Astronomy. — "*The Distribution of the absolute Magnitudes among the Stars in and about the Milky Way.*" (First Communication). By Dr. W. J. A. SCHOUTEN. (Communicated by Prof. J. C. KAPTEYN).

(Communicated in the meeting of October 26, 1918).

1. *Introduction.*

One of the most important problems of statistical astronomy is to examine, how in each part of space the stars with bright and faint absolute magnitudes are mixed and of which percentage of stars the luminosity lies between definite limits. Luminosity means the apparent brightness, which a star would have at the unit of distance, and for this unit we will take, in imitation of Professor KAPTEYN, the distance corresponding to an annual parallax $\pi = 0''.1$.

The first determination of the luminosity law was performed by Professor KAPTEYN and published in *Publ. Groningen* N°. 11 [1]). Afterwards several astronomers, employing different methods, have repeated this investigation. Besides the studies of COMSTOCK and WALKEY, who availed themselves of measured parallaxes, and whose results therefore are not much to be relied upon, unless great cautiousness is used, the researches of SEELIGER and SCHWARZSCHILD are well-known. In our dissertation [2]) we have discussed the three methods mentioned, and we have compared the results that were found. It appeared that serious objections may be raised against SEELIGER's investigation, so that we cannot attach much value to the frequency-function of absolute magnitudes found by him. We think we have also demonstrated in our work just cited that the method of KAPTEYN is for various reasons greatly to be preferred to that of SCHWARZSCHILD.

After we had finished this inquiry an earnest wish arose in us to establish the luminosity law according to the method which we think the best. It was known to us, that such a determination had been

1) Also in these Proceedings, Vol. III, p. 658.

2) On the Determination of the Principal Laws of Statistical Astronomy. Amsterdam, Kirchner 1918.

in course of preparation at the Astronomical Laboratory at Groningen for a long time already.

In this preparation are included the countings of stars of determined magnitude which are published in *Publ. Groningen* N^os^. 18 and 27, and the mean parallaxes of stars of determined magnitude and galactic latitude, which will soon be published and have been kindly lent us for our use by Prof. KAPTEYN and Dr. VAN RHIJN.

It is a matter of course that the preliminary results obtained by us, should be replaced by those of the definitive solution as soon as these are available. Yet we thought we might publish our results as they are partly based on other data, because they give a notion of the exactness that is to be obtained now already, and our preliminary results might be of some service perhaps until the time that the definitive shall have appeared.

In this communication we mention the results that we found, when we tried to determine the luminosity law according to KAPTEYN's method for the whole sky and for five zones of different galactic latitude.

We intend to publish in a second communication the results that are found, when SCHWARZSCHILD's method is applied to the same data. At the same time we hope to compare our results with those of other investigators.

In this article we also gratefully tender our sincere thanks to Prof. KAPTEYN for his kind help, which favoured our investigation.

2. *The Data of Observation.*

In applying KAPTEYN's method we have to take from the observations the following data:

1. the numbers $N_{m,\mu}$ i.e. the numbers of stars of determined apparent magnitude and proper motion,

2. the mean parallaxes $\pi_{m,\mu}$ of stars of determined magnitude and proper motion,

3. the value of ϱ, the probable error of the error curve log. $^{\pi}/_{\pi_0}$ in which π is the real and π_0 the probable parallax.

In our investigation we divided the sky into 5 zones. The galactic latitude we shall indicate by b.

Zone I = part of the sky between $b = -10°$ and $b = +10°$.
Zone II = „ „ „ „ with b from $-10°$ to $-30°$ and $+10°$ to $+30°$.
Zone III = „ „ „ „ „ b „ $-30°$ „ $-50°$ „ $+30°$ „ $+50°$.
Zone IV = „ „ „ „ „ b „ $-50°$ „ $-70°$ „ $+50°$ „ $+70°$.
Zone V = „ „ „ „ „ b „ $-70°$ „ $-90°$ „ $+70°$ „ $+90°$.

We have derived the numbers $N_{m,\mu}$ from the numbers of stars of determined magnitude and galactic latitude by performing countings in catalogues of stars with known proper motions. We have computed these numbers N_m for the different zones from the tables in *Publ. Groningen* N°. 18, as these have been corrected in *Publ. Groningen* N°. 27.

