Astronomy. — "*On the Parallax of some Stellar Clusters*" (Second communication). By Dr. W. J. A. SCHOUTEN. (Communicated by Prof. J. C. KAPTEYN).

(Communicated in the meeting of February 23, 1918).

In a former communication it was shown, how it is possible to determine the parallaxes of stellar clusters from the numbers of stars of determined magnitude in the clusters by means of the luminosity curve of KAPTEYN. The calculation was performed for *Messier* 3 and *h and χ Persei*. Now the same method is used in order to determine the parallax of some other clusters.

The Small Magellanic Cloud.

H. S. LEAVITT. 1777 Variables in the Magellanic Clouds Annals Harvard Observ. Vol. 60, N°. 4.

A preliminary catalogue containing 992 stars of the Small Cloud and 885 of the Great Magellanic Cloud. The places of 28 stars of catalogues in the neighbourhood of the Small Cloud are also given.

We counted a number of stars and estimated their diameter on a photographic plate, taken at the Harvard Observatory. For orientation we used the catalogue-stars the position of which Miss LEAVITT communicates. In order to reduce the estimates of diameters to magnitudes, we

1^{stly} counted an area of 1000 □′ without the Cloud, and determined from the numbers of stars of every magnitude the magnitude corresponding to every diameter by means of Publ. Gron. N°. 27, Table IV,

2^{ndly} we estimated the diameters of 142 variable stars, the magnitudes of which occur in LEAVITT's catalogue and which are equally distributed over the Cloud, and we have compared these with the mean magnitude, i. e. the average of maximum and minimum, given by Miss LEAVITT,

3^{dly} we have estimated the diameters of the catalogue-stars mentioned above and compared these with the magnitudes in the C. P. D. and the A. G. C.

Finally the magnitude corresponding to each diameter was determined from all these data by graphical smoothing.

We counted an area of 240 □′ in the Cloud. The results are given in the table below. In it N_m represents the number of stars from the brightest star to the magnitude under consideration.

Diameter	Magn.	N_m	Magn.	N_m	A_m	Normal	Cluster
25	10.1	1	10 0	1	1	1	—
22	10.4	2					
20	10.7	4	10.5	4	3	—	3
17	11.2	5	11 0	5	1	1	—
16	11.3	6					
15	11.5	13	11.5	16	11	1	10
14	11.7	19					
13	12.0	26	12.0	33	17	1	16
12	12.2	39					
11	12 5	55	12.5	60	27	3	24
10	12.8	87					
9	13.1	122	13.0	122	62	4	58
8	13.4	172					
7	13.7	220	13.5	202	80	5	75
6	14.0	282	14.0	305	103	8	95
5	14.3	358					
4	14.6	467	14.5	438	133	11	122
3	14.9	568					
2	15.2	810	15.0	655	217	16	201
1	15 6	1104	15.5	1064	409	22	387
0	16.0						

The normal number of stars is calculated for the galactic latitude $b = 10°$. As we always use the luminosity curve for whole numbers as values of the argument m and have counted here by half magnitudes, we may deduce from the above table the following two tables:

m	A_m	A_{m+1} / A_m	m	A_m	A_{m+1} / A_m
12.0	33	3.24	11.5	18	3.39
13.0	107	1.81	12.5	61	2.48
14.0	194	2 35	13.5	151	1.79
15.0	455		14.5	270	

The irregular progress of the quotients $\frac{A_{m+1}}{A_m}$ is partly to be explained from our counting only a small part of the cluster.

These numbers give the following values for the parallax:

I	$\pi = 0''.0004$
II	4
III	7
IV	13
V	11
VI	4
Mean	$\pi = 0''.0007 \pm 0''.0002$

From 142 cluster variables that are equally distributed over the cluster and occur in Miss LEAVITT's catalogue, we find for the mean apparent magnitude of these stars $\overline{m} = 14.67$ and $5 \log. \pi = -15.77$, so that the mean absolute magnitude of these δ Cephei variable stars with a short period is $\overline{M} = 3.9$ according to our determination of the parallax.

From some δ Cephei variable stars with a long period HERTZSPRUNG found for the parallax of the Small Magellanic Cloud $\pi = 0''.0001$.

