of them. This was made possible by the installation which we have already described in page 14. At the beginning of the experiment the theodolite was always put in a certain zero-position, in which the sighting-apparatus was parallel to the antero-posterior larger screw. It

could be checked by means of the distant telescope. After the investigation of the series turning to the right, the apparatus was replaced in the zero-position, and checked. It was now easy to remove it to its new position for the investigation of the series turning to the left and to reset it so that in the telescope, the image of  $Kr'$  was again in the centre of the image of  $Kr$ . Moreover, the direction of the sighting-apparatus with regard to the antero-posterior screw could be easily checked.

The data obtained in some of the most reliable series are given in Tables I and II. The symbols have the same meaning as has been adopted in the rest of the paper. Angles turning to the right are taken as positive and those which turn to the left as negative. We have already dealt with the values of  $r_0$  and  $\varphi_0$ , when explaining the measurement of  $r$  and  $\varphi$ . As the screws of the smaller support were replaced in the same position at the beginning of each series, those values were the same in both cases, too.

Figure 7 is a graph constructed by using the values of  $b'$ ,  $c'$ ,  $\xi'$ , and  $\eta'$  of both series together. The intersection of the  $X$ - and  $Y$ -axes is made to fall on  $A_0'$ , i.e., the zero-position of  $A$ . Now  $-10, -9, \dots -1, -A_0', 1, \dots 9, 10$  is the transformed curve of the cornea for the positions from  $-30^\circ$  to  $+30^\circ$ ;  $-10, -9, \dots -1, 0, 1 \dots 8, 9$  represent the corresponding centres of rotation.

TABLE I (lengths in mm)

										RIGHT
$\alpha$	$\sigma$	$\bar{x}$	$\varphi$	$r$	$\bar{x}$	$\bar{y}$	$b'$	$c'$	$\xi'$	$\eta'$
0°							0	0		
3°	0		32°27'25	45.465+	-0.075	6.327	1.140	4.286	82.407	-19.624 <sup>5</sup>
6°	0.336		29°27'25	45.465+	0	6.190	2.581	8.338	79.230 <sup>5</sup>	-21.202
9°	0.525		26°13'7	45.749	0	6.446	4.092	12.526	83.303 <sup>5</sup>	-18.419
12°	1.496		22°52'9	46.193	0	6.393	6.428	16.356	78.389 <sup>5</sup>	-30.166
15°	1.736		18°55'6	47.471	0	6.527	8.840	20.062	78.396 <sup>5</sup>	-27.846
18°	1.878		14°53'	48.969	0	6.880	11.209	23.868	82.696 <sup>5</sup>	-23.269
21°	2.352		10°49'4	50.607	0	7.079	13.824	27.490	81.676 <sup>5</sup>	-24.252
24°	2.844		6°35'4	52.678	0	7.486	16.657	31.072	83.636 <sup>5</sup>	-24.812
27°	3.189		2°13'4	55.215	0	7.506	19.525	34.240	78.581	-22.106 <sup>5</sup>
30°	4.074		-2°10'	58.092 <sup>5</sup>	0	8.292	22.847	37.460	82.670	-27.581
Average ( $M_r$ ):										81.099
										-23.928

Distances between  $M_r$  and the points ( $b'$ ,  $c'$ ) successively:

84.556; 84.791; 84.887; 85.200; 84.846; 84.596; 84.671; 84.674; 84.721; 84.705; 84.627.

Radius of the right circle = average = 84.752.

TABLE II (lengths in mm)

LEFT

$\alpha$	$\theta$	$\bar{x}$	$\varphi$	$r$	$\bar{x}$	$\bar{y}$	$b'$	$c'$	$\xi'$	$\eta'$
0°	—3°	0.067	32°27'25	45.465+	0	—6.162	0	0	79.263	—26.026 <sup>5</sup>
—3°	—3°	0.610	35°24'5	45.523	0	—6.334	—1.253 <sup>5</sup>	—4.184	81.665	—21.928
—6°	—3°	0.616	38°0'1	46.038	0	—6.266	—2.068 <sup>5</sup>	—8.548	79.770	—28.183 <sup>5</sup>
—9°	—3°	1.126	40°36'	46.562	0	—6.456	—2.984	—12.858	81.017	—25.070 <sup>5</sup>
—12°	—3°	1.790	42°53'3	47.524	0	—6.643	—3.508	—17.271	80.543	—19.510 <sup>5</sup>
—15°	—3°	1.941	44°49'	49.068	0	—6.942	—3.510	—21.673	83.711	—23.985 <sup>5</sup>
—18°	—3°	2.825	46°43'6	50.762	0	—7.061	—3.511 <sup>5</sup>	—26.241	78.496	—15.672 <sup>5</sup>
—21°	—3°	2.164	48°15'9	53.257 <sup>5</sup>	0	—7.294	—2.846	—30.518 <sup>5</sup>	85.155	—35.507 <sup>5</sup>
—24°	—3°	0.558	50°14'2	55.189	2.483	—6.372	—2.986 <sup>5</sup>	—35.131	80.052	—23.538 <sup>5</sup>
—27°	—3°	—2.685	52°58'9	55.692 <sup>5</sup>	6.003	—4.520	—2.266	—39.461	77.984	—16.513 <sup>5</sup>
—30°							—0.954 <sup>5</sup>	—43.629 <sup>5</sup>		

