Physics. — "*On the use of the audion in wireless telegraphy*". By D. COSTER. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of March 29 1919).

In the recent successes in wireless telegraphy the three-electrode-relais or audion has played the most important part. The audion consists of a vacuum tube, in which are fused three electrodes: a hot wire-kathode k, a usually flat anode a and a third auxiliary electrode h, placed between the other two and consisting of a few parallel and mutually connected metal wires, which is therefore called the grid. The properties of the audion are determined by the audion-characteristics, which give the relation of the currents i_a and i_h on the one side and the potentials e and v on the other. (See fig. 1). The current i_h is usually very small compared to i_a and it may be neglected in many cases. A simple scheme for the determination of the characteristics is given by fig. 1, where for the sake of simplicity the measuring instruments are not indicated. Fig. 2 gives i_a as a function of e, for different values of v. The different characteristics may be deduced from one another by parallel translation. Fig. 3 gives i_a as a function of v, while e is a constant.

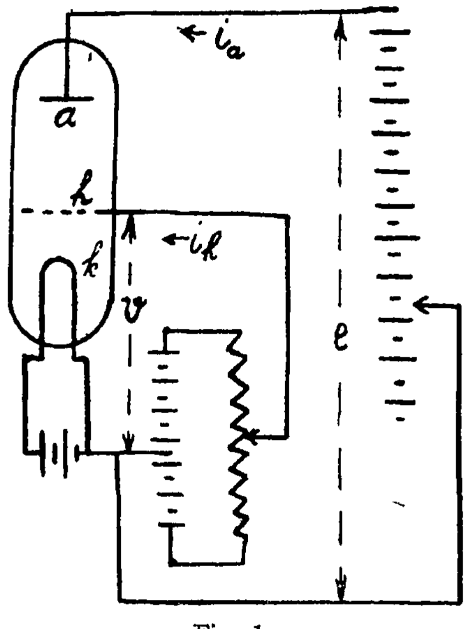

Fig. 1.

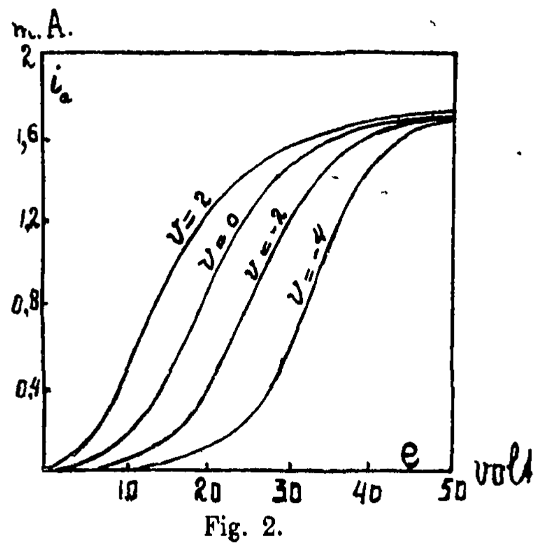

Fig. 2.

These characteristics are similar to those of fig. 2, as a rule however they are steeper. Figs. 2 and 3 give the essential features of the audion-characteristics; the different forms of audions show more or less important deviations.

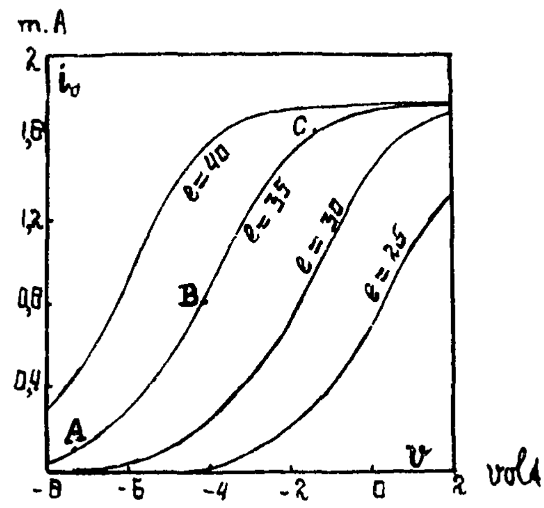

Fig. 3.

Though the character of the gas and the degree of its rarefaction are very important in the determination of the individual properties of the audion, they are problably not of essential signification. At any rate LANGMUIR [1]) has succeeded in constructing a normally functioning three-electrode-relais, which he calls pliotron, from which every trace of gas seems to have been removed. In the following discussion we may therefore assume that the electric conduction in the audion is exclusively performed by the thermoions.