Praesepe.

$\alpha_{1900} = 8^h 34^m 39^s$, $\delta_{1900} = +20°1'$, $b = +33°$, $l = 169°$.

Dr. P. J. VAN RHIJN. The proper motions of the stars in and near the Praesepe cluster, Publ. Groningen, N°. 26, 1916.

The measurement of 2 sets of plates, taken at Potsdam. The catalogue contains 531 stars. The diameters were reduced to photographic magnitudes by means of standard magnitudes, determined by HERTZSPRUNG. The probable error of a magnitude is $\pm 0^m.12$.

We have derived the visual magnitudes from the photographic ones in the same way as VAN RHIJN did on page 10 of his publication. The correction was determined from the value of the colour

index for each *apparent* magnitude that is based on PARKHURST and SEARES' researches. To this objections may be raised, as for the cluster stars we have to deal with *absolute* magnitudes. As, however, the relation between colour index and luminosity is only inaccurately known as yet and as moreover, it cannot be decided whether a given star belongs to the cluster or not, VAN RHIJN's method is the only one possible. VAN RHIJN found that the photographic magnitudes (international scale) between $m = 7.5$ and $m = 14.5$ wanted a constant correction $-0^m.5$ for reduction to the visual Potsdam scale. Therefore by a correction $-0^m.7$ they are reduced to the Harvard scale.

The number of cluster stars of each magnitude we find by diminishing the numbers counted by the normal number, which was determined for this cluster from Publ. Gron. N°. 27, Table V.

It appears at once that the Praesepe stars have faint luminosities. The declivities that we observe in the frequency curve of the magnitudes are partly smaller than the smallest declivity occurring in KAPTEYN's luminosity curve. That is why we could make only four determinations of the parallax notwithstanding the great interval of magnitudes. These give

$$\pi = 0''.024 \pm 0''.004.$$

This parallax is considerably greater than the one which we found for other stellar clusters.

Messier 52.

N. G. C. 7654; $\alpha_{1900} = 23^h19^m.8$, $\delta_{1900} = +61°3'$, $b = +1°$; $l = 81°$; class: $D\,3$.

F. PINGSDORF. Der Sternhaufen in der Cassiopeia. Diss. Bonn. 1909. Measurements of three plates, taken by KÜSTNER. The catalogue contains 132 stars up to $15^m.0$. The standard magnitudes have been determined by visual observations by means of gauzes of 25 stars by ZURHELLEN.

We find from 4 determinations:

$$\pi = 0''.002 \pm 0''.0003.$$

Messier 46.

N. G. C. 2437; $\alpha_{1900} = 7^h\ 37^m.2$, $\delta_{1900} = -14°35'$, $b = +6°$, $l = 200°$; class: $D1$.

W. ZURHELLEN. Der Sternhaufen Messier 46. Veröffentl. Kgl. Sternwarte zu Bonn, N°. 11, 1909.

Measurements of three plates, taken by KÜSTNER. The catalogue

contains 529 stars. For standard magnitudes 47 stars were used, the brightness of which was estimated by KÜSTNER or determined by means of gauzes by ZURHELLEN.

We find from 4 determinations:

$$\pi = 0''.002 \pm 0''.0001.$$

Messier 37.

N. G. C. 2099; $\alpha_{1900} = 5^h45^m.8$, $\delta_{1900} = +32°31'$, $b = +4°$, $l = 145°$; class: $D1$.

J. O. NORDLUND. Photographische Ausmessung des Sternhaufens Messier 37. Inaug. Diss. Upsala 1909, Arkiv för Matematik, Astronomie och Fysik, Band 5, N°. 17.

Dr. H. GIEBELER. Der Sternhaufen Messier 37. Veröffentl. Kgl. Sternwarte zu Bonn, N°. 12, 1914.

NORDLUND measures 4 plates and gives the places and magnitudes of 842 stars. The magnitudes are derived from the diameters according to the formula of CHARLIER by means of 214 standard magnitudes that have been determined photometrically by VON ZEIPEL. Many of the bright stars of the cluster are red (colour index $> 0^m.7$), *e.g.* some 50 or 70 % of the stars of the 10^{th} magnitude.