The results that VAN RHIJN published in the work last-mentioned, are very reliable, as they were confirmed by several independent studies. The results of NORT and SEARES, indeed, agree fairly well with those of VAN RHIJN and the numbers of CHAPMAN and MELOTTE too correspond to the Groningen countings as soon as they shall have been corrected for a mistake in the method of reduction which has been pointed out by VAN RHIJN.

In our countings the stars were divided according to their magnitudes into groups of 1—2.9, 3.0—3.9,4.0—4.9, ... 12.0—12.9. According to their proper motions they were counted between the limits 0—2″.9, 3″.0—4″.9, 5″.0—7″.9, 8″.0—9″.9, 10″.0—14″.9, 15″.0—19″.9, 20″.0—29″.9, 30″.0—49″.9 and >49″.9.

The following catalogues have been used:

1. L. BOSS. Preliminary Catalogue of 6188 Stars, 1910.

2. A. AUWERS. Catalog der Astronomischen Gesellschaft, Zone + 15° bis + 20°, 1896.

3. W. G. THACKERAY. Greenwich 1910 Catalogue of Stars, Zone + 24°0′ to + 32°0′. Analysis of Number and Percentages of Proper Motions. *Monthly Notices* **77**, 204—212, 1917.

F. W. DYSON and W. G. THACKERAY. The Systematic Motions of the Stars between Dec. + 24° and Dec. + 32°. *Monthly Notices* **77**, 581—596, 1917.

4. F. W. DYSON and W. G. THACKERAY. New Reduction of GROOMBRIDGE's Catalogue of Circumpolar Stars, 1905.

5. J. C. KAPTEYN and W. DE SITTER. The Proper Motions of 3300 Stars of different Galactic Latitudes, derived from photographic plates, prepared by Prof. ANDERS DONNER. *Publ. Groningen* N°. 19, 1908.

6. J. C. KAPTEYN. The Proper Motions of 3714 Stars derived from plates taken at the observatories of Helsingfors and the Cape of Good Hope. With the co-operation of Dr. H. A. WEERSMA. *Publ. Groningen* N°. 25, 1914.

7. Dr. A. A. RAMBAUT. A photographic Determination of the Proper Motion of 250 Stars in the Neighbourhood of Σ 443. *Monthly Notices* **73**, 616—630, 1913.

8. A. VAN MAANEN. Remarks on the Motion of the Stars in and near the double Cluster in Perseus. *Report of the Nineteenth Meeting*

of the American Astronomical Society. Popular Astronomy **25**, 108—110, 1917.

9. W. G. THACKERAY. Notes on some Proper Motions derived from a Comparison of CARRINGTON'S Catalogue with the Greenwich Places for 1900. *Monthly Notices* **67**, 146—148, 1906.

Boss' catalogue was only used for stars brighter than $5^m.8$.

AUWERS' catalogue was only used to determine $N_{m,\mu}$ for the whole sky.

The proper motions that have been derived from the Cape plates in *Publ. Groningen* N°. 25, are not very accurate. The plates had not been originally destined for deriving proper motions from them, and they had been measured absolutely. We have, therefore, in determining the mean values attached but very little weight to them.

No corrections have been applied in order to reduce our results for the different catalogues to the same scale of magnitudes or to one fundamental system or in order to correct the proper motions for errors of observation. These corrections may in our opinion be neglected, considering the comparative inaccuracy of the numbers. Moreover, we have always expressed the numbers $N_{m,\mu}$ for every magnitude in percentages of the numbers N_m and these percentages appeared to vary only little with the magnitudes.

Our countings in determining $N_{m,\mu}$ for the whole sky include 38818 proper motions, while moreover in the five zones resp. 8273, 10857, 6981, 3144 and 1488 stars were counted.

Now we are able to determine the numbers of stars of determined magnitude and proper motion. The results we found for the whole sky, are mentioned in table 1.

In our further research we did not use the numbers of stars with $\mu > 50''$. It is very difficult to determine these with sufficient accuracy from the data of known proper motions, so scanty as yet. It will appear that in consequence of this limitation we could not extend the luminosity curve found by us to the faintest stars.