For the number N of electrons, which in the unit of time enter the vacuum from the hot wire, RICHARDSON found the well-known formula:

$$N = aT^{\lambda} e^{-\frac{b}{T}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

here T is the absolute temperature of the hot wire, a and b are constants, λ is a quantity which differs but little from unity. The emerging electrons may be caught on an anode opposite the kathode; N is then determined by a current-measurement. When T is constant, the current increases at first with increasing potential-difference. It is only the maximum current, "the saturation current", which in its dependence on the temperature follows RICHARDSON's formula.

To explain this initial increase of the current with the impressed voltage LANGMUIR [2]) gave his theory of the space-charge. The electrons in the field between the kathode and the anode diminish the potential-gradient in the neighbourhood of the kathode. When this

[1]) LANGMUIR: General Electr. Rev. (1915) p. 327.
See also HUND: Jahrb. f. Drahtl. Tel. (1916) 10 p. 521.

[2]) Phys. Rev. (1913) p. 457.

gradient is zero part of the emerged electrons may fall back on the kathode. This theory led LANGMUIR to the formula[1]):

$$i = CV^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here i is the current as long as it remains below the saturation current, V is the impressed voltage, C is a constant which depends on the form and the distance of the electrodes. In the neighbourhood of saturation, i approaches a constant value.

This relation between the thermoionic current and the impressed voltage will be found in the characteristics of fig. 2 and 3. A complication is here caused by the presence of the third electrode, the grid, about which we may make the following observations. As a rule the audion is used with tensions of such values that we should have a saturation-current, if no grid were used. It is the function of the grid to retard more or less the electrons emitted by the kathode. The potential of the grid is therefore always chosen lower than the potential which we should have at that place, if the grid were removed. Usually the grid-potential is not much different from the average kathode-potential, in many cases it is even a little lower. Of the electrons, which reach the plane which we can draw through the grid, by far the greater part will escape between the grid-wires to the anode and but few will strike the grid. The usually small surface of the wires also contributes to this effect. Thus the grid current i_h (see fig. 1) is in normal working conditions small as compared to the anode-current i_a. The latter not only depends on the anode-potential e, but also on the grid-potential v. We cannot go far wrong in taking as "driving force" of i_a the mean potential in the plane of the grid. Denoting by φ this mean potential, φ is, as long as the anode-current has not yet reached its maximum, a linear function of e and v

$$\varphi = \alpha e + \beta v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

At first therefore we get for the anode-current the following relation:

$$i_a = C\,(\alpha e + \beta v)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

An analogous empirical formula is given by LANGMUIR[1]) for his pliotron.

In wireless telegraphy the audion is used as rectifier and as

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[1]) An analogous formula had been given before by CHILD for the transport of pos. ions. See Phys. Rev. (1911) p. 492.

[1]) See HUND: Jahrbuch f. Dr. Tel. (1916) 10 p. 521.

amplifier. Following the characteristics of fig. 3 we can easily check these functions. When the instantaneous condition of the audion is given by the point B on one of the characteristics, we see that small fluctuations of the grid-potential involve relatively large alterations of the anode-current. Owing to the linearity of the characteristics in the neighbourhood of B, these current-fluctuations are proportional to the variations of the grid-potential. Here it is of great importance, that the grid itself receives but little current "it reacts upon tensions"; very small energies are therefore sufficient to cause the alterations of the grid-potential. At the points A and C the audion acts at the same time as rectifier: to equal potential-variations in positive and negative sense correspond different current-variations.

Of late years it has been found that the audion can perform yet a third function. By a suitable arrangement we can form an unstable system, which gives rise to alternating currents of definite frequency, as in the so-called musical arc. Furthermore it has been found that the unstable connections increase to a high degree the amplifying action, so-called "back-coupling". For a good insight into the use of the audion in wireless telegraphy it is of importance to understand its generative action.

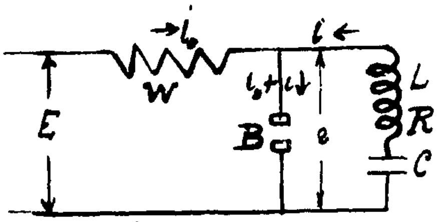

Fig. 4.