Average ( $M_l$ ): 80.766 —23.594<sup>5</sup>Distances between  $M_l$  and the points ( $b'$ ,  $c'$ ) successively:84.142; 84.283; 84.190; 84.434; 84.510; 84.290; 84.318<sup>5</sup>; 83.899<sup>5</sup>; 84.544; 84.535<sup>5</sup>; 84.141.

Radius of the left circle = average = 84.299.

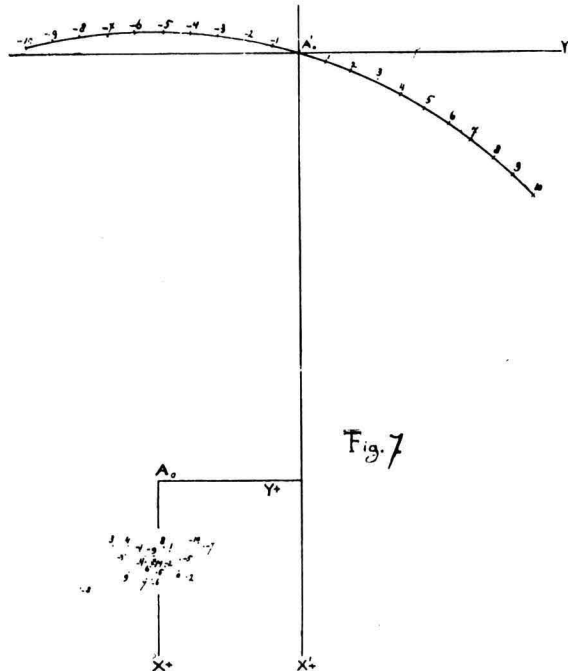
Distance between  $M_r$  and  $M_l$ : 0.472.

Distances between  $M$  (80.932; —23.761) and ( $b'_{-10}$ ,  $c'_{-10}$ ) ... ( $b'_{10}$ ,  $c'_{10}$ ) successively:  
 84.262; 84.666; 84.686; 84.050; 84.482; 84.468; 84.690; 84.622; 84.382; 84.487;  
 84.347; 84.578; 84.672; 84.976; 84.618; 84.366; 84.438; 84.440; 84.487; 84.469;  
 84.390.

Radius of the circle = average = 84.504.

It is obvious that the type of curve which fits the points of Fig. 7 best is the circle. The points  $M_r$  and  $M_l$  are taken as centre-points for the motions to the right side and to the left side respectively; they are plotted near  $M$ , the lower one being  $M_r$ , the upper  $M_l$ . The coordinates of these points are the averages of the corresponding coordinates  $\xi'_k$  and  $\eta'_k$  for the left and for the right side. The radii of the circles are the averages of the corresponding distances from  $M_r$  and from  $M_l$  to the computed points of the transformed curve of the cornea in the motion to the right and to the left side, as indicated in Tables I and II. The existence of the two points might lead to an assumption of slightly different centres of rotation for the right and for the left side, but is more probably to be explained by the fact that the right and the left side of the curve result from two different series of investigations.

In general the points lie very near to the circle, so that, in the graph, most of them cannot be discriminated from it; the arcs run really correctly



enough through the points. Nor is the way, in which the ever-changing centre of rotation moves, a systematic one, as the points seem to be scattered more or less haphazardly. May we conclude that there is really one centre of rotation? The relatively great distances at which some of the points lie from the others do not necessarily prove that there is actually more than one centre, for it is obvious that even small deviations from the circle may scatter the instantaneous centres of rotation considerably; on the other hand, the points  $M_r$  and  $M_l$  lie rather close together.