The numbers $N_{m,\mu}$ now being known for the different zones, we may also examine how the stars with determined proper motions are distributed over the sky with reference to the Milky Way. In order to do so we have calculated the numbers of stars with P.M. resp. $> 10''$, $5''$, $3''$ and $0''$ for every magnitude per 100 square degrees. It appeared that the numbers of stars with $\mu > 5''$ do not evince any galactic condensation, except perhaps for stars, fainter than $9^m.0$. The numbers with $\mu > 3''$ very clearly show a condensation for all magnitudes in the direction of the Milky Way, although to a less high degree than the numbers of stars with $\mu > 0''$.

TABLE 1. The numbers of stars of determined magnitude and proper motion for the whole sky.

m \ μ	0—2.9	3.0—4.9	5.0—7.9	8.0—9.9	10.0—14.9	15.0—19.9	20.0—29.9	30.0—49.9	> 49.9	Total
1—2.9	31	26	13	8	14	6	13	10	11	132
3.0—3.9	48	41	39	19	39	18	28	14	17	263
4.0—4.9	303	167	128	50	113	57	39	29	21	907
5.0—5.9	1090	665	575	222	329	140	117	72	52	3262
6.0—6.9	4019	2675	1528	564	728	349	185	123	82	10253
7.0—7.9	14664	8268	4150	1092	1716	530	374	281	125	31200
8.0—8.9	44996	27517	11992	2417	3718	930	744	464	186	92964
9.0—9.9	108257	78185	32478	8420	9142	2165	1203	480	241	240571
10.0—10.9	270748	172763	136019	29009	29653	2579	2579	645	645	644640
11.0—11.9	705391	436968	288711	74909	43697	4681	3121	1561	1561	1560600

This result agrees with that found by KAPTEYN in 1893 and with the conclusions, which DYSON and THACKERAY found lately from a discussion of the Greenwich 1910 catalogue.

The mean parallaxes $\pi_{m,\mu}$ for the whole sky and for the different galactic zones have not been determined by us directly. In our opinion they may be reduced for our investigation, preliminary as it is, with sufficient accuracy from former researches.

KAPTEYN found in *Publ. Groningen* N°. 8 for these mean parallaxes

$$\pi_{m,\mu} = a\, \mu^b\, \varepsilon^{m-5.0}$$

in which $a = 0''.0038$, $b = 0.71$ and $\varepsilon = 0.905$.

Use was made of the value 16.7 Kilometers a second for the velocity of the sun. If we accept the modern value, 19.5 Kilometers a second, the mean parallaxes π_m, used by KAPTEYN in deducing his formula, are lessened by 14 %. If we now suppose — as VAN RHIJN also does [1]) — that this correction for the mean parallaxes π_m changes only the value of a in $\pi_{m,\mu}$, we find $a = 0''.0032^5$.

KAPTEYN's parallaxes corrected in this way agree with the values, deduced by Dr. VAN RHIJN in his dissertation in an investigation, based on other and more modern data than those used by KAPTEYN, in which he partly also applied another method. Hence it follows that the parallaxes $\pi_{m,\mu}$, as given in *Publ. Groningen* N°. 8, are very reliable.

In order to derive from the parallaxes which may be applied to the whole sky, the $\pi_{m,\mu}$ for the different zones, we have assumed, that the $\pi_{m,\mu}$ of the various galactic zones are related in the same way as the π_m of these zones.

This assumption is rather arbitrary. Most probably it cannot be proved to be correct in all respects. There is reason to believe, however, that the error, made in this way, is not very great. Therefore we thought we might use this hypothesis in our preliminary research. We have already mentioned that we were able to use, thanks to the great kindness of Professor KAPTEYN and Dr. VAN RHIJN, the mean parallaxes of stars of determined magnitude and galactic latitude, which will soon be published in *Publ. Groningen* N°. 29. These parallaxes may be respresented by the following formulae:

Zone I	$\log. \pi_m$	$= 8.883 - 0.142\, m$
II		$= 8.904 - 0.142\, m$
III		$= 8.957 - 0.142\, m$
IV		$= 9.024 - 0.142\, m$
V		$= 9.066 - 0.142\, m$
Whole sky		$= 8.943 - 0.142\, m$

[1]) Diss. Groningen 1915, p. 35.

