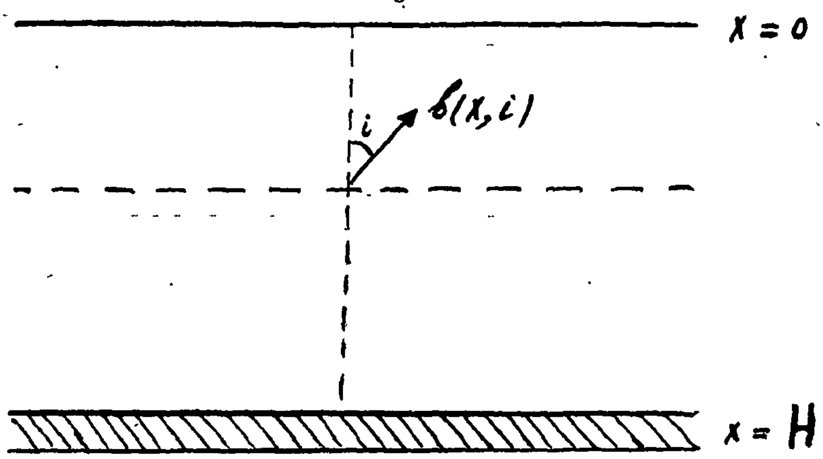
Physics. — *"On the Effective Temperature of the Sun"*. Some remarks in connection with an article by DEFANT: *"Diffusion und Absorption in der Sonnenatmosphäre."* By H. GROOT. (Communicated by Prof. W. H. JULIUS).

(Communicated in the meeting of March 29, 1919).

In a paper "Ueber Diffusion und Absorption in der Sonnenatmosphäre" (Sitz. Ber. d. Berl. Akad. 1914) SCHWARZSCHILD treated the problem of the radiation of a plain layer, which must be imagined as an absolutely black body, and above which there is an absorbing and dispersive atmosphere. When on a layer (see figure 1) bounded by the planes $x = 0$ and $x = H$ radiation of intensity S, starting from the black body ZZ' falls from all directions, SCHWARZSCHILD denotes by $b(x,i)$ the radiation which passes through the plane x in the same sense as the radiation S, and at an angle i with the normal to the boundary layers, and then tries to find a formula for $b(0,i)$.

Fig. 1.



Black body.

Accordingly $b(0,i)$ is the total intensity of light that passes out at an angle i at the boundary of the atmosphere, and is built up of direct light and light that is dispersed once, twice etc.

SCHWARZSCHILD succeeds in solving this problem for two special cases, and finds:

a. Limiting case of exclusive absorption ($s = 0$):

$$b(0,i) = a + \frac{b \cos i}{k}(1 - e^{-kH \sec i}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

b. Limiting case of exclusive dispersion ($k = 0$):

$$b(0,i) = \frac{0.5 + \cos i}{1+\sigma H} + \frac{0.5 - \cos i}{1+\sigma H} e^{-\sigma H \sec i} \quad . \quad . \quad . \quad . \quad (2)$$

Here k = coefficient of absorption, σ = coeff. of diffusion, H = height of the asmosphere, a and b are two numerical constants.

In his article: "Diffusion und Absorption in der Sonnenatmosphäre" (Sitz. Ber. d. K. Akad. zu Wien, Abh. IIa. Bnd. 125 (1914)). A. DEFANT by the aid of data which he derives from ABBOT's observations on the decrease of the intensity of radiation on the sun's disc from the centre towards the limb (Annals of the Astr. Observ. of Smithsonian Inst. Vol. III, Washington 1913, p. 158), tries to decide which of the two causes, absorption or dispersion, appears to be most active on the sun.

By means of a kind of "trial and error" method he succeeds in deriving a formula:

$$b(0,i) = \frac{0.5 + \cos i + e^{-0.0405\,\lambda^{-4} \sec i}(0.5 - \cos i) - 0.3804 + 0.3136 \cos i}{1 + 0.0405\,\lambda^{-4}} \quad . \quad (3)$$

which is halfway between (1) and (2) and yields numerically accurate values. This seems to point to this that the diffusion effect by far preponderates, but is yet influenced by a slight absorption.