The choice of these two points, however, is a more or less arbitrary one, for, as  $k$  increases, the values of  $\xi'_k$  and  $\eta'_k$  grow less exact; meanwhile all values of  $\xi'_k$  and  $\eta'_k$  influence the locations of  $M_r$  and  $M_l$  in the same way. In fact we can be sure that the arcs which we have drawn, do *not* represent the real paths of the point  $A$ , as these arcs do not pass through  $A_0'$  or, at least, through a point relatively very close to  $A_0'$ . If we draw arcs with radii  $M_r A_0'$  and  $M_l A_0'$ , the points deviate more systematically from them than they do from the circles we have just described. This will be much clearer, if we do the same in Fig. 8, which represents the motions of a point  $W$  of the cornea itself. If we could draw a circle through  $A_0'$  which should run correctly enough through the points, even this circle would not necessarily represent the real transformed curve of the cornea.

Let us assume a circular movement through  $A_0'$ , and let us suppose that,

because of errors in one or more of the first readings, all the following data are shifted in a certain direction, say downward and to the right. As the movement covers an angle of only 30 degrees on both sides, and as it is likely that the deviations will increase with the increasing value of the angle of rotation, it is perfectly possible, that these erroneous series of points may suggest a circular movement again. These new arcs, however, will have other centres; the centre for the movement on the right side will shift downward and to the left, the centre for the movement on the left side will shift upward and to the right. Consideration of a drawing of two intersecting circles will easily show it.

It is rather impossible to say, which circle most probably describes the real transformed curve of the cornea. However, if there were really only one centre of rotation, it would be likely to be very close to  $M_r$  and  $M_l$ .

We will now examine the location of several of the points of the  $A'$ -curve from the viewpoint we have developed generally when dealing with the errors, and we shall see if this location justifies us in the assumption of one centre of rotation.

Let us consider the first point of the  $A'$  curve in the movement to the right side, after a rotation of  $+3^\circ$ . The coordinates of this point are:  $b' = 1.140$  mm. and  $c' = 4.286$  mm. Now the coordinates of the corresponding point of the circle through  $A_0'$ , whose centre-point is  $M_r$  are  $x' = 1.364$  mm. and  $y' = 4.211$  mm. We take the circle through  $A_0'$ , because we consider the *relative* positions of the first and second points; we take  $M_r$  as a centre-point, because the centre should be at least very near to  $M_r$ . So, assuming for a moment that the coordinates of the right point are this  $x'$  and  $y'$ , we have to find out, if it is possible to explain a shifting of 0.075 mm. to the right side and a shifting of 0.224 mm. "upward" by means of the possible errors. For this computation we may set up the following assumptions.

10. We assume that both the first and the second adjustments of the theodolite have been erroneous; they are supposed to be erroneous in opposite directions, as the theodolite was turned too far; both readings contain the utmost error, i.e.  $10''$  (p. 22). This assumption gives a displacement of 0.002 mm. "downward", i.e. the change of  $r_0 \cos (\varphi_0 - \delta_0)$ , (see p. 8) and of 0.004 mm. to the left side, i.e. the change of  $r_0 \sin (\varphi_0 - \delta_0)$ . In the computation  $r_0 = 45.465$  and  $\varphi_0 = 32^\circ 27' 15''$ , as in Table II. This implies that, if we suppose that we describe the motion of the vertical spider-thread  $A$ , we have to assume that the theodolite was in the same erroneous position at the beginning of the measurement of  $r$  and  $\varphi$ . If this is not so, the values of  $r_0$  and  $\varphi_0$  of the spider-thread are slightly different, and we now describe the motion of another point of the "system" which is composed by the telescope  $K$  and the spider-thread  $A$ . This is, of course, perfectly allowed, as we are only discussing the question if the motion of this "system" is a circular one or not.

20. Then the antero-posterior larger screw was turned; we assume that its reading was erroneous. This gives a displacement of 0.002 mm. "downward".

30. Then the larger transverse screw was adjusted, and the subject sighted. We assume the utmost difference between the sighting in the zero-position and this new sighting; and, besides, the reading of the screw was erroneous. These assumptions together give a shifting of 0.042 mm. to the left side. (p. 13 and p. 21/22.)

40. After the sighting the observer looked into the telescope  $K$  and found that, by chance, it was adjusted. We will now assume, however, that it was not in the same position with regard to the eye as it was when in the zero-position. This assumption gives a shifting of 0.01 mm. "downward" and of 0.001 mm. to the left side.

It is easy to understand the possibility of the assumptions and their effect, if we consider Figure 6.

