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Physics. — *"Isothermals of monatomic substances and their binary mixtures. XX. Isothermals of neon from + 20° C. to —217° C."* By C. A. CROMMELIN, J. PALACIOS MARTINEZ, and H. KAMERLINGH ONNES. Communication N°. 154*a* from the Physical Laboratory at Leiden.

(Communicated in the meeting of June 29, 1918).

§ 1. *Introduction.* This paper is the continuation of a previous preliminary communication [1]). The reduction of the neon-isothermals has now progressed so far, that what follows may be looked upon as a pretty nearly completed whole. The measurements refer to pressures up to about 90 atmospheres and as regards the temperature go from + 20° C. down to — 217° C., they therefore embrace the region between the ordinary temperature down to the lowest temperature to be reached with liquid oxygen. The region from — 218° C. to —246° C., which can now also be covered by means of the hydrogen vapour cryostat [2]), is here left out of account; we hope shortly to be able to continue our measurements in this region. As regards the importance of such determinations as will be communicated on this occasion and the apparatus which have been used for the purpose we may refer to previous communications on the isothermals of hydrogen and of argon. [3])

For the sake of completeness and of a better survey of the whole work we have included in the table the material published in the previous communications on neon, quoted above, viz. the series I, II, III, IV, VI, VII, VIII and IX.

§ 2. *The results of the measurements* are given in table I, where

$\theta =$ the temperature on the *international* [4]) CELSIUS-scale, i.e. the temperature on the *international* KELVIN-scale diminished by 273.09,

$p =$ the pressure in international atmospheres (for Leiden 1 international atmosphere = 75.9488 cms mercury),

$d_A =$ the density expressed in the normal density (0° C. and 1 atm.);

$v_A =$ the volume expressed in the normal volume (0° C. and 1 atm.).

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[1]) H. KAMERLINGH ONNES and C. A. CROMMELIN, Comm. N°. 147*d*; these Proc. XVIII (1) p. 515.

[2]) H. KAMERLINGH ONNES, Comm. N°. 151*a*; these Proc. XIX (2) p. 1049.

[3]) Comp. H. KAMERLINGH ONNES and H. H. FRANCIS HYNDMAN, Comm. N°. 69; these Proc III p. 481; H. KAMERLINGH ONNES and C. BRAAK, Comm. N°. 97*a* these Proc IX p. 754; C. BRAAK, Dissertation, Leiden, 1908; C. A. CROMMELIN, Dissertation, Leiden, 1910.

[4]) Comp. H KAMERLINGH ONNES, Comm. Suppl. N°. 34*a*. § 5.

TABLE I.

Series.	N⁰.	p	d_A	pv_A
$\theta = + 20°.00$ C.				
VI	1	22.804	21.046	1.0835
VI	2	25.015	23.052	852
VI	3	26.575	24.464	863
VI	4	29.090	26.757	872
VI	5	32.572	29.891	897
VIII	1	34.887	32.002	902
VI	6	35.423	32.447	917
VI	7	37.812	34.601	928
VIII	2	39.168	35.843	928
VIII	3	44.762	40.862	955
VIII	5	54.149	49.213	1003
VIII	6	59.717	54.161	026
VIII	7	65.021	58.797	059
VIII	9	77.360	69.338	131
VIII	10	82.545	73.967	160
VIII	11	88.239	78.886	186
VIII	12	93.298	83.154	220
$\theta = 0°.00$ C.				
VII	1	22.064	21.869	1.0089
VII	2	23.555	23.314	103
VII	3	25.867	25.558	121
VII	4	28.468	28.089	135
VII	5	30.790	30.345	147
IX	1	39.753	39.098	168
IX	2	44.892	44.030	196
IX	5	59.777	58.234	265
IX	6	66.104	64.135	307
IX	7	74.059	71.495	359
IX	8	79.108	76.127	392
IX	9	84.662	81.347	408

TABLE I (*Continued*).