The question of the instability of electrodynamical systems mechanically at rest has been studied among others by SIMON and his pupils[1]). These investigators have succeeded in establishing some general rules which are easily obtained by the aid of a simple diagram (see fig. 4).

E is a constant electromotive force, W is a resistance which is so large, that compared with it the variable resistance of the arc B is negligible, L is a selfinduction, whose resistance is R, C is a capacity. Owing to the assumption with regard to W, the current i_0 may be considered as a constant. We assume that the arc-tension is only a function of the arc-current $i_0 + i$, which is linear for small values of i. For this problem we therefore arrive at linear differential equations, whose general solution consists in a "continuous-current-solution" and an "alternating-current-solution", which may be considered independently of each other. Thus the tension e may

[1]) Phys. Zeitschr. (1902) III p. 282.
" " (1903) IV p. 366 and p. 737.
See also Jahrb. f. Dr. Tel. (1918) 1 p. 16.

be represented by a constant tension e_0 augmented by an alternating tension e_1; for the current the division into i_0 and i has already been made.

For the oscillation-circuit, consisting of arc, selfinduction and capacity we have:

$$e + Ri_1 + L\frac{di_1}{dt} + \int\frac{i_1 dt}{C} = 0, \quad \ldots \ldots \quad (5)$$

giving by differentiation:

$$L\frac{d^2 i_1}{dt^2} + \left(R + \frac{\partial e}{\partial i_1}\right)\frac{di_1}{dt} + \frac{i_1}{C} = 0 \quad \ldots \ldots \quad (6)$$

Hence it follows that continual alternating currents can only exist, if:

$$\frac{\partial e}{\partial i_1} = -R \quad \ldots \ldots \ldots \quad (7)$$

It is thus necessary that an increase in the current involves a decrease of tension and inversely, in other words: the condition for the generation of alternating currents is a falling arc-characteristic. For the alternating current the arc behaves as a "negative resistance" hence the quantity $-e$ may be considered as the electromotive force for the alternating current.

Applying this to the audion, we see from the characteristics of fig. 2, that for a *constant* gridpotential, it has a *rising* characteristic, hence it is *stable*. Only by coupling the grid to the oscillation-circuit it is possible to make the system *unstable*.

The anode current i [1]) is a function of the anode-potential e and the grid-potential v;

$$i = f(e, v) \quad \ldots \ldots \ldots \quad (8)$$

If here also we assume a linear relation between current and tensions, the general solution of the differential equations for this system consists of the sum of a "continuous-current-solution" i_0, e_0, v_0 and an "alternating-current-solution" i_1, e_1, v_1. Here again the quantity $-e_1$ is to be considered as the electromotive force for the alternating current.

From (8) it follows that

$$i_1 = \frac{\partial f}{\partial e} e_1 + \frac{\partial f}{\partial v} v_1 \quad \ldots \ldots \ldots \quad (9)$$

Putting $\dfrac{\partial f}{\partial e} = \dfrac{1}{r}$ and $\dfrac{\dfrac{\partial f}{\partial v}}{\dfrac{\partial f}{\partial e}} = \lambda$, this becomes:

1) For the sake of convenience we henceforth leave out the index a.

$$-e_1 = \lambda v_1 - r i_1 \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

From (10) we see in connection with the above discussion that, if we are at the proper point of the characteristic (e.g. B in fig. 3) and the grid is subjected to potential-fluctuations v_1, we may consider the audion as an alternating current-generator with an electromotive force λv_1 and an internal resistance r. The potential-fluctuations of the grid may be caused by an external electromotive force. But they may also be produced by coupling the grid in a proper manner to the anode-circuit, by which an oscillation when once arising, maintains itself. Both methods find manifold application in wireless telegraphy. The second method will be especially discussed here. In doing so we shall make use of the method of "complex resistances", which is customary in alternating-current-theory; the following remarks on this method may be useful.

We suppose an arbitrary electric system, consisting of self-inductions, capacities and resistances, in which somewhere an electromotive force $E \cos pt$ is applied. The currents which arise in the system, satisfy a set of linear differential equations [1]) of the form:

$$\Sigma\, R_h i_h + L_h \frac{di_h}{dt} + \int \frac{i_h\, dt}{C_h} = \left\langle \begin{array}{l} 0 \\ E \cos pt \end{array} \right. \; . \; . \; . \; . \quad (11)$$

where the summation is to be extended over all the currents occurring in any closed circuit which can be described in the system. The solution of (11) consists of the general solution of a set of homogeneous linear equations, which are obtained from (11) by putting $E \cos pt = 0$, and a particular solution. The first part of the solution gives the (damped) free vibrations of the system; the second part the forced vibration. To discover the particular solution use can be made of the complex notation by putting $E e^{jpt}$ for $E \cos pt$, where $j = \sqrt{-1}$; and by trying for i_h a solution of the form $A_h e^{jpt}$; A_h is complex and gives not only the amplitude but also the phase-shift of i_h.

