Physics. — *"The Propagation of Light in Moving, Transparent, Solid Substances. II. Measurements on the* FIZEAU-*Effect in Quartz".* By Prof. P. ZEEMAN and Miss A. SNETHLAGE.

(Communicated at the meeting of May 3, 1919)

1. In communication I the apparatus has been described that has proved suitable for the investigation of the FIZEAU effect in solid substances.

We have now carried out experiments with quartz, which was traversed by beams of light in the direction of the optical axis. We were led to the choice of this substance by the consideration that in general, crystals are the most homogeneous bodies that we know, and the scattering of light in a crystal must be exceedingly slight on account of its regular structure [1]).

It appeared to us later that the best optical glass, for our purpose, can be compared in some respects with quartz, in others it is even preferable.

In some series of experiments 10 quartz rods were used, supplied by the firm of STEEG and REUTER, with endplanes normal to the optical axis, and of the dimensions $10 \times 1.5 \times 1.5$ cm. Later on four similar rods supplied by the firm of A. HILGER, Ltd. were added to them. For a series of experiments the rods were joined together to form a column of a length of 100 cm.; in a second series of 140 cm. They were placed one behind another in a groove which was milled in a wooden beam, fastened to the driving apparatus by means of four solid screws. The different rods are separated from each other by rubber discs with round apertures of a diameter of about 13 mm. Each quartz rod rests in a groove 13 mm. deep, and is pressed down by two brass plates, fastened with screws in the upper surface of the wooden beam, a thin piece of cork being placed under the plates. The space remaining at the ends of the groove is filled up by a piece of brass tubing. Solid brass plates, which clasp the beam, shut off the ends of the groove.

[1]) LORENTZ. Théories statistiques en thermodynamique, p. 42.

2. In order to place the quartz rods in the groove, we proceeded in the following way. After the interference-bands had been produced with great distinctness, and the beam had been placed on the apparatus, one quartz rod was put in the groove, and if necessary the interference-lines were made distinct anew. It was then ascertained which of the four positions obtained by rotating the rod round its longitudinal axis, gives bands that change least, when the machine is made to assume different positions. Then the second rod is placed behind the first, likewise in four positions etc., till all the rods have been arranged. In order to prevent reflected light from entering the interferometer, each of the rods is placed in a somewhat sloping position by putting a piece of thin cardboard at one end. The rods are put in one by one. After each addition it is tried, whether the correct position has been obtained.

We may still remark in this connection that the glass cylinders with which we have made experiments (see the following communication) have been manufactured so exceedingly well by the firm of ZEISS, that on rotation about the longitudinal axis in a cylindrical groove there does not appear an appreciable change of the interference bands. Hence the optical control becomes a great deal simpler than for quartz. The interference bands finally photographed through the quartz column are decidedly less distinct than the interference bands that are observed when the column has been removed. The lines have become slightly diffuse. This is not the case when the glass cylinders of ZEISS have been introduced. The diameter amounted to 25 mm. with a length of 20 cm. As there were used six cylinders, there were twelve reflecting planes for a total length of glass of 120 cm. In the experiments with the quartz column of 140 cm. length the number of reflecting planes amounted to twenty-eight. Though this great number of reflecting planes must have an unfavourable influence on the distinctness of the system of fringes, yet it was beyond all doubt that it was not owing to this cause that the quartz column had a more unfavourable influence than the glass column. We might still have eliminated the reflections on the interfaces by introducing a liquid of the mean index of refraction of quartz between the successive rods. The complication of the apparatus, which would ensue from this, and the unfavourable experience which we had with moving liquids, made us resolve to put up with the reflections.

3. As source of light a 12 Ampères arc-lamp was used, the light of which was made sufficiently monochromatic by means of filters.

Experiments were carried out with three different colours, the effective wave-lengths of which amounted to 6510, 5380, 4750 Å.U.

4. When white light traverses the apparatus, we easily distinguish the central band. Its centre is the point whose displacement we should wish to measure in an experiment with white light. Also for incident monochromatic light we can speak of the centre of the central band. It is the point that remains fixed when the interference lines rotate, or become narrower or wider through any cause that does not depend on the Fizeau effect. The position of the centre can be determined by means of the horizontal and movable vertical cross-wires in the telescope, by subjecting the interference bands to some modification with the compensator, thus causing the centre to be observed clearly.

