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Physics. — "*Some experiments on gravitation. The ratio of mass to weight for crystals and radioactive substances.*" By Prof. P. Zeeman.

(Communicated in the meeting of Sept. 29, 1917).

1. Our ideas concerning gravitation have been so radically changed by Einstein's theory of gravitation that questions of the utmost interest in older theories are now simply discarded or at least appear in a changed perspective. We cannot try anymore to form an image of the mechanism of the gravitational action between two bodies, and we must return to the older theories in order to justify the suspicion, that the structure of substances might influence their mutual attraction. In most crystalline substances the velocity of propagation of light, the conduction for heat and the dielectric constant are different in different directions, and we might then suspect that the lines of gravitative force spread out from a crystal unequally in different directions.

A. S. Mackenzie[1]) in America, and Poynting and Gray[2]) sought for evidence of a directive gravitational attraction.

Mackenzie proved with an apparatus like that used by Boys in his beautiful researches on the gravitation constant, that when the axes of calcspar spheres were set in various positions the maximum difference of attraction amounted to less than $\frac{1}{200}$th part of the total attraction.

Poynting and Gray proved that the attraction between two quartz spheres with parallel axes, differs less than 1 in 16000 from the attraction between these spheres with crossed axes.

Kreichgauer[3]) sought for a change of weight of sodium acetate when this substance crystallized from the fluid (supersaturated) state. It appeared that the change amounted to less than $\frac{1}{2}.10^{-7}$ of the total weight.

2. *The weight of quartz spheres in different positions.*

Determinations of the weight of crystals in different orientations have, I believe, never been published. Some years ago I decided upon

1) Physical Review **2**. 321. 1895.
2) Philosophical Transactions. A. **192**. 245. 1899.
3) Kreichgauer. Verhandl. Berliner Physik. Ges. **10**. 13. 1892.

carrying out such experiments, but only now, in connection with connected material I intend to give an account of the results. Weighings were made with different crystals, but the greatest accuracy was obtained in a comparison of the weights of two quartz spheres, 42 m.m. in diameter and weighing about 127 gms. Each sphere was mounted in a ring of argentan, and could be turned about a horizontal axis, the ends of which were supported by the suspending wires of the pans of the balance. The spheres were of nearly equal weight, so that it was only necessary to nearly cancel the difference of the weight of the spheres. The optical axes of the spheres hanging from the left and right arms of the balance were placed alternatively in vertical and horizontal positions by means of a simple mechanism, *allowing the necessary operations without opening of the balance case.*

From 15 different series of weighings it resulted that the *double* effect sought for is less than 0.01 mg in 127 gms. or less than 1 in 13000000.

I have much pleasure in thanking Miss C. M. PEEREBOOM, phil. nat. cand., who has taken part in the investigation, and made many weighings.

Experiments with the torsion balance.

3. By means of the common balance we are able to ascertain the equality of two weights. In how far equal weights correspond to equal masses, in the meaning introduced in the science of mechanics, can be found out only by experiments. A rough experiment to prove the proportionality of weight and mass consists in observing the equality of the time of fall of various bodies. Much more accurate results were obtained by NEWTON's pendulum experiments (descent along a circular arc). Pendulums of equal form and hanging by equal threads, but of various composition, have the same time of oscillation. In his fundamental experiments NEWTON [1] was able to ascertain the equality of the times with an accuracy of one part in 1000 and this therefore is also, at the same place, the accuracy of the proportionality of weight and mass. BESSEL [2]) refined NEWTON's method, and came to the conclusion that a difference of attraction, experienced by various bodies of equal mass, must be less than 1/60.000 of the total attraction.

A much more accurate result was obtained by VON EÖTVÖS with CAVENDISH's torsion balance, which he brought to a high degree of

1) NEWTON. Principia.

2) BESSEL. Abh. Berliner Akademie. 1830.

perfection, after many years of continuous, very refined, studies on the local variations of gravity, and which he applied also to the problem now under consideration.[1]) The force acting on a body at the surface of the earth is the resultant of two forces: the attraction of the earth and the centrifugal force. The direction of the resultant is dependent upon direction and magnitude of these components. At a given place of the earth the centrifugal force is directed perpendicularly to the earth's axis and dependent upon the mass. If for various substances of equal masses the attractions were different, then the resulting force for these substances would have different direction, and a couple would act on a torsion balance, the rod of which is placed perpendicularly to the meridian and carries at its ends different substances.