Instead of (11) we thus obtain a set of linear algebraic equations of the form:

$$\Sigma \left\{ R_h + j\left(p L_h - \frac{1}{pC_h} \right) \right\} A_h = \left\langle \begin{array}{l} 0 \\ E \end{array} \right. \; . \; . \; . \; . \quad (12)$$

Equations (12) are analogous to KIRCHHOFF's equations for a direct-current-system, only in the present case complex resistances occur of the form $Z_h = R_h + j\left(pL_h - \dfrac{1}{pC_h} \right)$. Therefore the same rules may

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[1]) At the same time they satisfy algebraic equations of the form $\Sigma i_k = 0$.

be applied as in direct-current-problems. For instance two parallel resistances z_1 and z_2 may be replaced by one single resistance $\frac{Z_1 Z_2}{Z_1 + Z_2}$. If there are also mutual inductions M_r in the system, the left-hand part of (12) is to be completed by terms of the form $jp_r M_r A_r$. In that case the method of complex resistances is still applicable though the analogy with direct-current-problems now does not hold entirely.

We shall now apply the method to the audion. By a system of self-inductions, capacities and resistances the anode is connected with the kathode; the grid is coupled with this circuit either by a direct contact, or by means of one or more mutual inductions. Further there are two batteries in the system, which provide for the mean tensions e and v being such that we are operating at the proper point of the characteristics (e.g. B in fig. 3). For alternating currents the batteries behave as ordinary resistances. Since we are only concerned with the alternating currents and tensions we shall henceforth for the sake of convenience omit the indices to these quantities. (See e.g. (10)).

Hence we obtain for the audion a same set of equations as (11), $E \cos pt$ now having the value λv, where v is the alternating part of the grid-potential. These can again be reduced to a set of equations (12). The grid-potential, however, in its turn is a function of the currents in the anode-circuit. Hence (12) is to be completed by one equation of the form:

$$E = \lambda \Sigma B_r A_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where the B_r's in general are complex quantities.

If for a given connection we can find a set of A's which satisfy (12) and (13) for a real value of p, this connection has a generative action. Many of such connections have been published in almost confusing abundance. A summary is for instance given by ARMSTRONG [1]) and by ECCLES [2]).

From the above we may deduce the following general rules for the generative audion-connections:

A. If a connection is found, which gives alternating currents of a definite frequency, we can deduce from it others, which give currents of the same frequency by replacing the "alternating resistances" by others which are equivalent for this frequency. In this

[1]) See Jahrb. f. Dr. Tel (1918) 12, p. 241.

[2]) Electrician July 1916, p. 573, Aug. 1916, p. 595.
See also: Yearbook of Wireless Tel. (1917) p. 674.

manner two parallel alternating resistances Z_1 and Z_2 may be replaced by a single one of the value $\dfrac{Z_1 Z_2}{Z_1 + Z_2}$.

B. We can also find other connections by replacing all the alternating resistances by their conjugate complex values. Indeed in this case there is a set of A's conjugate complex to the former A's which also satisfy (12) and (13) for a real value of p. In the case that there are no mutual inductions in the circuit, this can be obtained by replacing every self-induction L_h by a capacity C'_h and inversely, such that $L_h C'_h = L'_h C_h = p^2$. If mutual inductions also occur this simple substitution is no longer applicable [1]. Besides in this case the mutual induction must change its sign. This may be arrived at by changing the terminals on the primary or the secondary side.

On the ground of the above general rules we may draw the following conclusions with regard to the generative connections:

There are two types of connections:

I. Those with direct coupling. Here the grid is immediately connected to a point of the oscillation-circuit.

II. Those with *indirect* coupling. Here the grid is coupled by means of one or more mutual inductions with the anode-circuit.

I. *Direct coupling.*

The general type of these connections, to which they can all be reduced according to rule A, is given by fig. 5. $z_1 = z'_1 + z''$ and z_2 [2]) are alternating-current-resistances; r and λ have the same signification as in (10).