GIEBELER discusses 2 plates taken by KÜSTNER and measured by STROELE. The catalogue contains 1231 objects. The magnitudes have been joined with NORDLUND's scale by comparing those of 450 stars. For the red stars too the photographic magnitude is given.

For our purpose it is a drawback that for the red stars the photographic magnitude is mentioned. This is why the brightest stars, among which many red ones occur, could not be used by us. Excluding these we find from 4 determinations:

$$\pi = 0''.002^5 \pm 0''.0004.$$

Messier 36.

N. G. C. 1960; $\alpha_{1900} = 5^h\,29^m.5$, $\delta_{1900} = +34°\,4'$, $b = +2°$, $l = 142°$; class: $D2$.

Dr. S. OPPENHEIM. Ausmessung des Sternhaufens *G. C.* N°. 1166. Publ. der v. Kuffner'schen Sternwarte in Wien, Bd. III, pag. 271-307, 1894.

Measurements of three photographic plates. The catalogue contains 200 stars. The magnitudes were derived from the diameters, measured in connection with estimates of visual magnitudes found by Dr. PALISA for the greater part of the stars.

The interval of magnitudes is small. We find from 3 determinations:

$$\pi = 0''.005 \pm 0''.001.$$

20 *Vulpeculae.*

N. G. C. 6885; $\alpha_{1900} = 20^h7^m.6$, $\delta_{1900} = 26°10'$, $b = -5°$, $l = 31°$.

H. SCHULTZ. Micrometrisk bestämning af 104 stjernor inom teleskopiska stjerngruppen 20 Vulpeculae. Kongl. Svenska Vetenskaps-Akademiens Handlingar, Bandet 11, N°. 3, 1873.

The magnitudes have been determined by a photometer in accordance with ARGELANDER's scale.

A. DONNER und O. BACKLUND. Positionen von 140 Sternen des Sternhaufens 20 Vulpeculae nach Ausmessungen photographischer Platten. Bulletin de l'Acad. Imp. des Sciences de St. Pétersbourg, Série V, Volume II, pag. 77-92, 1895.

Measurements of 2 plates taken by DONNER at Helsingfors. The magnitudes were taken from SHILOW.

M. SHILOW. Grössenbestimmung der Sterne im Sternhaufen 20 Vulpeculae. Bulletin etc. ut supra, pp. 243-251.

The magnitudes of the 140 stars, the position of which was determined by DONNER and BACKLUND, were found by measuring the diameters of the images. As standards those 100 magnitudes were used that SCHULTZ had determined already. SHILOW uses CHARLIER's formula $m = x - y \log D - zD$. The probable error of a difference $m - m_{\text{SCHULTZ}}$ is $\pm 0^m.25$.

We have not reduced the magnitudes based on ARGELANDER's scale, to the HARVARD scale, because SHILOW's magnitudes differ considerably from those of SCHULTZ. We find for the parallax from 7 determinations:

$$\pi = 0''.005 \pm 0''.001.$$

Messier 5.

N. G. C. 5904; $\alpha_{1900} = 15^h13^m.5$, $\delta_{1900} = +2°27'$, $b = +45°$, $l = 333°$; class: $C3$.

M. SHILOW. Positionen von 1041 Sternen des Sternhaufens 5 Messier, aus photographischen Aufnahmen abgeleitet. Bulletin de l'Acad. Imp. des Sciences de St. Pétersbourg, Série V, Vol. VIII, pag. 253-312, 1898.

Measurements of 2 plates, taken resp. by BELOPOLSKY and KOSTINSKY. The magnitudes have been determined in a rather inaccurate manner, *viz.* by comparing the diameters with the images of stars of 20 Vulpeculae, the magnitudes of which are known.

S. I. BAILEY. Variable Stars in the Cluster Messier 5, Annals Harvard Observ., Vol. 78, Part. II, 1917.

Ninety-two stars are dealt with. For 72 the period is mentioned.

Among these 3 have long periods. Moreover the magnitudes are given for 25 comparison-stars.