Series.	N⁰.	p	d_A	pv_A
$\theta = -\ 103^\circ.01$ C.				
XV	1	35.558	56.40	0.6304
XV″	1	36.697	58.23	6302
XV	2	40.610	64.21	6324
XV″	2	42.107	66.53	6329
XV″	4	55.136	86.57	6369
XV	4	58.583	91.76	6384
XV	5	78.110	120.52	6481
$\theta = -\ 141^\circ.22$ C.				
XVI″	1′	33.840	69.83	0.4846
XVI	2	37.707	77 71	4852
XVI″	2	38.581	79.50	4853
XVI	3	43.319	88.97	4869
XVI″	4	49 881	102.32	4875
XVI	4	51.916	106.42	4878
XVI	5	66 471	134.91	4927
XVI	6	78.558	158.06	4970
$\theta = -\ 182^\circ.60$ C.				
X	2	32.067	99.89	0.3210
X″	2	32.988	102.84	3208
X	3	36.438	113.69	3205
X″	3	36.880	115.07	3205
X	4	41.371	129.44	3196
X″	4	42.533	133.15	3194
X	5	49.943	156.61	3189
X″	5	50.514	158.55	3186
X″	6	63.320	199.21	3179

TABLE I (*Continued*).

Series.	N⁰.	p	d_A	pv_A
$\theta = - 200^\circ.08$ C.				
XI	1′	26.214	105.10	0.2494
XI	2′	28.402	114.38	2483
XI	3″	31.417	127.24	2469
XI	1	34.268	139.81	2451
XI	4	34.285	139 88	2451
XI	5	39.843	164.30	2425
XI	2	39.891	164 63	2423
XI	3	46.517	194.30	2394
XI	3′	46.529	194.51	2392
XI	6	47.951	200.79	2388
III	1	61.657	263.77	2338
III	2	67.456	291.10	2317
III	3	73.850	320.35	2302
III	4	79.923	348.59	2293
$\theta = - 208^\circ.10$ C.				
XII	1	24.071	111.90	0.2151
XII	3	28.844	136.44	2114
XII	4	31.948	153.00	2088
XII	5	37.856	185.47	2041
XII	6	41.798	207.95	2010
IV	1	58.472	308.32	1897
IV	2	64.451	345.22	1867
IV	3	69.692	377.89	1844
IV	4	74.532	409.18	1822
IV	5	79.228	439.12	1804

TABLE I (*Continued*).

Series.	N⁰	p	d_A	pv_A
$\theta = -213^\circ.08$ C.				
XIII	1	23.086	119.92	0.1925
XIII	2	24.810	129.82	1911
XIII	3	26.673	140.90	1893
XIII	4	29.365	157.70	1862
XIII	5	32.441	177.37	1829
XIII	6	37.418	210.68	1776
II	1	53.896	334.59	1611
II	2	59.769	382.03	1565
II	3	66.271	435.46	1522
II	4	72.858	484.75	1503
II	5	79.698	534.62	1491
$\theta = -217^\circ.52$ C.				
XIV	1	21.349	123.40	0.1730
XIV	2	22.997	134.72	1707
XIV	3	24.686	146.67	1683
XIV	4	26.848	162.51	1652
XIV	5	30.042	186.94	1607
XIV	6	32.795	209.68	1564
I	1	49.930	358.51	1393
I	2	53.528	395.62	1353
I	3	59.618	458.40	1301
I	4	64.975	511.85	1269
I	5	71.649	571.69	1253
I	6	79.417	632.23	1256

A graphical representation of the observations will be found in fig. 1 in a $\left(\frac{pv_A}{T}, d_A\right)$ diagram.