When we add the values of all those shiftings, we get 0.014 mm. "downward" and 0.047 mm. to the left side. This makes  $b' = 1.154$  mm. and  $c' = 4.239$  mm. We now have to compare this new location with a new point of the circle, which corresponds to  $3^\circ 20''$ . If we take  $M_r$  as a centre-point again, the coordinates are  $x' = 1.366$  mm. and  $y' = 4.219$  mm. So the point  $A$  still lies 0.212 mm. too far "upward" and 0.020 mm. too far to the right. If we assume a rotation around an average centre-point  $M$  whose coordinates are the averages of those of  $M_r$  and  $M_l$ , we get  $x' = 1.357$  mm. and  $y' = 4.211$  mm., so that the differences are 0.203 mm. and 0.028 mm.

We may consider the first point on the other side in the same way. Its coordinates are:  $b' = -1.254$  mm. and  $c' = -4.184$  mm. The coordinates of the corresponding points of the circle of which  $M_l$  is the centre-point are:  $x' = -1.124$  mm. and  $y' = -4.259$  mm. We assume: 10. that the whole angle over which we have rotated the theodolite is  $20''$  smaller than 3 degrees; 20. that there is the utmost difference between the sighting in the zero-position and this new sighting, and that the reading of the screw was erroneous; 30. that the smaller antero-posterior screw was erroneously adjusted by the observer in focussing the eye in the telescope  $K$ , and that the reading on the head of the screw involved the maximal error. All these assumptions together permit us to shift the point  $A$   $0.003 + 0.012 = 0.015$  mm. "downward" and  $0.004 + 0.042 - 0.001 = 0.045$  mm. to the left. Then we get for  $b'$ :  $-1.239$  mm., and for  $c'$ :  $-4.229$  mm. We have to compare this location with a point of the circle which corresponds to  $-2^\circ 59' 40''$ . The coordinates of this point are:  $x' = -1.122$  mm. and  $y' = -4.251$  mm., if we take  $M_l$  as a centre-point again. So the point  $A$  lies 0.117 mm. too far "upward", and 0.022 mm. too far to the right. If we assume a rotation around the average centre-point  $M$ , which we previously mentioned, we get  $x' = -1.131$  and  $y' = -4.260$ , so that the differences are 0.108 mm. and 0.031 mm.

It is unlikely that all the errors accumulate in the way we have assumed. On the other hand, it is obvious that, even with all these assumptions, we cannot explain the differences between the coordinates of the points experimentally found and those of the theoretical points with which they may correspond.

The treatment of the points which lie farther on upon the curve is more complicated because of the accumulation of the errors of the second category (pag. 21) in the course of the computation. However, if we consider the *relative* location of two successive points, the errors which could have been made in the movement from one point to the other would be of primary importance, for the errors which the coordinates of the first point involve enter into those of the second point too. For instance, the coordinates of the point *A* for  $-18^\circ$ , as given in Table II, are :  $b' = -3.512$  and  $c' = -26.241$ . If we assume  $M_1$  as a centre of rotation and if we consider a rotation over  $-3^\circ$ , which starts from this point, we shall obtain for the coordinates of the point *A* for  $-21^\circ$  :  $x' = -3.258$  and  $y' = -30.648$  ; the coordinates of the point *A*, as given in the table, are  $b' = -2.846$  and  $c' = -30.519$ , so that this point would lie 0.412 mm. too far "downward" and 0.129 mm. too far to the right. It is obvious that these deviations cannot be explained by assumptions either about another location of the points and of the centre of rotation, or about errors in the observations and in the readings.

After all, the conclusion to which we have come, is, that the deviations are real, at least to a certain amount ; *that the path of the point A is not a circle, and that, in reality, there is certainly not only one centre of rotation for the whole motion.*

Up to this point the way in which the point *A* oscillates around the circle seems not to be a very systematic one. In other series the points appeared to be scattered in a somewhat different way.

*Practically, we may speak of one centre of rotation,* by which we mean the centre-point of the circle which passes in the most correct way through the points ; its coordinates will be the averages of the coordinates of the instantaneous centres of rotation.

On the other hand, I would state emphatically, that such a description does not take into account what really happens. In fact, the importance of the deviations grows clearer when we consider not only the transformed curve which the thread *A* describes, but also the curves which the points of the eye itself have consequently to describe.