In SHILOW's catalogue the magnitudes of 1006 stars are mentioned. The interval of magnitudes is small and the magnitudes are inaccurate. Nor did we succeed in reducing them to a more exact scale by means of BAILEY's magnitudes. We find the results $\pi = 0''.0002$ and $\pi = 0''.0009$; consequently as average value:

$$\pi = 0''.0005^5 \pm 0''.0002.$$

According to SHAPLEY the average photogr. magnitude of the variable stars is $15^m.25$ and we found $5 \log. \pi = -16.3$; therefore $M = 15^m.25 - 11^m.3 = 4^m.0$. So we get for the mean absolute magnitude of the variable cluster stars 4.0.

If we determine the parallax from the variable stars with a known period, we find, when making use of HERTZSPRUNG's numbers:

$$\pi = 0''.0002.$$

Messier 13.

N. G. C. 6205; $\alpha_{1900} = 16^h 38^m.1$, $\delta_{1900} = 36°39'$, $b = +40°$, $l = 26°$; class: C 3.

J. SCHEINER. Der grosze Sternhaufen im Hercules Messier 13, Abhandl. Kgl. Akad. Berlin 1892.

The catalogue contains 823 stars. The magnitudes are uncertain.

H. LUDENDORFF. Der grosze Sternhaufen im Hercules Messier 13. Publ. Astroph. Observ. Potsdam, Bd. XV, N°. 50, 1905.

This catalogue contains 1118 stars. The brightness is not expressed in magnitudes; but the diameters are estimated in 16 "Helligkeitsstufen".

H. SHAPLEY. Studies etc. Second Paper: Thirteen hundred stars in the Hercules Cluster (Messier 13). Contrib. MT. WILSON Observ. N°. 116, 1915.

The photogr. and photovis. magnitudes of 1300 stars have been determined; but of only 650 stars they have been published. For the statistical investigation 1049 magnitudes and colour indices were used.

We make use of LUDENDORFF's catalogue and we availed ourselves of SHAPLEY's results in reducing the "Helligkeitsstufen" to magnitudes. First we can express the "Stufen" in photographic magnitudes by means of a table in SHAPLEY's work (p. 25, Table VIII) and these may be reduced to photovisual ones by means of the Tables XIV and XVI. No correction is wanted for the difference between the scales of HARVARD and MOUNT WILSON, because the visual Harvard

scale is continued only up to $12^m.0$ and for this magnitude agrees with the MT. WILSON scale.

Now we determine the numbers A_m. For the brightest magnitudes we find then a declivity, which surpasses by far the greatest declivity, found in KAPTEYN's curve. This value, great as it is, may perhaps be explained from the manner, in which the diameters have been reduced to magnitudes. Excluding of these values being undesirable *a priori* and not possible on account of the small interval of magnitudes, we have smoothed the numbers observed by a continuous curve. Then we find from 4 determinations:

$$\pi = 0''.00075 \pm 0''.00006.$$

From SHAPLEY's research (l. c. p. 79) we derive for the mean photographic magnitude of the variable cluster stars which are probably δ Cepheids, $m = 15.2$ and we found $5 \log. \pi = -15.4$, so that according to our determination of the parallax their mean absolute magnitude $= 4.8$ [1]).

From 2 variable stars with known period SHAPLEY (l. c. p. 82) found for the parallax the value:

$$\pi = 0''.00008.$$

Messier 67.

N. G. C. 2682; $\alpha_{1900} = 8^h45^m.8$, $\delta_{1900} = +12°11'$, $b = +34°$, $l = 183°$; class: *D* 2.

E. FAGERHOLM. Ueber den Sternhaufen Messier 67. Inaug. Diss. Upsala, 1906.

The catalogue contains 295 stars. The magnitudes were derived from the diameters by means of CHARLIER's interpolation-formula, after the visual magnitudes of 15 stars had been determined photometrically.

H. SHAPLEY. Studies etc. III. A catalogue of 311 Stars in Messier 67, Contrib. MT. WILSON Observ. N°. 117, 1916.

For all stars the photogr. magnitudes have been determined and also the photovisual ones for all stars within 12′ of the centre. In this way 232 colour indices were found. SHAPLEY finds a much greater number of back-ground stars than would be expected.

OLSSON's catalogue cannot be used on account of the inaccuracy of the magnitudes.

We first make use of FAGERHOLM's catalogue. The magnitudes that are expressed in the *P. D.* scale, are reduced to the Harvard scale by adding a correction $-0^m.2$.