In how far the considerations through which he arrives at formula (3), are of value, must be left undecided here. It is certain that the numerical values are pretty accurate, as table I shows convincingly.

TABLE I.

$\cos i$	$\lambda = 0.433\,\mu$			$\lambda = 0.604\,\mu$			$\lambda = 1.031\,\mu$		
	$b(0,i)$	$b(0,i) \times 355$	Observed value	$b(0.i)$	$b(0,i) \times 375$	Observed value	$b(0.i)$	$b(0\,i) \times 117$	Observed value
1.0	1.2752	453	456	1.0643	399	399	0.9486	111	111
0.9	1.1906	423	419	1.0164	381	380	0.9175	107	107
0.8	1.0996	390	384	0.9656	361	360	0.8838	103	105
0.7	1.0006	355	348	0.9097	341	337	0.8491	99.4	100
0.6	0.8932	317	309	0.8476	318	313	0.8137	95.2	95.8
0.5	0.7764	276	277	0.7764	291	288	0.7765	90.9	90.0
0.4	0.6506	231	238	0.6917	259	265	0.7366	86.2	86.2
0.3	0.5180	184	192	0.5863	220	230	0.6912	80.9	80.9

Explanation of table I:
In this table $b(0,i)$ calculated for the values of $\cos i$ is given in the first column for three different values of λ by the aid of (3). In column 3 the found values of $b(0,i)$ have been multiplied by a factor in order to render a comparison with ABBOT's values, recorded in the fourth column, possible.

By the aid of (3) and ABBOT's values, which I subjoin, DEFANT tries to draw a conclusion on the effective temperature of the sun.

Wavelength in μ	Radiation in the centre of the sun's disc.
0.323	144
0.386	338
0.433	456
0.456	515
0.481	511
0.501	489
0.534	463
0.604	399
0.670	333
0.699	307
0.866	174
1.031	111
1.225	77.6
1.655	39.5
2.097	14.0

(ABBOT's values).

His reasoning is as follows:

For $i = 0$ we obtain $b(0,0)$ i.e. formula (3) then gives for every wavelength λ the intensity of radiation passing out in the centre of the sun's disc, when that of the area of the photosphere for this λ is put equal to 1. What we measure is, however, not the quantity $b(0,0)$, but the radiation i_λ, actually passing out, which is in relation with $b(0,0)$ through the formula:

$$I_\lambda = \frac{i_\lambda}{b(0,0)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

in which I_λ is the intensity of radiation in the spectrum of the photosphere (considered as absolutely black body) for the wavelength λ.

By the aid of (3) and (4) and ABBOT's values the following table can, therefore, be calculated for I_λ (table II). According to our

supposition the photosphere radiates as an absolutely black bo(
that PLANCK's formula may be applied, according to which[1]:

$$I_\lambda = \frac{7.211\times10^{8}}{\lambda^{5}\left(10^{\frac{2\,1563\times2890}{\lambda T}}-1\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

The quantities I_λ from the table are expressed in an unknown ı
When we consider this unity and T as unknown quantities,
T can be solved from two values of I_λ (for λ_1 and λ_2 e.g.).

If our basis is correct, we must find the same temperature all combinations in pairs of I_λ.

DEFANT calculates T from the combinations

$$\left.\begin{array}{ll}\lambda_1 = 0.5 & I_\lambda = 700 \\ \lambda_2 = 0.9 & I_\lambda = 180\end{array}\right\} T = 8900^\circ$$

and

$$\left.\begin{array}{ll}\lambda_1 = 0.6 & I_\lambda = 350 \\ \lambda_2 = 1.2 & I_\lambda = 90\end{array}\right\} T = 8700^\circ$$

and considers the agreement "genügend". (loc. cit. p. 517).

TABLE II.