When the centre has been determined, the movable vertical wire is displaced over a few bands, so that this wire can have no disturbing influence on the measurement on the photo.

A series of photos is then taken on one photographic plate, in which the directions of the movement alternated.

The observed effect is derived from the displacement of the centre. Of course plates on which a notable rotation of the interference bands has occurred, are rejected.

5. The following table may serve as an example of the results obtained by measurement of a plate taken with:

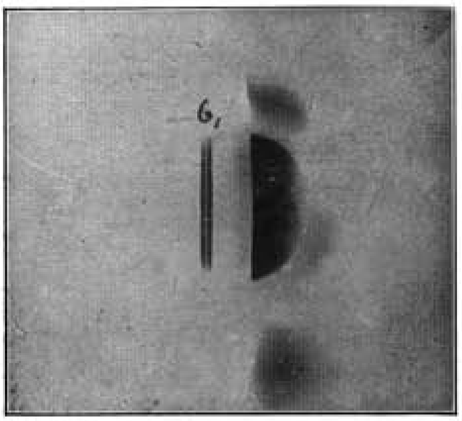
Green light $\lambda = 5380$ Å.U.

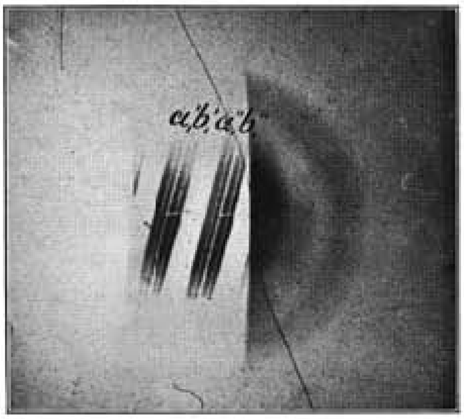
Number of the plate	Maximum velocity in cm. per sec.	Length of column of quartz in cm.	Observed effect	Effect reduced to 1 m. of quartz and max. velocity 10 meters	Mean value for the plate
48	750	100	92	123	
			137	183	
			118	157	152
			99	132	
			125	167	

The effects are given in thousandths of the distance of the fringes.

Altogether photos have been taken on eleven plates with green light $\lambda = 5380$ Å.U. In all fifty-one values have been obtained in this way

P. ZEEMAN and A. SNETHLAGE: "The Propagation of Light in Moving Transparent Solid Substances. II. Measurements of the Fizeau-Effect in Quartz".





for the observed effect, but not the same number of photos have been taken on all plates as on Nr. 48. The velocities used ranged between 750 and 950 cm./sec. the length of the quartz column being 100 cm. for nine plates, and 140 cm. for two. The obtained values may now be used in two ways to derive a final result from them. For each plate a mean value can be derived, and the arithmetical mean may be taken of the eleven values thus obtained. In this way:

$$0{,}146 \pm 0{,}012,$$

as final result of the effect reduced to a velocity of 1000 cm./sec. and a length of quartz of 100 cm., the mean error being recorded after the $\pm$ sign.

Another way in which the values can be combined is by taking the arithmetical mean of the fifty-one values. Thus we find:

$$0{,}148 \pm 0{,}006.$$

From formula (4), which was given in our communication I, and will be proved presently, follows for the theoretical value of the effect:

$$0{,}143.$$

6. With red light $\lambda = 6510$ Å. U. twenty-seven values have been obtained for the effect on six plates. To eight of them corresponds a quartz column of 140 cm., to nineteen one of 100 cm. The velocities range between 750 and 960 cm./sec.

The result, when the mean values of the different plates are combined, is:

$$0{,}123 \pm 0.014.$$

The arithmetical mean of the twenty-seven separate values yields:

$$0{,}125 \pm 0.007.$$

The calculation gives for the expected effect:

$$0.115.$$

7. The results with violet light $\lambda = 4750$ Å. U. should be received with some diffidence, as it appeared afterwards that the violet filter transmitted some red light, which had not been detected at first. Hence it is possible that this cause slightly vitiated the later series. It must be said, however, that no trace of change could be ascertained in the values of the later series.

When the eight mean values of the different plates are combined, the result becomes:

$$0{,}156 \pm 0.008$$

The arithmetical mean of the thirty-one separate values differs very little from this:

$$0{,}156 \pm 0.007$$

The theoretical value is 0.166.