[1]) The values of the parallax and the mean absolute magnitude given here, are to be preferred to the preliminary results published in the first communication.

Now we derive from 2 determinations (the interval being only 2 magnitudes):

$$\pi = 0''.001 \pm 0''0007.$$

According to SHAPLEY (l. c. p. 10) the difference FAG.-MT. W. is constant $= + 0^m.24$ and as HARV. = MT. WILSON photovis., we have also: FAG.-HARV. $= + 0^m.24$. We have taken FAG.-HARV. $= + 0^m.2$, so that the magnitudes used should be correct. Upon closer inquiry, however, the difference FAG.-SHAPLEY appears not to be constant, but to vary with the magnitude. We have determined the errors of FAGERHOLM's scale by comparing the magnitudes of 156 stars, and afterwards we have calculated the numbers A_m for the corrected magnitudes. Now we derive for the parallax from only one determination that can be used:

$$\pi = 0''.002.$$

By telling off SHAPLEY's catalogue we find for the parallax the values $\pi = 0''.001$ and $\pi = 0''.002$. Summing up, we may assume for the parallax of this cluster:

$$\pi = 0''.002.$$

For this cluster SHAPLEY determined the colour indices of *all* the stars, perceptible on the plate within a circle with a radius of 12′. But here, too, no great value can be attached to a comparison of the distribution of colours, found by SHAPLEY for every M, with SCHWARZSCHILD's table. For it is not certain that all stars up to $13^m.0$ are visible on the plate, and just here the separation of cluster stars and back-ground stars offers great difficulties. According to SHAPLEY the distribution of colours, expressed in percentages of the numbers of stars of determined absolute magnitude, is as follows:

Type \ *M*	+ 4.0	+ 3.0
B	0	0
A	0	15
F	38	30
G	51	20
K	11	30
M	0	5

Messier 11.

N. G. C. 6705; $\alpha_{1900} = 18^h\ 45^m.7$, $\delta_{1900} = -6°23'$, $b = -4°$, $l = 355°$; class : $C3$ [1]).

W. STRATONOFF. Amas stellaire de l'écu de Sobieski (Messier 11), Publ. de l'Observ. de Tachkent N°. 1, 1899.

The catalogue contains 861 stars. From the estimates and measurements of diameters the magnitudes have been derived by means of the Southern B. D.

H. SHAPLEY, Studies etc. IV. The galactic cluster Messier 11, Contrib. MT. WILSON Observ. N°. 126, 1916 (A. P. J. Vol. 45, 1917).

For 458 stars the photogr. and photovis. magnitudes have been determined. For statistical research 364 stars were available, after the uncertain magnitudes and the stars upon which the EBERHARD-effect may be of influence had been excluded.

We tell off STRATONOFF's catalogue and we determine the quotients $\frac{A_{m+1}}{A_m}$. It then appears that the magnitudes are too inaccurate and cannot be used.

Now we reduce STRATONOFF's magnitudes to SHAPLEY's scale. In order to do so we compare the magnitudes of 293 stars. The results are given in the table subjoined.

$m_{SHAPLEY}$	Sh—Strat.	Number of comp. stars
10.0	+ 1.53	30
5	1.94	44
11.0	1.68	38
5	1.39	26
12.0	1.45	11
5	1.21	14
13.0	0.91	50
5	0.80	27
14.0	0.70	53