Wavelength λ	$1 + 0.0405\,\lambda^{-4}$	$b\,(0,0)$	i_λ	I_λ
0.323	4.721	0.299	144	481.8
0.386	2.824	0.479	338	705.7
0.433	2.152	0.593	456	769.5
0.456	1.937	0.644	515	799.8
0.481	1.757	0.682	511	748.9
0.501	1.646	0.710	489	688.7
0.534	1.498	0.737	463	628.1
0.604	1.304	0.816	399	500.2
0.670	1.201	0.853	333	390.5
0.699	1.170	0.864	307	355.2
0.866	1.072	0.903	174	192.8
1.031	1.036	0.918	111	120.9
1.225	1.018	0.926	77 6	83.8
1.655	1.005_4	0.931	39.5	42.4
2.097	1.002_1	0.932	14.0	15.0

[1]) The constants are those used by DEFANT.

Unfortunately, however, a fatal error has slipped in. For to $\lambda = 0.6$ does not correspond $I_\lambda = 350$, but — (interpolating graphically) — $I_\lambda = 506$, which yields $T = 6600°$ instead of 8700°, so that agreement is out of the question.

A serious objection to the whole method seems perfectly obvious to me, namely this:

The assumption that all kinds of light come to us from one photospheric surface, in other words that light of various wavelengths should come from the same depth of the sun, appears more and more untenable in the light of recent researches (see e.g. the thesis for the doctorate of J. SPIJKERBOER "Verstrooiing van licht en intensiteitsverdeeling over de zonneschijf" (1917) (Dispersion of light and Distribution of Intensity over the Sun's Disc)). If, however, in reality light of different wavelengths originates from different parts of the sun, it becomes very questionable whether we shall be allowed to apply PLANCK's formula, as we saw DEFANT do. For this would mean that we supposed every kind of light to have, as it were, a kind of "photosphere of its own", which radiates as a black body, the photosphere for the greater wavelengths lying deeper than that for the smaller. It might then be expected that the temperature determined with PLANCK's formula, becomes a function of λ, i. e. *T would be the greater as λ increases.*

In this latter remark we have a means to investigate whether the hypothesis that the photospheres overlap each other like scales can find a semblance of justification.

By graphical interpolation from the values of table II I construed table III:

TABLE III.

λ	I_λ	λ	I_λ	λ	I_λ	λ	I_λ
0.40	709	0.70	342	1.00	134	1.60	46
0.45	791	0.75	284	1.10	104	1.70	39
0.50	714	0.80	239	1.20	87	1.80	32
0.55	604	0.85	201	1.30	74	1.90	25
0.60	506	0.90	174	1.40	64	2.00	19
0.65	418	0.95	153	1.50	55		

As we do not know the unity in which I_λ is expressed, we require, as was already remarked before, always two values of I_λ (a_1 and a_2) to find T.

The calculation comes to this:

Let A be $= 7.210 \times 10^8$, $\beta = 2.1563 \times 2890$, a_1 and a_2 the values of I_λ corresponding to λ_1 and λ_2, f an unknown factor dependent on the unities in which I_λ has been measured Then the following equations hold:

$$a_1 f = \frac{A}{\lambda_1^5\left(10^{\frac{\beta}{\lambda_1 T}} - 1\right)} \quad \text{and} \quad a_2 f = \frac{A}{\lambda_2^5\left(10^{\frac{\beta}{\lambda_2 T}} - 1\right)} \quad . \ . \ . \quad (6)$$

When we choose the values of λ so that $\lambda_2 = 2\lambda_1$, and when we put $10^{\frac{\beta}{\lambda_2 T}} = x$, we easily get:

$$\left(\frac{a_2}{a_1}\right)\left(\frac{\lambda_1}{\lambda_2}\right)^5 (x^2 - 1) = x + 1 \quad . \ . \ . \ . \ . \ . \ . \quad (7)$$