We have calculated — if we call the parallaxes, used formerly by KAPTEYN, $\pi_{m_{\text{KAPTEYN}}}$, and if for convenience' sake we indicate those meant above by $\pi_{m_{\text{VAN RHIJN}}}$ — the $\pi_{m,\mu}$ for the whole sky and for the 5 zones by multiplying the numbers of table G of *Publ.*

TABLE 2.

The mean parallaxes of stars of determined magnitude and proper motion for the whole sky.

m \ μ	1.5	4.0	6.5	9.0	12.5	17.5	25	40
2.6	0″.0065	0″.0132	0″.0187	0″.0237	0″.030	0″.037	0″.049	0″.068
3.6	.0057	.0115	.0163	.0206	.026	.033	.042	.059
4.6	.0049	.0100	.0142	.0178	.023	.029	.037	.051
5.6	.0043	.0087	.0123	.0155	.020	.025	.033	.045
6.6	.0037	.0076	.0107	.0135	.017	.022	.028	.039
7.6	.0033	.0066	.0093	.0117	.014	.019	.025	034
8.6	.0029	.0057	.0081	.0103	.012	.016	.022	.029
9.6	.0025	.0050	.0071	.0090	.011	.014	.019	.026
10.6	.0022	.0043	.0061	.0077	.009	.012	.016	.023
11.6	.0019	.0036	.0051	.0065	.008	.010	.014	.020

Groningen N°. 8 by a factor $\frac{\pi_{m_{\text{VAN RHIJN}}}}{\pi_{m_{\text{KAPTEYN}}}}$. The parallaxes, which may be applied to the whole sky, are given in table 2.

We ourselves have not tried to determine once more the probable deviation of the error curve log. π/π_0. Moreover we have supposed that ϱ does not vary with the galactic latitude. We made use of the value 0.19, found by KAPTEYN both in our solution for the whole sky and in that for the 5 galactic zones. It has, however, been proved in *Publ. Groningen* N°. 11 that the value of ϱ has only little influence on the result.

3. *Applying* KAPTEYN's *Method.*

We have applied KAPTEYN's method without any modification. For a description in detail of this method we refer to his treatise in *Publ. Groningen* N°. 11, which we cited already above. We shall limit ourselves to a short discussion of the hypotheses that have been made and an explanation of the tables mentioned in this essay.

The hypotheses, made in KAPTEYN's investigations, are three in number:

1stly the density is only a function of r;

2ndly the luminosity curve is independent of the distance from the sun and there is no absorption of light in space;

3dly the quantities $z = \log. \pi/\pi_0$ are distributed according to the law of errors.

The first hypothesis is necessary if we want to derive frequency-functions that may be applied to the whole sky. We seek for mean values for the unknown quantities and so we cannot take into account the variations in the values with the galactic latitude and longitude. KAPTEYN's method may, however, be used just as well, if we want to reckon with the influence of the galactic latitude by making separate solutions for the different galactic zones.

The second hypothesis can hardly be dispensed with. If we have certainty that the frequency-function of absolute magnitudes is everywhere the same in space, KAPTEYN's method offers the means to examine, whether there is a perceptible extinction of light and, on the other hand, when we know that there is no absorption we can examine if the luminosity curve varies with the distance from the sun. As neither one thing nor the other, however, is certain, we are obliged for the time being to make the supposition in question.

If we establish the frequency-function for the different galactic zones separately, this difficulty is diminished considerably. We can then compare the luminosity curves, found for the various zones. If now, it appears that these curves correspond, it is highly improbable that the distribution of luminosities varies with the distance to the sun. If the luminosity law depends on r and not on b, we should find a fan-shaped composition of the sidereal system, which is not inconceivable, but highly improbable all the same.

If we, may assume that the distribution of the luminosity is independent of the distance to the sun, KAPTEYN's method enables us to determine the absorption, or at least to examine if it has any influence on the distribution of stars in space which we found.

The third hypothesis was, indeed, when KAPTEYN made it for the first time a rather arbitrary assumption, and it must be conceded that other results would have been found, if another hypothesis had been made. The hypothesis was made, because at that time there were still too few measured parallaxes, to enable us to deduce the form of the frequency curve $log.\ ^{\pi}/_{\pi_0}$ directly from the data. Perhaps this will be possible when applying the method once more. It is, however, of little importance to discuss in the present stage the question whether this hypothesis could be justified or not, as SCHWARZSCHILD has proved that the above-mentioned relation exists for a special form of the density law and the luminosity law, which form obtained a great amount of probability owing also to his investigations.