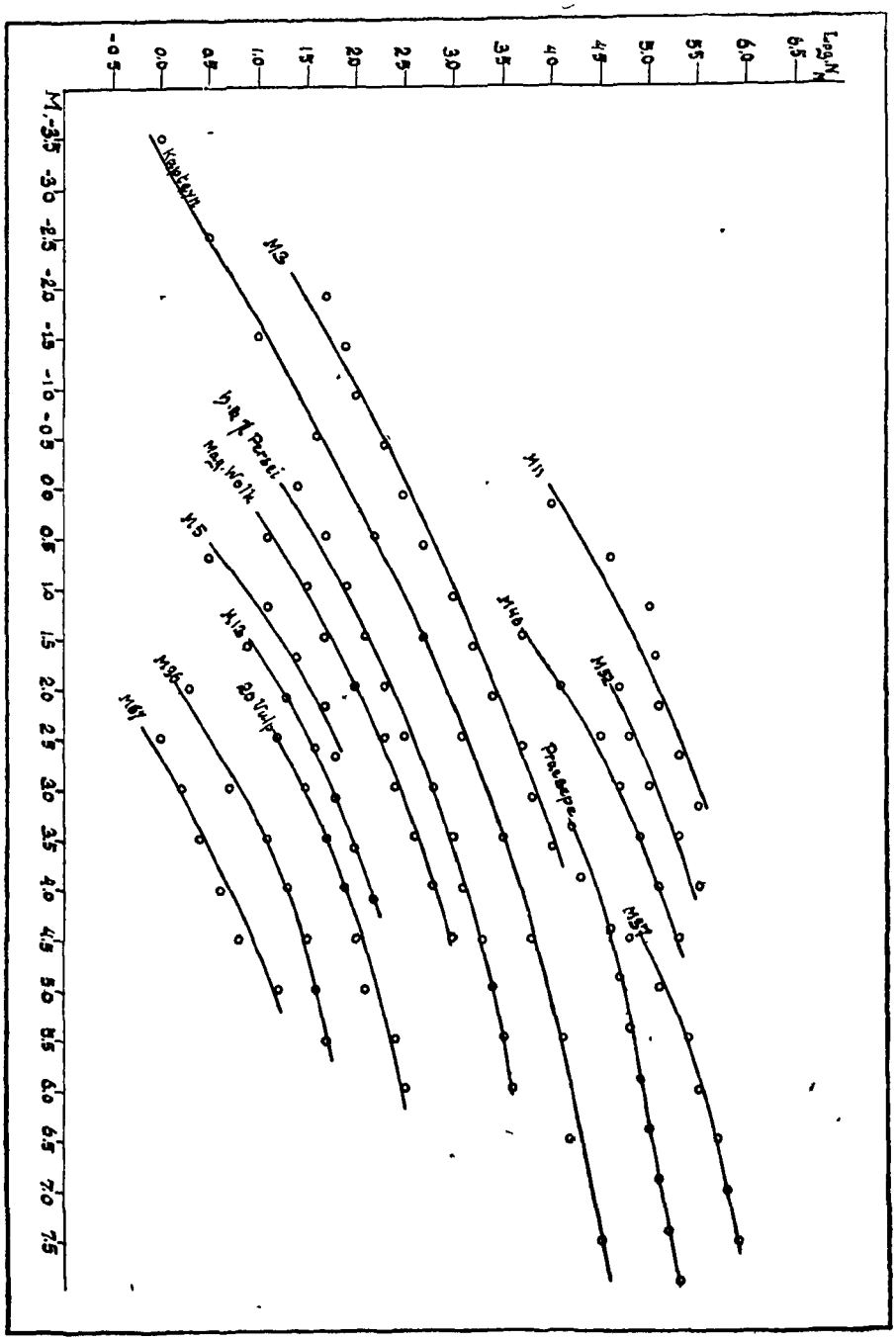
Afterwards we determine by interpolation A_m for the corrected magnitudes. In this way we find for the parallax from 2 determinations:

$$\pi = 0''.00055 \pm 0''.00003$$

[1]) SHAPLEY reckons Messier 11 among the open clusters.

The mean parallax of the globular clusters is 0″.0006 and that of the open clusters (Praesepe excluded) is 0″.003.

The number of parallaxes, determined at present, is still too small to derive conclusions from them as regards the distribution of clusters in space. Perhaps this will be possible, when we shall have extended



our research to more clusters. It will then also be possible to investigate, how far our results give support to the well-known theory of giant and dwarf stars.

From the figure subjoined it is evident that the luminosity curves of the various clusters greatly resemble that found by Prof. KAPTEYN for the stars in the neighbourhood of the sun. And so this method of determining the parallax, proposed by Prof. KAPTEYN, is justified.

In the graphical representation N_M means the number of stars from the brightest star to the absolute magnitude under consideration. As it is only our purpose to compare the relative frequencies of the various absolute magnitudes, we added in each curve a constant amount to log. N_M.

Amsterdam, December 1917.