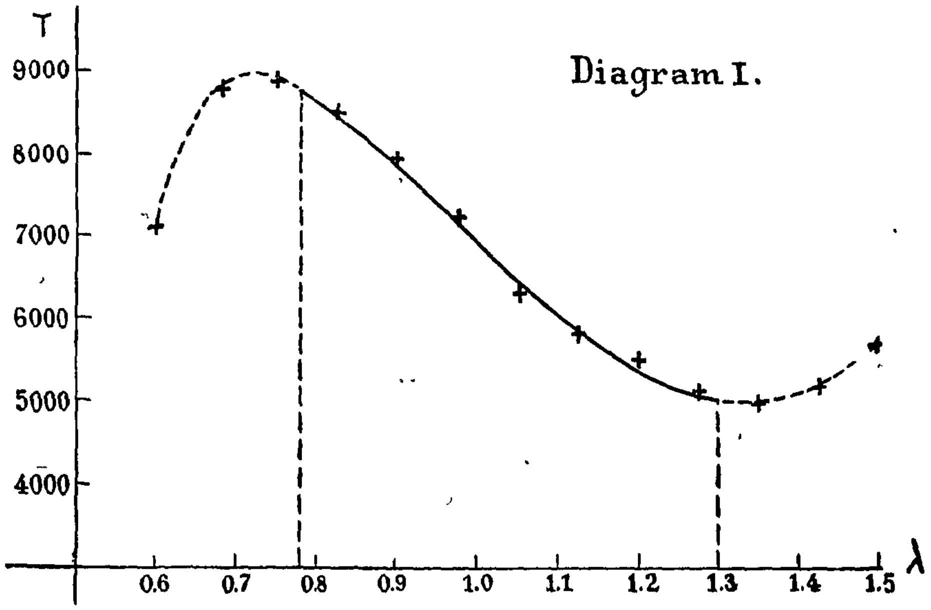
The root $x = 1$ yields $T = \infty$, has therefore no physical meaning, so that we find T from:

$$\left.\begin{aligned} x &= 32\left(\frac{a_2}{a_1}\right) - 1 \\ T &= \frac{\beta}{\lambda_2 \, lg \, x} \end{aligned}\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

In this way I found:

$\lambda_2 =$ 0.8	0.9	1.0	1.1	1.2	1 3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
T = 7200	8800	8900	8500	7900	7200	6300	5800	5500	5100	5000	5200	5700

hence on an average *a decrease of T with increase of λ* (see diagram I).



. Nor is this manner entirely satisfactory; for now we do not know to what λ the found T should properly speaking belong, because the two values of λ (λ_1 and λ_2), which are required, can lie pretty far apart in this way of calculation. Does for ($\lambda_1 = 0.9$, $\lambda_2 = 1.8$) T e. g. belong to λ_1, to λ_2, or to a value lying somewhere between λ_1 and λ_2?

When we want to avoid this difficulty, we may treat the equations (6) as follows:

Let $\lambda_1 : \lambda_2$ be $= n : m$ or

$$\left.\begin{aligned} \lambda_1 &= \frac{\varrho}{m} \\ \lambda_2 &= \frac{\varrho}{n} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (9)$$

we find easily:

$$10^{\frac{m\beta}{\rho T}} - 1 = \left(\frac{a_2}{a_1}\right)\left(\frac{m}{n}\right)^5\left(10^{\frac{n\beta}{\rho T}} - 1\right) \quad \ldots\ldots \quad (10)$$

Put:

$$10^{\frac{\beta}{\rho T}} = z \qquad \left(\frac{a_2}{a_1}\right)\left(\frac{m}{n}\right)^5 = C \quad \ldots\ldots \quad (11)$$

then (10) passes into:

$$z^m - Cz^n + (C-1) = 0 \quad \ldots\ldots \quad (12)$$

When we take care that m is $= n + 1$, the shape becomes somewhat more suitable for numerical approximation, namely:

$$z^n(z - C) + (C-1) = 0 \quad \ldots\ldots \quad (12a)$$

When z has been sufficiently closely approximated, T follows from:

$$T = \frac{\beta}{\varrho \lg z} \quad \ldots\ldots\ldots\ldots \quad (13)$$

In this way λ_1 and λ_2 can be brought close enough together to exclude indefiniteness in the choice of the λ to which T belongs.