VON EÖTVÖS used a torsion balance with a rod, 25—50 cm. long; the torsion wire was of platinum, 0,04 mm. thick, and charged with various substances all of 30 gms. weight. The rod is placed perpendicularly to the meridian and its position relative to the case of the intrument determined accurately by means of mirror and scale. The whole intrument, rod with case, is then rotated through 180 degrees, the substance that first hung at the east side, now hanging at the west side. The position of the rod relatively to the instrument is now read again. The kind of effect considered must produce a torsion of the suspension wire. With a brass ball at one end, with glass, cork or stibnite crystals at the other end of the rod no effect was to be observed. A difference of weight of various substances of equal mass, must be for brass, stibnite and cork less than one twentymillionth, for air and brass less than one hundred thousandth.

4. The astonishing fact of the equivalence of mass and weight, the expression of the narrow tie between the phenomena of inertia and gravitation is of fundamental importance for EINSTEIN's theory of gravitation. This theory, only possible, if there exists a field of force giving the same acceleration to all bodies, even enables us to "create" a gravitation field by a transformation of coordinates.[2]) .

The fact mentioned therefore merits to be tested in all possible directions. It has been my aim to extend the work of VON EÖTVÖS in two directions, viz.: by the investigation of *orientated* crystals and of radioactive substances. I also hoped, that I might be able to

[1]) v. EÖTVÖS. Ann. d. Phys. **59.** 354 1897, especially p. 372—373, and Mathem. u. Naturw Berichten aus Ungarn. **8. 64.** 1891.

[2]) EINSTEIN. Ann. d. Phys. **49.** 769. 1916.

introduce some changes, securing, at any rate, the independence of my results.

An investigation of crystals seemed important to me, because von Eötvos in his investigation of stibnite does not mention, whether attention was paid to a definite orientation. The orientation relative to the vertical however might be of supreme importance. An investigation with radioactive substances is of interest, because it enables us to verify the proposition that energy possesses mass. We know that if ΔE denotes a change of energy of a body, c the velocity of light, then to ΔE corresponds a change Δm of the mass given by the formula $\Delta m = \frac{\Delta E}{c^2}$. Because c^2 is extremely great, we can by ordinary methods only obtain inappreciable changes of mass. We cannot hope ever to be able to measure the changes of mass caused by the effect of temperature or by chemical transformations

In the case of radioactive bodies the processes of disintegration entail losses of energy of another order of magnitude than in the case of chemical transformations. During the transformation of uranium into lead and helium an enormous amount of energy must be released. This is already the case during part of the necessary transformations, for in the course of its life one gram of radium with its transformation products including radium F emits about 3,7 10^9 calories [1]).

This corresponds to a change of mass equal to $\frac{3{,}7 \times 10^9 \times 4{,}18 \times 10^7}{9 \times 10^{20}} =$

$= 0{,}6 \times 10^{-4}$ g per gram.

If this energy possesses mass, but no weight, then pendulums with lead, helium, uranium must give values for the acceleration, differing by more than 1 part in 10,000.

Already several years ago these considerations were given by J. J. Thomson, who also made experiments with a pendulum the bob of which was made of radium. It was, however, impossible to obtain a high degree of accuracy as the quantity of radium available was very small. Afterwards Southerns [2]) made experiments in Thomson's laboratory with pendulums with uranium oxide and red lead. He came to the conclusion that the ratio of mass to weight for uranium oxide, does not differ from that for lead oxide by more than one part in 200,000.

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1) Rutherford. Radioactive Substances. p. 582.

2) Southerns. Determination of the ratio of mass to weight for a radioactive substance, Proc. Royal. Soc. London, A. **84**, 325. 1910.

Hence we must conclude, within the limits of experimental errors, that if energy possesses mass it also possesses weight. Now we can considerably restrict these limits by the use of the torsion balance and this justifies us, I think, in applying it to the investigation of radioactive substances.

5. *New experiments.* My own experiments were made with an apparatus, principally after the design of that of v. Eötvös, but of much smaller dimensions. The weights at the end of the torsion rod were each 30 grams in v. Eötvös' experiments; in my apparatus the weights were each of 1 gram. The weight of the torsion rod with mirror was only about 1,5 grams. The distance between the centres of the cylindrical weights at the ends of the torsion rod is about 10 cms. The smallness of these weights enabled me to take advantage from the properties of fine quartz wires, not yet discovered, indeed, at the time v. Eötvös began his researches.