8. We collect the results in the following table.

λ	Δ_w	Δ_{th}
4750	0.156 ± 0.007	0.166
	0.156 ± 0.008	
5380	0.148 ± 0.006	0.143
	0.148 ± 0.012	
6510	$0.125 \pm 0\ 007$	0.115
	0.123 ± 0.014	

The observed displacement of the bands is indicated under Δ_w. The mean error has been calculated in two ways, as was discussed above. The second values are those derived from the average of the mean values of the individual plates.

Under Δ_{th} the theoretical value is given calculated by the aid of the data for the index of refraction for the ordinary ray in quartz, taken from KOHLRAUSCH's data.

It is not to be denied that taking the particular difficulties of the experiments into consideration, the agreement between theory and observation is very satisfactory.

The change of the effect with wave-length as well as the absolute value of the effect are represented very well. In the discussion of the experiments with glass, for which the dispersion is greater than for quartz, we shall have an opportunity to point out the very pronounced influence of the dispersion term.

9. *The formula for the optical effect.* We consider two of the rays which bring about the interference phenomenon, and which have passed over opposite paths. We shall denote quantities which refer to the first ray, by one accent, and those belonging to the second ray by a double accent. Each of the paths traversed, consists of three parts: 1 a path 1 in the air, 2 a path 2 in the quartz column, 3 a path 3 in the air.

The times expressed in seconds, which the light requires to pass over each of the parts we call resp. t'_1, t'_2, t'_3, t''_3, etc.

If the quartz is at rest, of course $t'_1 = t''_1$ etc. If, however, the quartz moves with the velocity w, the time required to traverse the quartz column (length l) in the direction from 1 to 2

$$t'_2 = \frac{l}{\frac{c}{\mu'} + \left(1 - \frac{1}{\mu'^2}\right) w - w} = \frac{l}{\frac{c}{\mu'} - \frac{w}{\mu'^2}} \quad . \; . \; . \; . \; . \; . \quad (1)$$

in which the difference between the velocity of the light in quartz, and of that of the column itself must be taken into account. While the light is passing through the quartz, the quartz moves on, hence t_2 is changed by an amount:

$$-\frac{lw}{\frac{c}{\mu'} - \frac{w}{\mu'^2}} \cdot \frac{1}{c} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

We get for the ray in the opposed direction:

$$t''_2 = \frac{l}{\frac{c}{\mu''} - \frac{w}{\mu''^2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

and for the other quantity:

$$+\frac{lw}{\frac{c}{\mu''} + \frac{w}{\mu''^2}} \cdot \frac{1}{c} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

For the first ray the entire difference of time becomes, therefore:

$$\frac{l}{\frac{c}{\mu'} - \frac{w}{\mu'^2}} - \frac{lw}{\frac{c}{\mu'} - \frac{w}{\mu'^2}} \cdot \frac{1}{c}$$

or expressed in periods for one ray

$$\frac{l}{\lambda}\left(\mu' + \frac{w}{c} - \frac{w}{c}\mu'\right) \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

and for the other

$$\frac{l}{\lambda}\left(\mu'' - \frac{w}{c} + \frac{w}{c}\mu''\right) \quad . \; . \; . \; . \; . \; . \; . \quad (6)$$

When we now consider that:

$$\mu' = \mu + \lambda \frac{d\mu}{d\lambda} \frac{w}{c} \quad \ldots \ldots \ldots \quad (7)$$

and

$$\mu'' = \mu - \lambda \frac{d\mu}{d\lambda} \frac{w}{c} \quad \ldots \ldots \ldots \quad (8)$$

we find, after substitution of this in the formulae (5) and (6), and after subtraction for the entire phase difference of the two rays that: —

$$\Delta = \frac{2lw}{c\lambda}\left(-\lambda\frac{d\mu}{d\lambda} - 1 + \mu\right)$$

or, on reversal of the direction of motion, an optical effect: —

$$\Delta = \frac{4lw}{c\lambda}\left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right) \quad \ldots \ldots \quad (9)$$

With regard to the dispersion term it is still noteworthy that in FIZEAU's experiment with water the light is transferred from standing *water* to moving *water*, and $\frac{w}{c/\mu}$ must be written in the formulae instead of $\frac{w}{c}$.