I have already pointed out that, in measuring the position of the spider-thread *A* in relation to the eye, the precision of measurement is smaller than that of the location of *A* itself. In the investigations described in Section 7 (page 18), *AR* (fig. 5) was found to be 22.95 mm. ; the point *A* was found to lie 66.78 mm. in front of the foremost point of the cornea and 21.91 mm. to the right of it. As we explained, the point *P* is not the real foremost point of the cornea, because *PQS* is not exactly perpendicular to

the visual line. The computation took account of this fact. The "practical" centre of the rotation for the left side  $M_l$  lies  $80.766 - 66.78 = 13.99$  mm. behind the foremost point of the cornea and  $23.594 - 22.95 = 0.65$  mm. left to the point of intersection of the cornea and the visual line. For the point  $M_r$  those values are:  $81.099 - 66.78 = 14.32$  mm. and  $23.928 - 22.95 = 0.98$  mm.

Now let us consider the movements of a point of the cornea which lies at a distance  $a$  from the point  $A_0'$ . This point may be the point  $W$  in fig. 5. The  $X$ - and  $Y$ -axes have the same direction as in the former part of the paper; but their point of intersection is made to fall on the zero-position of  $W$ , i.e.  $A_0$  in fig. 7 and in fig. 8. The angle between  $A_0 A_0'$  and the negative  $X$ -axis is called  $\theta$ . We call the coordinates of the point  $W$ :  $b$  and  $c$ ;  $\xi$  and  $\eta$  are the coordinates of the instantaneous centres of rotation. It is easy to relate these coordinates to the corresponding ones in the former system, where the intersection of the axes fell on  $A_0'$ . In fact, we have:

$$\begin{aligned} b &= b' - a \{ \cos \theta - \cos (\theta + a) \} \\ c &= c' - a \{ \sin (\theta + a) - \sin \theta \} \\ \xi &= \xi' - a \cos \theta \\ \eta &= \eta' + a \sin \theta, \end{aligned}$$

if  $a$  is the angle of rotation.

TABLE III (lengths in mm)  $\theta = 18^\circ 57' 9$   
 $a = 70.6088$

RIGHT

$\alpha$	$\sigma$	$b$	$c$	$\xi$	$\eta$
0°	3°	0	0	15.631	3.3235
3°	3°	-0.152	0.822	12.4545	1.745
6°	3°	-0.184	1.484	16.5275	4.528
9°	3°	-0.320	2.362	11.6135	-7.219
12°	3°	0.199	2.974	11.6205	-4.899
15°	3°	0.625	3.561	15.9205	-0.322
18°	3°	0.850	4.356	14.9005	-1.305
21°	3°	1.1655	5.083	16.8605	-1.865
24°	3°	1.550	5.896	11.805	0.8405
27°	3°	1.829	6.425	15.894	-4.634
30°	3°	2.427	7.146		
Average ( $M_r$ ): 14.323					-0.981

Distances between  $M_r$  and the points ( $b, c$ ) successively:

14.36; 14.59; 14.72; 15.02; 14.67; 14.43; 14.49; 14.49; 14.51; 14.52; 14.41.

Radius of the right circle = average = 14.565

Computations have been made for a point in which  $a = 70.609$  mm. and  $\theta = 18^\circ 58'$ . The results are shown in Tables III and IV for the right and

TABLE IV (lengths in mm)  $\theta = 18^\circ 57'9$   
 $a = 70.6088$

LEFT

$\alpha$	$\sigma$	$b$	$c$	$\xi$	$\eta$
$0^\circ$		0	0		
$-3^\circ$	$-3^\circ$	$-0.145^5$	$-0.658$	12.487	$-3.079^5$
$-6^\circ$	$-3^\circ$	$-0.036^5$	$-1.442$	14.889	1.019
$-9^\circ$	$-3^\circ$	$-0.216$	$-2.130$	12.994	$-5.236^5$
$-12^\circ$	$-3^\circ$	$-0.197$	$-2.886$	14.241	$-2.123^5$
$-15^\circ$	$-3^\circ$	0.154	$-3.608$	13.767	$3.436^5$
$-18^\circ$	$-3^\circ$	$0.312^5$	$-4.483$	16.935	$-1.038^5$
$-21^\circ$	$-3^\circ$	0.943	$-5.064$	11.720	$7.274^5$
$-24^\circ$	$-3^\circ$	$0.574^5$	$-5.987$	18.379	$-12.560^5$
$-27^\circ$	$-3^\circ$	0.875	$-6.644$	13.276	$-0.591^5$
$-30^\circ$	$-3^\circ$	$1.573^5$	$-7.167^5$	11.208	$6.433^5$
Average ( $M_l$ ): 13.990					$-0.647$

Distances between  $M_l$  and the points ( $b, c$ ) successively:

14.00<sup>5</sup>; 14.13<sup>5</sup>; 14.05; 14.30; 14.36; 14.15; 14.20<sup>5</sup>; 13.77; 14.44; 14.42; 14.02<sup>5</sup>.