We have deduced the luminosity law from the data of observation mentioned in § 2. Space was divided into a number of shells the radii of which had been selected for convenience' sake in such a way that $log.\,r$ increases with 0.2. Afterwards the mean parallax was determined for each of the numbers $N_{m,\mu}$ that had been found by means of the formula for $\pi_{m,\mu}$. Then with the aid of the value found for the probable deviation of the error curve $log.\ ^{\pi}/_{\pi_0}$, it was calculated which part of the numbers found occur in every shell.

The numbers $N_{m,\mu}$ which we found for the whole sky are given in table 1, and the corresponding parallaxes $\pi_{m,\mu}$ in table 2. In table 3 we have mentioned, how these numbers of stars are divided over the various shells. Tables in accordance with this we have calculated for the five zones.

Now we have derived from these tables others, indicating the numbers of stars of every absolute magnitude per unit of volume in the different shells. For the whole sky we communicated our results in table 4. Between the fat-faced lines the numbers

TABLE 3. The number of parallaxes for the whole sky.

π	Mag. / $\bar{\pi}$	2.6	3.6	4.6	5.6	6.6	7.6	8.6	9.6	10.6	11.6	Total
0″.00000—0″.00100		0	1	6	33	228	83	4325	17045	55568	212210	289499
.00100— .00158	0″.00118	1	2	20	94	469	3339	9806	28249	90900	245902	378782
.00158— .00251	.00187	3	6	55	247	1170	4603	15720	45170	112684	341132	520790
.00251— .00398	.00296	7	14	99	435	1781	6153	20643	52073	119250	328492	528947
.00398— .00631	.00469	14	25	147	579	1957	6700	18139	46380	139905	234121	447967
.00631— .0100	.00743	16	38	150	597	1780	4721	12972	28374	77002	133400	259050
.0100 — .0158	.0118	24	47	146	503	1430	3155	6896	15180	34288	43900	105569
.0158 — .0251	.0187	21	46	121	389	806	1030	2715	5702	11082	17370	39282
.0251 — .0398	.0296	15	26	65	171	358	678	1222	1752	2806	2247	9340
.0398 — .0631	.0469	11	28	56	117	147	553	267	347	465	202	2193
.0631 — .1000	.0743	6	10	17	27	37	47	62	53	45	47	351
.1000 — .158	.118	2	2	4	7	7	10	11	5	0	16	64
> .158	.204	1	1	0	1	1	3	0	0	0	0	7
		121	246	886	3200	10171	31075	92778	240330	643995	1559039	2581841

TABLE 4. The log. of the number of stars per unit of volume.

$\bar{\pi}$ \ M	—7.0	—6.0	—5.0	—4.0	—3.0	—2.0	—1.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
0″.00118	3.503	3.804	4.804	5.476	6.274	7.027	7.494	7.954	8.462	8.894										
.00187		4.580	4.881	5.843	6.496	7.171	7.766	8.299	8.758	9.145	9.636									
.00296			5.548	5.849	6.703	7.341	7.954	8.492	9.018	9.420	9.779	0.220								
.00469				6.449	6.701	7.470	8.066	8.595	9.129	9.562	9.969	0.449	0.672							
.00743					7.107	7.483	8.079	8.679	9.153	9.577	0.016	0.356	0.797	1.028						
.0118						7.883	8.175	8.667	9.205	9.658	0.002	0.342	0.684	1.038	1.145					
.0187							8.425	8.766	9.186	9.693	0 009	0.116	0.537	0.859	1.148	1 343				
.0296								8.879	9.118	9.516	9.936	0.257	0.534	0.790	0.947	1.151	1.055			
.0469									9.344	9.750	0.051	0.371	0.470	1.046	0.730	0.843	0.970	0.608		
.0743										9.681	9.903	0.133	0.334	0.471	0.575	0.695	0.627	0.556	0.575	
.118											9.804	0.804	0.105	0.348	0.348	0 503	0.544	0.202	—	0 707
.204												9.979	9.979	—	9.979	9.979	0.456	—	—	—

of stars of the apparent magnitudes 3.0 to 10.0 are found. These are most reliable.