10. *Derivation of formula* (9) *from the theory of relativity.*

After we had communicated formula (9) to Prof. LORENTZ, he had the kindness to give us a derivation strictly from the theory of relativity, which will follow here.

Let x', t' be a system of coordinates, in which the rod AB is at rest; length of the rod l'.

Light motion on the lefthand side of A:

$$a_1 \cos n'\left(t' - \frac{x'}{c} + p'_1\right) \quad \ldots \ldots \ldots \quad (1)$$

In the rod:

$$a_2 \cos n'\left(t' - \frac{x'}{v'} + p'_2\right)$$

On the righthand side of B:

$$a_3 \cos n'\left(t' - \frac{x'}{c} + p'_3\right) \quad \ldots \ldots \ldots \quad (2)$$

v' velocity of propagation belonging to n'.

We easily find:

$$n'(p'_3 - p'_1) = n' l'\left(\frac{1}{c} - \frac{1}{v'}\right) \quad \ldots \ldots \quad (3)$$

Through the relativity transformation:

$$x' = ax - bct \quad , \quad t' = at - \frac{b}{c}x \qquad a^2 - b^2 = 1 \quad . \quad . \quad (4)$$

and

$$x = ax' + bct' \quad , \quad t = at' + \frac{b}{c}x'$$

we may pass to a system, in which the rod moves with a velocity:

$$w = \frac{bc}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From (2) and (3) we derive

$$\alpha_1 \cos\left[n\left(t - \frac{x}{c}\right) + n'p'_1\right] \text{ and } \alpha_2 \cos\left[n\left(t - \frac{x}{c}\right) + n'p'_2\right] \quad . \quad (6)$$

$$n = (a + b)\, n' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The phase difference between (1) and (2), i.e. the change of phase brought about by the presence of the rod, is given by (3) in angular measure, and this same difference of phase still exists between the expressions (6).

Expressed in wave-lengths or periods, it is:

$$\Delta = \frac{n'l'}{2\pi}\left(\frac{1}{c} - \frac{1}{v'}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Here, however, we must express n' and v' in the n and v corresponding to it. When we neglect the terms of the second order,

$$a = 1 \ , \ b = \frac{w}{c} \ ,$$

follows from (4) and (5), hence according to (7):

$$n = \left(1 + \frac{w}{c}\right)n' \ , \ n' = \left(1 - \frac{w}{c}\right)n$$

$$v' = v + (n' - n)\frac{dv}{dn} = v \quad \frac{w}{c}\, n\, \frac{dv}{dn}$$

$$l' = l$$

After substitution in (8) we find for the part of Δ that depends on w:

$$\frac{nl}{2\pi}\cdot\frac{w}{c}\left\{\frac{1}{v}-\frac{1}{c}-\frac{n}{v^2}\frac{dv}{dn}\right\},$$

or after introduction of $n=\frac{2\pi c}{\lambda}, v=\frac{c}{\mu}$

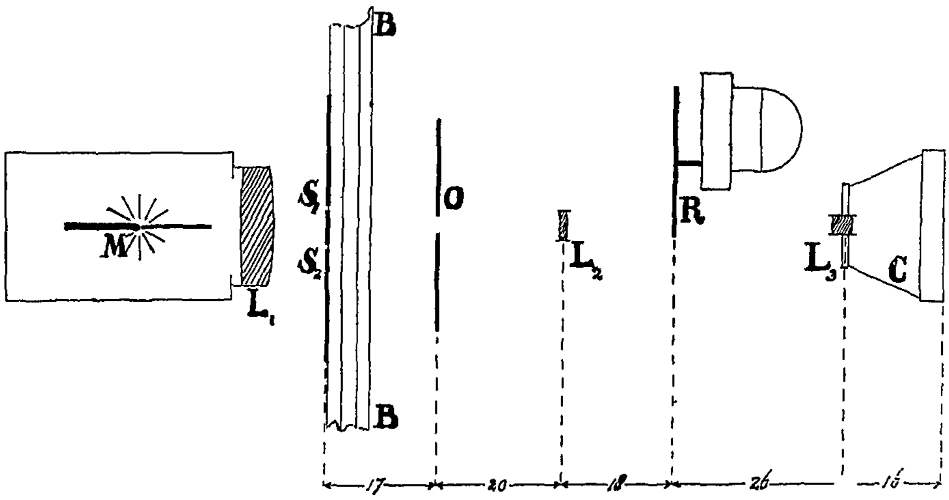
$$\frac{l}{\lambda c}\left(\mu-1-\lambda\frac{d\mu}{d\lambda}\right)w$$

by the aid of which (9) of § 9 follows immediately.