Radius of the left circle = average = 14.17.

Distances between  $M$  (14.156<sup>5</sup>;  $-0.814$ ) and ( $b_{-10}, c_{-10}$ )...( $b_{10}, c_{10}$ ) successively:

14.09; 14.50<sup>5</sup>; 14.53; 13.88; 14.32; 14.28; 14.50; 14.43; 14.24; 14.30; 14.18;  
14.40; 14.52; 14.82; 14.46; 14.22; 14.27<sup>5</sup>; 14.27; 14.28; 14.29<sup>5</sup>; 14.17<sup>5</sup>.

Radius of the circle = average = 14.33.

for the left side respectively. The results are plotted in Fig. 8, where  $-10, -9, \dots, A_0, \dots, 9, 10$  is the curve of the point  $W$ , and where  $-10, -9, \dots, 0, \dots, 8, 9$  represent the corresponding centres of rotation. The arcs which have been drawn through the points have centre-points  $M_r$  and  $M_l$ , whose coordinates are the averages of the instantaneous centres. It is obvious that the path of the point  $W$  is less "circular" still, and the deviations are more marked than in fig. 7. We know from our former considerations, that these deviations are, to a great extent, real and so also is the scattered location of the instantaneous centres of rotation. On the other hand, the circle passes rather correctly through the points; we may say that there is "practically" one centre of rotation, if we do not forget that the real motion is a much more irregular one.

Further investigations may disclose whether the amount of these

deviations is different in different individuals or even different in one individual in different circumstances. They may also elucidate the causes of the deviations.

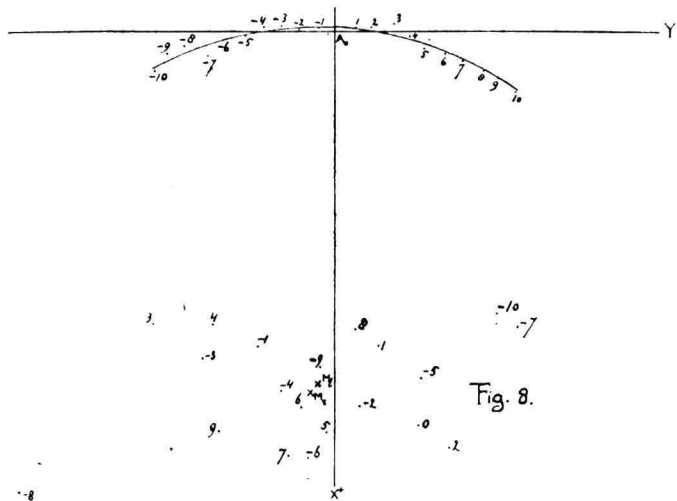


Fig. 8.

I have still some remarks to make about a publication which came out since I published my former paper <sup>1)</sup>.

H. HARTINGER used an apparatus like KOSTER's, and he measured, in a more accurate manner, the shiftings along the line of sight. On the assumption of a circular movement, centres of rotation were computed for rotations over  $0^\circ - 10^\circ$ ,  $0^\circ - 20^\circ$  and  $0^\circ - 30^\circ$  to the right and to the left. These centres of rotation lie rather close together. In one case they lie within a circle with a radius of 0.5 mm. It is obvious that such a treatment must secure a similar set of points because their location is less influenced by the instantaneous deviations of the circle. If I treat my data in a similar way, the centres of rotation come closer together too :

Interval		$\xi'$	$\eta'$	$\xi$	$\eta$
$0^\circ$	$-30^\circ$	80.936	-23.596	14.160	-0.649
$0^\circ$	$-15^\circ$	80.557	-24.168	13.781	-1.221
$0^\circ$	$+15^\circ$	80.613	-23.542	13.837	-0.595
$0^\circ$	$+30^\circ$	81.326	-23.904	14.550	-0.957

Such a treatment, however, does not give an analysis of what really may happen.

<sup>1)</sup> H. HARTINGER (Jena), Zur Bestimmung des Augendrehpunkts, Berichte ueber die 47 Zusammenkunft der deutschen ophthalmologischen Gesellschaft in Heidelberg, 1928.