The densities in the different shells were determined by comparing each time in two successive shells the numbers of stars of a determined absolute magnitude. The relative density of two successive shells was found as the average of three determinations, based respectively on the stars of the apparent magnitudes from 3.0 to 8.0, 3.0 to 9.0 and 3.0 to 10.0. The density in the fifth shell, *viz.* the one for which the mean parallax is 0".0296, was supposed to be *one*, after which the density at every distance from the sun, expressed in this unity, is known.

The average density for the whole sky varies about in the same way with the distance to the sun as KAPTEYN found. In zone I the density at great distances is considerably more than the average, in zones III, IV and V on the other hand it is much less.

By means of the densities found we now calculated from table 4 and the similar ones for the galactic zones the number of stars per unit of volume, after the density had been reduced to the one for $\pi = 0''.0296$. In order to do so each number in the last-mentioned tables was diminished by the logarithm of the density of the shell pertaining to it. In this way for instance table 5 has been calculated from table 4.

From the tables that were found last of all, the luminosity curves for the whole sky and the 5 zones may be deduced at once. The numbers, standing between the fat lines in each column, correspond pretty well.

In table 5 we took the averages of these numbers (not of the logarithms), and noted down the logarithm of these averages in the last line of each table. In taking the mean equal weight was ascribed to all numbers, except to those of the first four shells. These numbers are not very reliable, because they are small, but especially because we were obliged to exclude from our investigation stars with a proper motion $> 50''$ per century. Consequently the luminosity curves that have been found are only of value up to $M = 5.0$.

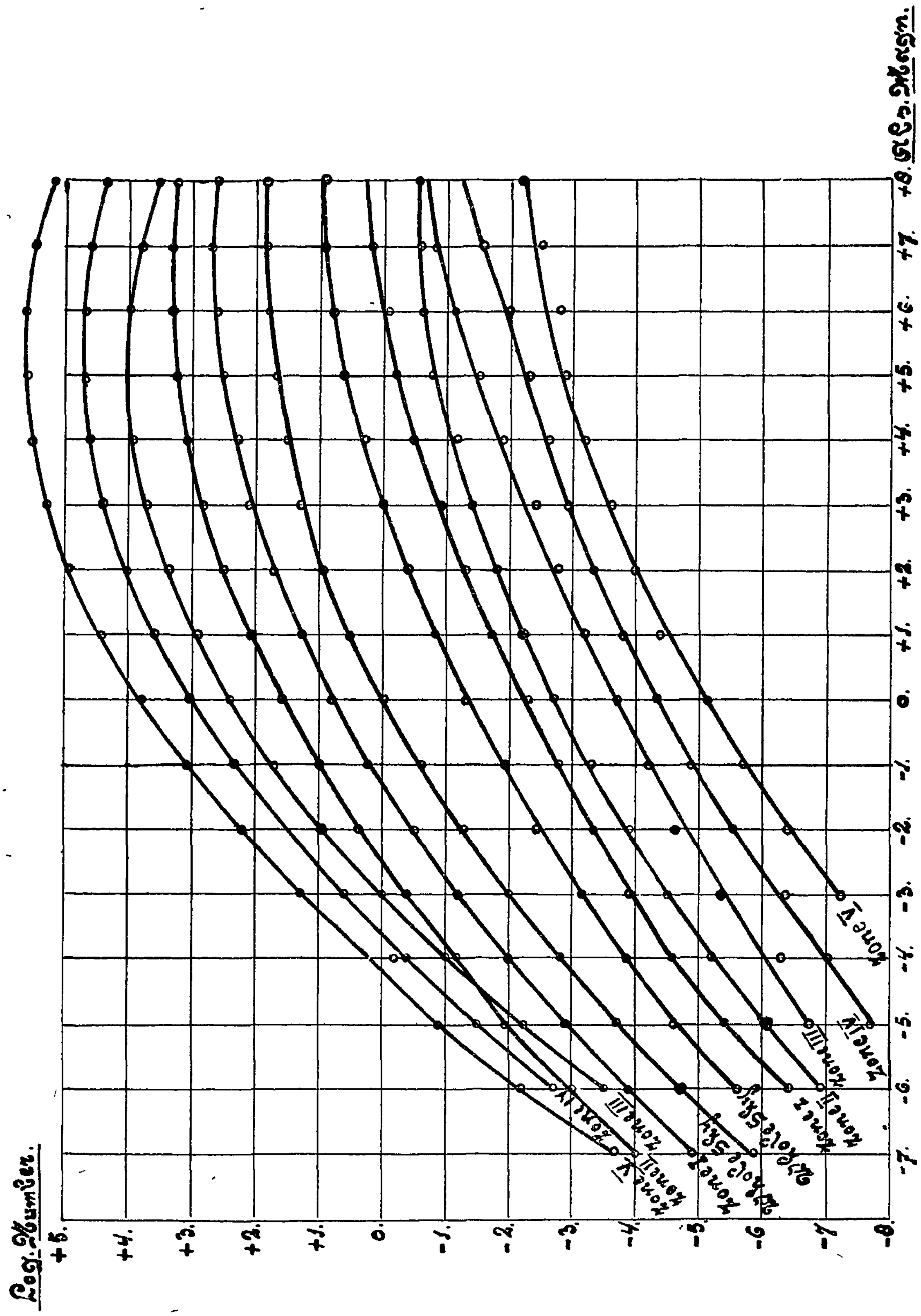
It is of interest to point out that our result for the whole sky corresponds beautifully to KAPTEYN's found in *Publ. Groningen* N°. 11.