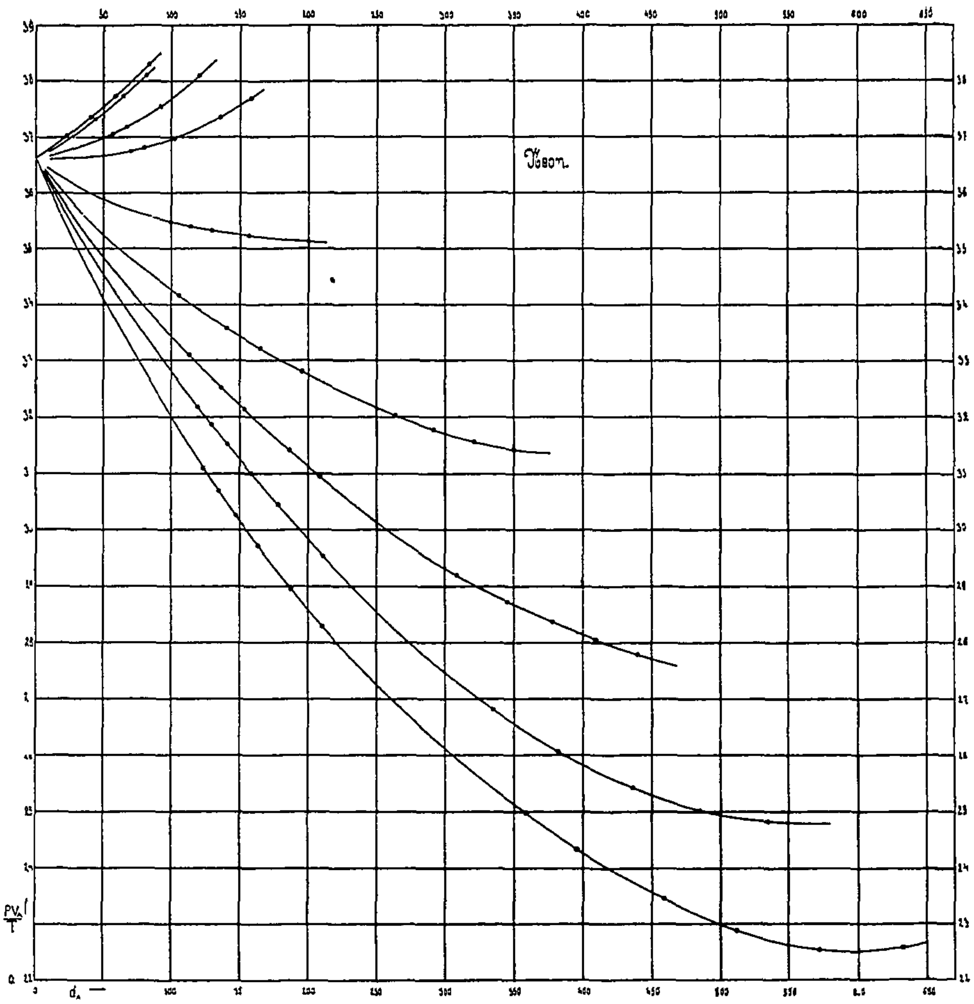

Fig. 1.

§ 3. *Virialcoefficients.*

By means of the above results some of the coefficients in the empirical equation of state

$$pv_A = A_A + B_A d_A + C_A d_A^2 + D_A d_A^4 + E_A d_A^6 + F_A d_A^8 \quad ^{1})$$

could be computed. These calculations only embrace the coefficients B_A, C_A, D_A and in one case E_A; for the densities which were reached are not so high as would be necessary for the deduction of F_A and in most cases of E_A also; these were therefore found from the reduced equation of state VII. A. 3 [2]) or VII. 1. [3]), in which the coefficients $\mathfrak{E}$ and $\mathfrak{F}$ are identical. In some of the calculations,

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1) H. Kamerlingh Onnes, Comm. N⁰. 71; these Proc. IV p. 125.

2) H. Kamerlingh Onnes and C. A. Crommelin, Comm. N⁰. 128; these Proc. XV (1) p. 273.

3) Suppl. N⁰. 19.

as will appear presently, D_A, in others D_A and C_A were assumed according to VII. A. 3. For this purpose use was made of the critical constants of neon as published on a former occasion [1])

$$\theta_k = -228°.35\ C. \qquad p_k = 26.86\ \textit{int. atm}$$

The calculations were conducted in three different ways:

a. for all temperatures only B_A was calculated from the observations, the remaining coefficients being assumed, viz. C_A and D_A according to VII. A. 3., E_A and F_A according to VII. 1 or to VII. A. 3- (as noticed above, this comes to the same);

b. for the lowest 4 temperatures B_A and C_A were computed from the observations, further as under *a*;

c. for all temperatures B_A and C_A were deduced from the observations, for —200°.08 C., —208°.10 C. and —213°.08 C. also D_A and for —217°.52 C. also E_A, further as under *a*.

The calculations *c* were made first, with a view to obtaining the best possible accordance with the observations, the coefficients therefore bearing a purely empirical character. When it appeared that the values of C_A could not be connected by a smooth curve, much less those of D_A, which proved the observational material to be insufficient for the deduction of C_A and D_A as functions of the temperature, we proceeded to the methods given under *a* and *b*, in which the values of D_A and partly even those of C_A were assumed. Naturally the accordance with the observations is very much inferior with the methods *a* and *b* than with *c*.

The results of the calculations which were all conducted by the method of least squares are found in tables II and III. Table II gives the individual virial-coefficients, as calculated from the observations according to *a*, *b*, and *c*, table III the coefficients borrowed from VII. A. 3 as well as the values of A_A computed from the equation

$$A_A = A_{A_0}(1 + 0.0036618\,\theta), \text{ }^{2})$$

where for A_{A_0} the value + 0.99986, as published on a former occasion [3]), was taken as a basis.