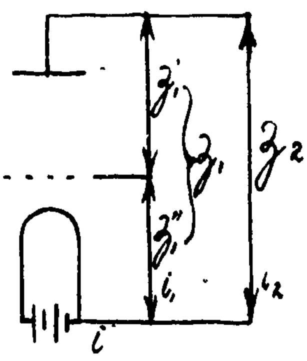

Fig. 5.

The relations (12) and (13) change into:

$$\left(r + \frac{z_1 z_2}{z_1 + z_2}\right) i = \lambda v \quad . \quad . \quad (14)$$

$$v = - z_1'' i_1 = - \frac{z_1'' z_2}{z_1 + z_2} i \quad . \quad . \quad . \quad . \quad . \quad (15)$$

By combination of (14) and (15) we get:

$$r + \frac{z_1 z_2}{z_1 + z_2} + \lambda \frac{z_1'' z_2}{z_1 + z_2} = 0 . \quad . \quad . \quad . \quad . \quad (16)$$

[1]) (Note to the English translation). Prof. ELIAS has kindly pointed out to me, that in this case a wholly symmetrical substitution is obtained by changing the mutual induction into a "mutual capacity" (two condensers telescoped into each other).

[2]) In the fig. denoted by gothic letters.

By substituting $z_1 = x_1 + jy_1$ etc., where $j = \sqrt{-1}$ and putting the real and the imaginary parts of (16) separately equal to zero, we obtain:

$$x_2 \{ x_1' + (1+\lambda) x_1'' \} + r (x_1 + x_2) - y_2 \{ y_1' + (1+\lambda) y_1'' \} = 0 \quad (17)$$
$$y_2 \{ x_1' + (1+\lambda) x_1'' \} + r (y_1 + y_2) + x_2 \{ y_1' + (1+\lambda) y_1'' \} = 0 \quad (18)$$

The y's and possibly also the x's contain the frequency p. By eliminating p from (17) and (18), we obtain a relation for the constants of the circuit:

$$\psi (r, R_1 L_1 C_1 \ldots) = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

which must be satisfied, in order that permanent alternating currents may exist. Besides either of the equations (17) and (18) can be employed for the determination of the frequency. (19) gives a *necessary* condition; it is also *sufficient*, if (17) or (18) contains one real root.

If p has a real value, x_1, x_2 etc. are essentially positive as having the character of ohmic resistances; r and λ are positive audion-constants; $y_1, y_2 \ldots$ are either positive (of the nature of a self-induction) or negative (capacity). From (17) and (18) it is obvious, that connection I can only be made in two essentially different manners. From (17) it may be concluded, that y_2 and $y'_1 + (1+\lambda) y''_1$ must have the same sign, from (18) it follows. that $y_1 = y'_1 + y''_1$ and y_2 must be of a different sign.

First manner:

y_2 and y''_1 positive; y_1' negative, whereas

$$y_1'' + y_2 < - y_1' < (1+\lambda) y_1'' \quad . \; . \; . \; . \; . \; . \quad (20)$$

This connection in its simplest form is given by fig. 6. Now (20) assumes the form:

$$p (L_1 + L_2) < \frac{1}{pC_1} < (1+\lambda) p L_1$$

hence it follows that

$$L_2 < \lambda L_1 .$$

Instead of (17) and (18) we obtain:

$$R_1 R_2 (1+\lambda) + (R_1 + R_2) r + \frac{L_2}{C_1} - (1+\lambda) p^2 L_1 L_2 = 0$$

$$p L_2 R_1 (1+\lambda) + r \left\{ p (L_1 + L_2) - \frac{1}{pC_1} \right\} + R_2 \left\{ -\frac{1}{pC_1} + (1+\lambda) p L_1 \right\} = 0$$

whence by elimination of p the rather complicated condition for oscillation can be deduced.

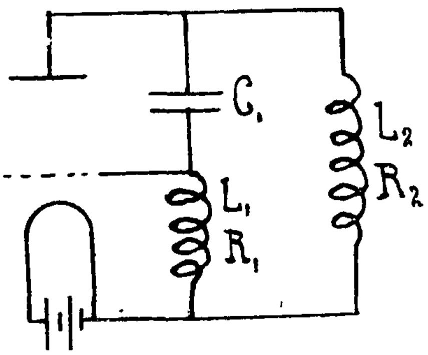

Fig 6.