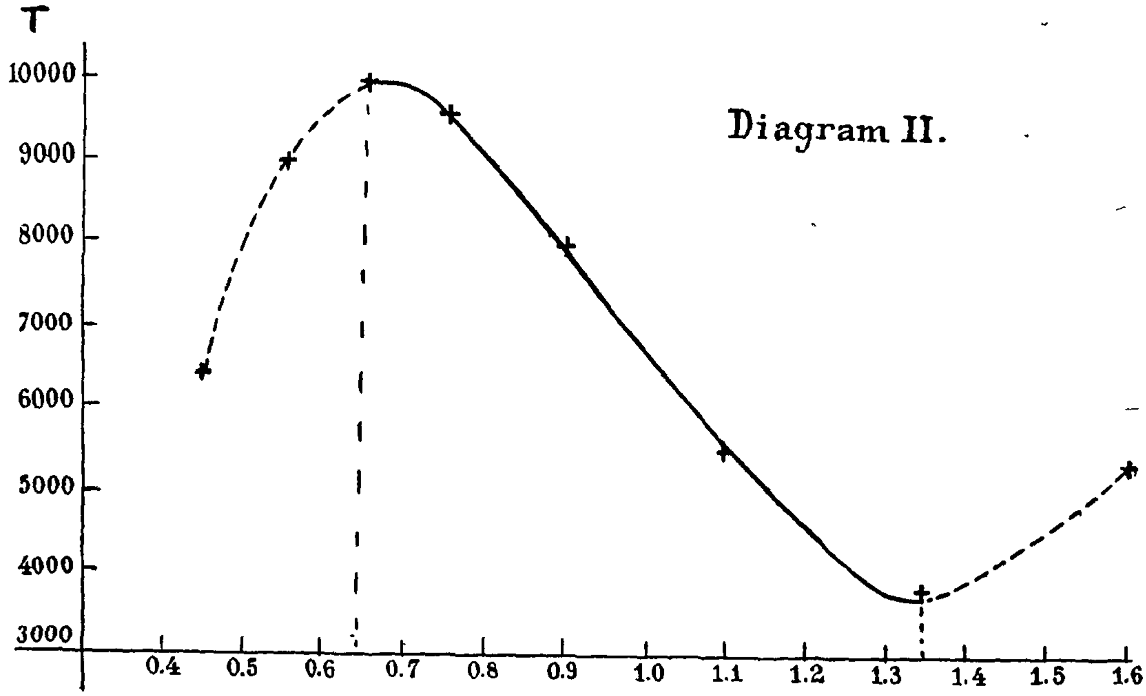
Thus we found:

λ_1	0.4	0.5	0.6	0.7	0.8	1.0	1.2	1.5	1.8
λ_2	0.5	0.6	0.7	0.8	1.0	1.2	1.5	1.8	2.0
T	(6400)	9000	10.000	9600	8000	5500	3800	5400	—

so that on an average:

$\lambda = 0.5-0.7$	$T = 9500$
$0.7-1.2$	6000
$1.2-1.8$	4600

hence a similar result as for the first method. (See diagram II). The deviations inter se are now much larger, as was, indeed, to be



expected, as on the small intervals λ_1—λ_2 the inevitable errors in I_λ (an experimental quantity!) make themselves very greatly felt.

Thus $\left.\begin{matrix}\lambda_1 = 1.8\\ \lambda_2 = 2.0\end{matrix}\right\}$ give an imaginary value for T, but when for $\lambda_2 = 2{,}0$ $I_\lambda = 22$ is taken instead of $I_\lambda = 19$, then T would become $= 18000°$.

In this manner particularly the smaller values of I_λ are unfavourable, hence the values for $\lambda_1 = 1{,}5$ and $\lambda_1 = 1.8$ are not much to be trusted.

The values of I_λ for $\lambda < 0{,}5$ are strictly speaking also unreliable, because the graphical interpolation — as indeed every other too — becomes very inaccurate here.

When we leave all these doubtful values of T out of consideration we come to the result that particularly in the region of the reliable values of T (the full line in the diagram) *there is an unmistakable tendency of T to decrease on the increase of λ, hence exactly the reverse of what we thought we might expect a priori.*

In a following paper I propose to discuss the question to what this unexpected result is to be attributed.

Utrecht, March 1919.