The torsion wire in my apparatus was 22 cms. long, and about 0,01 mm. thick. The time of oscillation ranged from about 350 to 400 seconds.

In order to protect the apparatus from thermal and electrical perturbations we used, as also did v. Eötvös, double and even triple walls, of brass, about 3 mms. thick.

Manipulations with an apparatus of so great a sensibility as this torsion balance, requires exceptional stability of the surroundings. The mounting on the brick piers of the Amsterdam laboratory proved to be quite insufficient. I, therefore, resolved to construct an arrangement, probably securing the apparatus against vibrations and permitting its rotation, with scale and telescope, about a vertical axis through 180°.

The principle of this arrangement is the one used by Michelson and by Einthoven for similar purposes.

I have much pleasure to thank here Mr. W. M. Kok, phil. nat. cand. for his assistence in the construction of this arrangement and during the continuation of the present investigation.

The principal part of the arrangement is an iron basin floating in a tank with thick oil. Tank and basin are of annular form; the central part being open, it becomes possible to suspend an apparatus from a vertical bar through the centre of the annular basin and to fix it at different heights.

A more detailed description is reserved for another occasion.

It was found that this arrangement gave excellent protection against vibrations of short period and permitted also to give slow and smooth rotations.

On the contrary the protection against vibrations of long period was very bad. Experiments with the suspended torsion balance soon taught that, even during the most quiet hours of the night, the torsion rod was never at rest. Sometimes the amplitude of the oscillations gradually diminished to zero, but then the amplitude increased again to 5 m.m., not to mention the extremely annoying nutations of the mirror, which, indeed, never ceased. Apparently vibrations of a period of 300 or 400 seconds (the period of the torsion balance) are never failing in the marshy land of Amsterdam, at least in the neighbourhood of the physical Laboratory.

It was therefore hopeless to work with the torsion balance in *Amsterdam*, and I resolved to continue my experiments in the cellar of a country house near *Huis ter Heide* (prov. *Utrecht*).

To my surprise I found that the stability of the balance, at the new station, was most excellent. The motion of the mirror, about a horizontal axis, was entirely absent and the amplitude of the oscillations always decreased with time. After about one hour the image seen in the telescope was at rest. The apparatus was placed upon a wooden table, resting on the cellar floor. Even hard stampings upon the floor in the neighbourhood of the apparatus had not the slightest effect.

Of course the temperature of the cellar was very constant. One disturbance had an effect on the observations, viz. the magnetic action due to the iron beams of the cellar vault. The constant displacement of 0,3 m.m., noticed in the experiments with quartz and recorded later on, is probably due to this cause.

In view of the accuracy aimed at in the experiments, this amount could not be neglected, though in some experiments its influence is eliminated. I therefore transferred the apparatus, first to a second place in the cellar, where presumably the perturbations would be less.

Afterwards the apparatus was placed in the vestibule. Also here the stability was excellent, but of course the constancy of temperature, though satisfactory, somewhat less. Several excellent series of observations were obtained. As they extended, however, over the whole day and the principal entrance of the house was then put out of use, I restricted these observations to a rather limited number of days.

6. For *Amsterdam* the latitude $\varphi = 52,4$ and $g = 981,3$ cm/sec The angle α between the attraction of the earth b, and the resultant of attraction and centrifugal force a, becomes $5'42'' = 342''$.

As is easily seen from figure 1, $\alpha = \frac{a \sin \varphi}{b}$ and hence $d\alpha = = \frac{d a \sin \varphi}{b^2} db = \alpha \frac{db}{b}$, if the attraction b changes with the amount db.

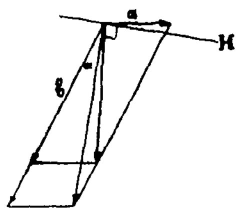


Fig 1.

By the change of direction $d\alpha$, the force acting in the horizontal direction H becomes $Rd\alpha$, R being the resultant of a and b.

If there is a change of mass a with da, then $d\alpha = \frac{da \sin \varphi}{b} = \alpha \frac{da}{a}$.

Let the difference of attraction for two substances be 1/1000000, then $d\alpha = 0{,}000342''$ or in radians 1/600000000.

In order to give an idea of the sensibility of the apparatus and to calculate the effect to be expected with a given value $d\alpha$, I now give some details of the arrangement, a sketch of which is given in figure 2.