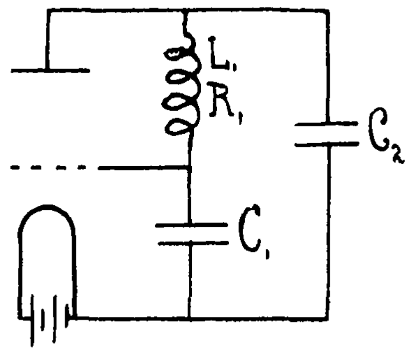

Fig. 7.

Second manner.

This can be obtained from the former by the substitution B, y_2 and y''_1 are here negative and y_1' positive, whereas

$$-(y_1'' + y_2) < y_1' < -(1 + \lambda)\, y_1''$$

Fig. 7 gives the simplest form. The relations (17) and (18) here give respectively:

$$p^2 = \frac{1+\lambda}{L_1\, C_1 + r\, R_1\, C_1\, C_2} \quad \ldots\ldots\ldots \quad (21)$$

$$p^2 = \frac{1}{L}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{R}{rC_2}\right) \quad \ldots\ldots\ldots \quad (22)$$

Connections, which belong to the type of fig. 7, are frequently applied. They have been dealt with theoretically by VALLAURI [1]). Connections of the type of fig. 6 also occasionally find an application [2]).

II. *Indirect coupling.*

The simplest case to deal with is that, where the grid-circuit is currentless. The reduced type of this connection is given by fig. 8. Here

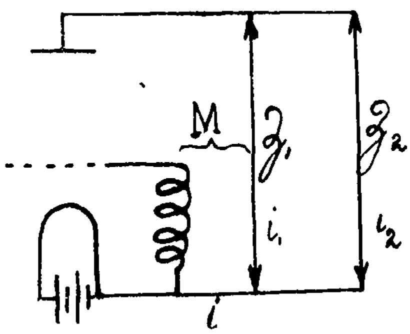

Fig. 8.

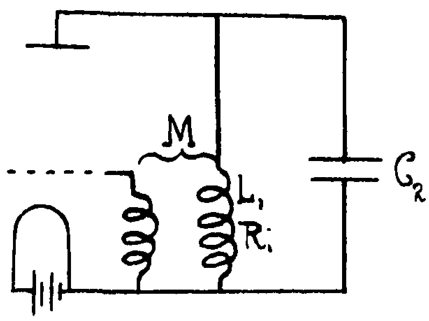

Fig. 9.

1) See Jahrb. f. Dr. Tel. (1918) 12 p. 381.

2) See e.g. ARMSTRONG l.c., fig 13. The capacity, which we have called C_1 in fig. 6, is absent here. The anode and the grid, however, which are sealed in at the same end of the audion, have sufficient capacity with respect to each other.

$$\lambda v = \left(r + \frac{z_1 z_2}{z_1 + z_2}\right) i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$v = -j p M i_1 = -j p M \frac{z_2}{z_1 + z_2} i \quad . \quad . \quad . \quad . \quad . \quad (24)$$

From (23) and (24) it follows that

$$\lambda p M y_2 = r (x_1 + x_2) + x_1 x_2 - y_1 y_2 \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$-\lambda p M x_2 = r (y_1 + y_2) + x_1 y_2 + x_2 y_1 \quad . \quad . \quad . \quad . \quad . \quad (26)$$

These equations can only be satisfied in two manners, which by substitution B can be deduced from each other:

First manner: y_1 pos., y_2 neg., M neg.

Second manner: y_1 neg.. y_2 pos., M pos.

They are given by fig. 9 and 10.

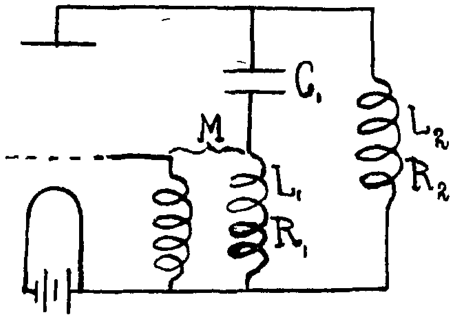

Fig. 10.

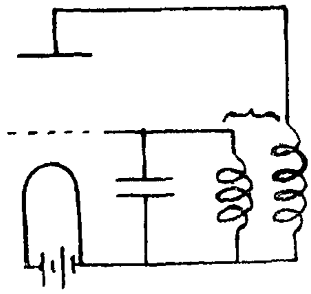
Fig. 11.