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[1]) H. Kamerlingh Onnes, C. A. Crommelin and P. G. Cath, Comm. N⁰. 151*b*, these Proc. XIX (2) p 1058.

[2]) Comm. N⁰. 71.

[3]) Comm. N⁰. 147*d*.

TABLE II.

θ	$B_A \times 10^3$	$B_A \times 10^3$	$C_A \times 10^6$
	According to *a*.	According to *b*.	
+ 20°.00	+ 0.54880		
0°.00	47148		
— 103°.01	16653		
— 141°.22	055249		
— 182°.60	— 0.093113		
— 200° 08	15746	— 0.18779	+ 0.21531
— 208°.10	19553	21706	18307
— 213°.08	22305	24084	18407
— 217°.52	24028	25880	19649

TABLE II (*Continued*).

θ	$B_A \times 10^3$	$C_A \times 10^6$	$D_A \times 10^{12}$	$E_A \times 10^{18}$
	According to *c*.			
+ 20°.00	+ 0.51578	+ 0 82778		
0°.00	41334	1.1538		
— 103°.01	069193	1.1515		
— 141°.22	— 0.025378	0.71945		
— 182°.60	13435	33607		
— 200°.08	19667	27847	— 0.24096	
— 208°.10	22926	25304	0.16102	
— 213°.08	24625	21123	0.005848	
— 217°.52	29313	36427	0.46739	+ 0.57517

TABLE III.

θ	A_A	$C_A \times 10^6$	$D_A \times 10^{12}$	$E_A \times 10^{18}$	$F_A \times 10^{27}$
		According to VII. A. 3.			
+ 20°.00	+ 1.0731	+ 0.29747			
0.00	0.99986	25440			
— 103°.01	62271	0.072156	+0.37445	—0.1373	+ 40.29
— 141°.22	48281	39576	28409	0.03754	12.51
— 182°.60	33131	58524	12718	2409	— 4.190
— 200°.08	26731	96581	11124	4293	5.666
— 208°.10	23795	0.12219	0.081145	+0.04550	5.367
— 213°.08	21971	14073	60843	4599	4.836
— 217°.52	20345	15882	41215	4576	4.160

§ 4. *Discussion and comparison with other observations.* The differences between the pv_A-values calculated from these equations and the observed values are represented graphically in fig. 2 as functions of the densities d_A, the ordinates being the observed minus the calculated pv_A-values, expressed as percentages of the latter. In this manner the character of the deviations is more easily grasped than would be the case, if the numbers were given in the tables.

The correspondence between the new and the old series is very satisfactory on the whole; only in the isothermal for — 217°.52 C. a marked deviation may be noticed. Whereas for the isothermals of —200°.08 C. the deviations of the observations from the most closely corresponding formula (method *c*) are within 0.1 $^0/_0$, differences of almost $^1/_2$ $^0/_0$ occur in the isothermals for — 217°.52 C.

The differences between the various sets of B_A-values obtained on this occasion from the smoothed B_A-values according to VII. A. 3, viz. $\Delta B_A = B^{=}_{A}$ (calc.) — B_A (VII. A. 3) are represented in fig. 3; the corresponding deviations of the B_A-values obtained by Cath and one of us[1]) from measurements at low pressures are included in the figure.

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[1]) P. G. Cath and H. Kamerlingh Onnes, Comm. N⁰. 152*e*, presented to the Meeting of the Academy some time ago and shortly to be published in the Proceedings; preliminary values are given by P. G. Cath, Dissertation, Leiden 1917, p 77.