11. *Direct determination of the velocity of the beam.*

In I § 3 the method has been indicated by which the maximum velocity was determined. We have, however, also measured it directly by the following method, which was, not applied until the experiments with glass were undertaken, but which is described here, because it has confirmed the velocity determination on the supposition of a fly-wheel revolving with a constant angular velocity.

To the beam BB is attached a black screen with two slits S_1 and S_2, across which threads are stretched for accurate refer-



ence. Consider only the slit S_1, which moves in the field of light of the condenser L_1. By the aid of the achromatic lens L_2 a sharp image of S_1 is projected on the plane R of the circular plate of a light interruptor used for a large galvanometer. This latter apparatus, to which our attention was drawn by Mr. Wertheim Salomonson, and which was put at our disposal by him, consists of an electromotor with a centrifugal speed indicator. One of the five axes of the apparatus revolves 25 times per second. The aluminium plate R (diameter 30 cm) has 40 slits in its circumference, each about 1 mm. wide, so that 1000 flashes arise per second. The distance between two slits is about 23 mm., so that light is allowed to pass during 1/20000 second.

An image of the stationary slit S_1, projected on the photographic plate of the camera C, when the disc rotates, is reproduced in b, on the Plate, fig. 1: the two cross-wires are seen, and on the righthand side of the slit the rim of the circular plate R, the centre of which lies on the left side. The dark circles are caused by reflections, but are of no consequence.

When S_1 moves and the disc revolves, after every thousandth of a second the light is let through the slit in R, and S_1 is photographed by means of the lens L_2.

The images of S_1 assume an oblique position, because during the displacement of the beam, the slit in the disc gets continually higher in its movement.

It is easy to see that the slope of the image is determined by the ratio of the velocity of the slit to that of the beam, or rather to that of the image on the disc. From the distances of S_1 and R to L_2, and from L_2 to R and the plate, the reduction which the velocity of the beam undergoes in the image, can be immediately estimated, or it can be directly measured by photographing a divided scale in the plane S_1 S_2. The amount of obliqueness of the slit image (Plate fig. 2) immediately gives an approximate value of the velocity of the beam, which can, indeed, be found in a still simpler way from the distance of corresponding points of b'_1 b''_1.

For a more accurate determination of the velocity the second slit S_2, which is at 4,15 cm. distance from S_1, can be of service. Let us suppose that about 1000 cm./sec. have been found for the approximate value of the velocity of the beam, then S_1 has shifted about 1 cm. after every thousandth of a second, and S_2 gets about to the first position of S_1 after four thousandths of a second. Hence between the images of S_1 an image of S_2, viz a_1'', will appear in general on the photo, from which the position of the slit images can be accurately derived, and then we know that in order for the beam to move a distance 4,15 cm., 4 + a fraction thousandths of seconds are required, which can be measured from the relative positions. In order to distinguish the slits, a cross-wire has been stretched only over S_1.

We only get two images of each of the slits on the photo, because it is only possible that images are formed by the different lenses within a limited cone.

As it is our intention to determine the velocity of the beam with a definite direction of motion, the shutter (I § 4), which as a rule is before the objective of the telescope, has been placed at 0 (cf. the above figure). This ensures that the light passes through the apparatus at the right moment. The experimental error of the determination of the velocity appears to amount to at most 0.75 %.

We still wish to draw attention to a peculiarity of the described method of the velocity determination, which seems interesting from a theoretical, though not from a practical, point of view.

The measurement actually takes place with a *moving measuring rod* (length at rest $= S_1 S_2$), a peculiarity which we do not remember having seen used in practice with any other method.

We are indebted to Messrs. W. DE GROOT and G. C. DIBBETZ JR. for their assistance in the theoretical and experimental work, and to Mr. J. VAN DER ZWAAL for his help in the difficult adjustment of the apparatus and the manufacture of the auxiliary appliances.