The connection of fig. 9 is also frequently made use of. It was thoroughbly discussed by VALLAURI[1]). That of fig. 10 so far has apparently not been used.

If the indirect coupling is applied and there is also a current in the grid-circuit, the arbitrariness is so great, that it seems rather difficult to establish any general rules, except the substitution-rules A and B. Still for every special case the above calculation leads directly to the result and gives a better survey than the solution of a set of simultaneous differential equations. A simple instance of these connections is given by fig. 11; they occur very often.

The same connections, which will make the audion generate, are also exceedingly suitable for giving a good amplifying action. Whether an audion acts as a generator, depends in the end on the roots of an algebraic equation of the n^{th} degree:

$$a_0 p^n + a_1 p^{n-1} \ldots . + a_n = 0. \quad . \quad . \quad . \quad . \quad . \quad (27)$$

This equation has been obtained from a homogeneous linear differential equation of the n^{th} order:

$$a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1} x}{dt^{n-1}} + \ldots . + a_n = 0, \quad . \quad . \quad . \quad . \quad . \quad (28)$$

[1]) See l.c.

by trying a solution of the form $x = Ae^{pt}$. Here the a's are functions of the alternating current-resistances; x indicates a current or a tension.

If at least one root $p = p_0$ is a pure imaginary quantity, free undamped vibrations can occur. Where the audion is applied in a receiving station for wireless telegraphy, the grid-potential is subjected to a forced vibration on account of the coupling with the antenna. Instead of (28) we now obtain an equation with a right-hand side of the following form, in complex notation [1]):

$$a_0'\frac{d^n x}{dt^n} + a_1'\frac{d^{n-1}x}{dt^{n-1}} \dots + a'_n = Ee^{p't}. \quad . \quad . \quad . \quad . \quad (29)$$

where p' is a pure imaginary quantity.

The particular solution, which gives the forced vibration is found by putting $x = A'e^{p't}$. To determine the amplitude $|A'|$, we have:

$$|A'| = \frac{E}{|a_0' p'_n + \dots a'_n|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

If p' is equal to an (imaginary) root p_0 of (27), we can make the denominator of the right-hand side of (31) as small as we like, by making the constant a' but little different from a_0 etc. in (27). A limit is only given by the condition, that the natural vibrations determined by:

$$a'_0 p'^n + \dots a'_n = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

have to be sufficiently damped; therefore it is necessary that the roots of (32) have a sufficiently large real part. Hence by a coupling as that of fig. 12 the object is attained of the system having but little friction for the forced vibration.

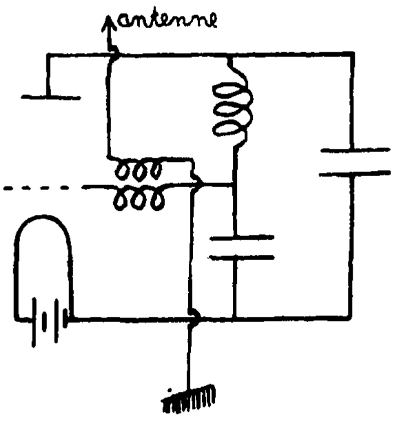

Fig. 12.

The audion is exceedingly well adapted to receive undamped waves. According to the heterodyne-principle local oscillations are then excited in the receiving station, which give rise to beats of audible frequency which can be detected in the ordinary manner by rectifier and telephone. The audion is then tuned in such a way, that the natural frequency differs but slightly from that of the forced vibration. It is then obvious that the system for this vibration has but little "friction".

1) We reserve the a's without accents for the special case, that these quantities are chosen in such a manner that (27) has one root, which is a pure imaginary quantity.

In the foregoing considerations we have assumed that a linear relation holds between current and tension. From the fig. 2 and 3, however, it appears that this it only true for a limited part of the characteristics. The current-amplitude cannot rise above a definite value. It follows that actually we have not to ask for the pure imaginary roots of (27), but for the roots with a positive real part. By an investigation as given above we only get to know the points, where the real part of the roots p changes its sign. It appears to me that in most cases this will be sufficient to discover the different coupling-possibilities.

In an American patent of Nov. 1917 (N° 102, 503) the use of the audion as generator is described, where especially the *quadratic* terms in the current-tension relation seem to assume the most important part. It is rather difficult to draw conclusions from patent-descriptions and as far as I know a discussion of such a coupling has thus far not been published.