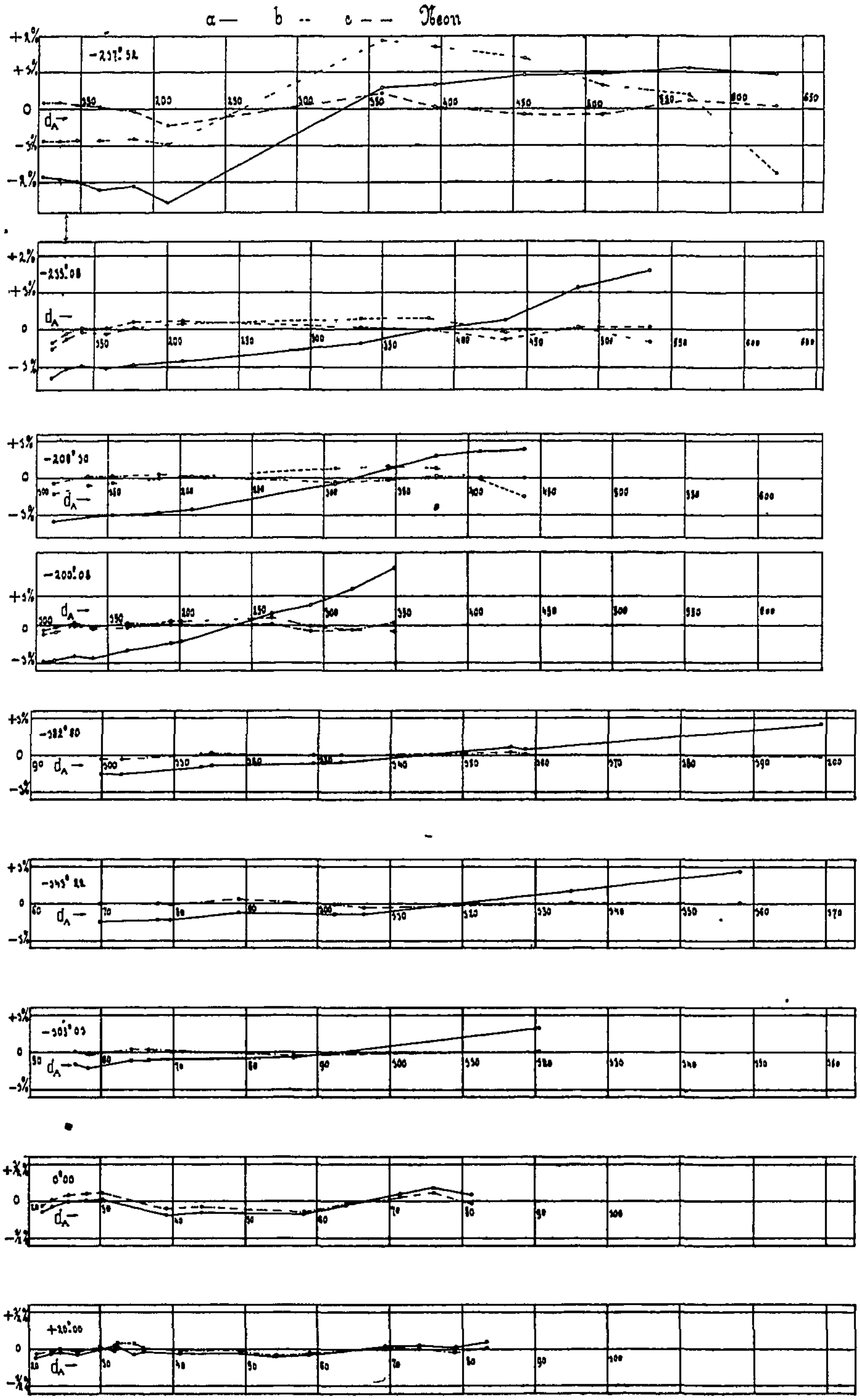
a — b -- c --- Neon
-237°52
-235°08
-208°30
-200°08
-182°80
-143°22
-103°03
0°00
+20°00
d_A →
0
+5%
-5%

Fig. 2

It may be noticed, that the B_A-values according to *a* agree fairly

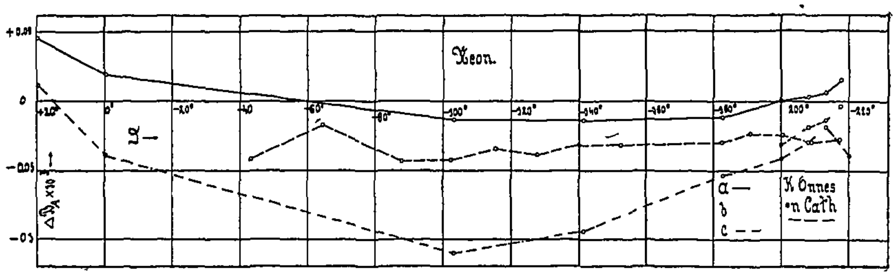

Fig. 3.

well with those according to VII. A. 3. Those obtained by method *c* deviate much more markedly, as might be expected; especially at the lower temperatures they show much smaller values.