Furthermore it is remarkable that the curves found for the various zones differ only a little. We already observed that from this we may conclude with a certain probability that the luminosity curve does not change with the distance to the sun either.

In table 5 and the corresponding tables for the 5 galactic zones

TABLE 5. The log. of the number of stars per unit of volume, reduced to $\pi = 0''.0296$.

M / $\bar{\pi}$	—7.0	—6.0	—5.0	—4.0	—3.0	—2.0	—1.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
0″.00118	4.141	4.442	5.442	6.114	6.912	7.665	8.132	8.592	9.100	9.532										
.00187		4.976	5.277	6.239	6.892	7.567	8.162	8.695	9.154	9.541	0.032									
.00296			5.741	6.042	6.896	7.534	8.147	8.685	9.211	9.613	9.972	0.413								
.00469				6.527	6.779	7.548	8.144	8.573	9.207	9.634	0.047	0.527	0.750							
.00743					7.153	7.529	8.125	8.725	9.159	9.623	0.062	0.402	0.843	1.074						
.0118						7.902	8.194	8.686	9.224	9.677	0.021	0.361	0.703	1.057	1.164					
.0187							8.426	8.767	9.187	9.694	0.010	0.117	0.538	0.860	1.149	1.344				
.0296								8.879	9.118	9.516	9.936	0.257	0.534	0.790	0.947	1.151	1.055			
.0469									9.269	9.675	9.976	0.296	0.395	0.971	0.655	0.768	0.895	0.533		
.0743										9.932	9.846	0.384	0.585	0.722	0.826	0.946	0.878	0.807	0.826	
.118											0.249	0.249	0.650	0.793	0.793	0.948	0.989	0.647	—	1.152
.204												0.712	0.712	—	0.712	0.712	1.189	—	—	—
Mean	4.141 :	4.442	5.367	6.140	6.873	7.572	8.146	8.679	9.182	9.629	0.017	0.299	0.600	0.827	0 947	0.870 :	1.038 :	0.647 :		


Log. Number.
+5. +4. +3. +2. +1. 0. -1. -2. -3. -4. -5. -6. -7. -8.
Abs. Magn.
+8. +7. +6. +5. +4. +3. +2. +1. 0. -1. -2. -3. -4. -5. -6. -7.
Zone V
Zone IV
Zone III
Zone II
Zone I

the numbers in each column, as has been already observed, agree with each other. If our conclusion that the frequency curve of the absolute magnitudes does not vary with r, is right, we may derive from the agreement of the luminosity curves in the different shells that there is no perceptible absorption of light in space.

In figure 1 the frequency curves that were found have been drawn. The six lines in the lowest part of the figure refer to the determination of the luminosity curve, discussed in this essay. The six lines in the upper part refer to an application of SCHWARZSCHILD's method to the same data, which will be explained in a following communication.

The line representing our determination of the luminosity curve for the whole sky, indicates the logarithm of the number of stars of every M per unit of volume in the neighbourhood of the sun. For the other curves we added, in order to make comparison possible, a constant amount to each number.

Amsterdam, June 1918.