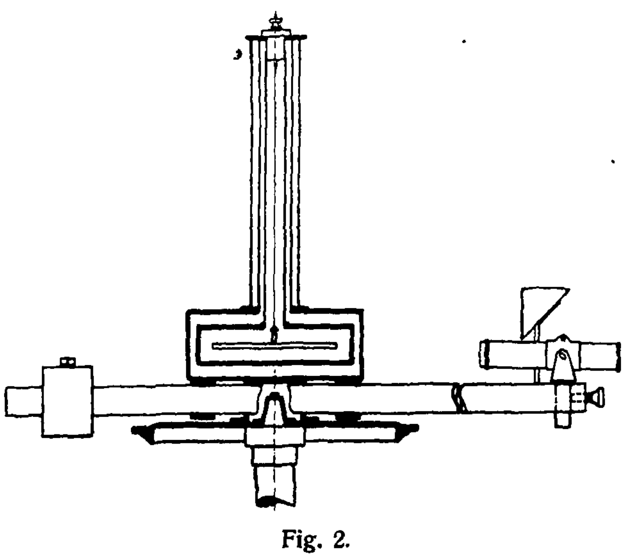


Fig. 2.

Experiment with quartz cylinders, mean weight of each 0,888 g.

Time of oscillation (complete) of torsion rod with quartz cylinders 350 sec.

Time of oscillation (without cylinders) 186 sec.

Distance of the centres of quartz cylinders 7,6 cm.

Moment of inertia of quartz cylinders $K_1 = 2 \times 3{,}8^2 \times 0{,}888 = = 25{,}6$ g. cm².

Moment of inertia rod + cylinders $K = 25{,}6 + 10{,}1 = 35{,}6$ g.cm².

For the torsion couple S we find from $S = \frac{4\pi^2\ 35,7}{350^2} = 0,0115$ $cm^2\ g.\ sec.^{-2}$.

Hence force per radian twist $0,0115/3,8 = 3,03 \times 10^{-3}$ dynes $= 3,09 \times 10^{-6}$ g.

Distance scale to mirror $= 540$ mm.

A displacement of 1 mm. observed in the telescope corresponds to a rotation of $\frac{1}{2 \times 540}$.

A difference of the positions of the balance equal to 1 mm., when pointing first West than East, corresponds for the single effect to a force measured by an angle of $\frac{1}{4 \times 540}$, and therefore equal to $\frac{3,09 \times 10^{-6}}{2160} = 1,43 \times 10^{-9}$ g.

The vertical force acting on the cylinder is 0,888 g.

If the effect is $\frac{1}{1000000}$, then we have (see above) $Rda =$ $= \frac{0,888}{6} 10^{-8} = 1,44 \times 10^{-9}$ g.

We therefore see that with the sensibility used, 1 mm. of the scale corresponds to an effect of 1 in 1000,000. In many cases the result is certainly smaller than 0,1 mm.

7. *Results.* We will now summarize the results obtained. In the first experiments the position of rest of the rod was determined from three succeeding extremities of vibration. The presence of the observer, however, brings about a marked disturbance by convection currents. Preference was given in the subsequent observations to the noring of the final position of rest, actually attained after about one hour.

Experiments were made with quartz, calc-spar, lead oxide, uranium oxide, uranyl nitrate.

Quartz. The 2 cylindrical quartz rods were 25 mm. long and of 4,5 mm. diameter. The axis of the cylinders lay in the vertical plane through the rod of the balance. The crystallographic axis was perpendicular to the axis of the cylinder. I determined its position before the beginning of the experiments, by means of observations with the polariscope, and noted it by means of a small cross, cut by a diamond, in the surface of the cylinder.

Experiments *in the cellar.* The annexed table gives an example of results for cylinders with crossed and with parallel axes.

Quartz cylinders: axes ⊥. Date: August. 2.

	Telescope West	Telescope East.
Reading scale in m.m.	66.8	67.2
	66.8	67.2
	67.0	67.3
	67.0	67.4
	67.0	
	66.92	67.28, hence E > W 0.36.

The results are exhibited as a curve in fig. 3. The abscissae give the hours of the observations. The temperature of the instrument diminished in the course of the day a few tenths of a degree.

Quartz cylinders. Axes //. Date: August. 5.

	Telescope West	Telescope East.
Reading scale in m.m.	39.7	40.0
	39.7	40.2
	39.8	40.1
	39.8	40.1
	39.75	40.10, hence E > W 0,35

These results are represented in fig. 4.

Fig. 3.

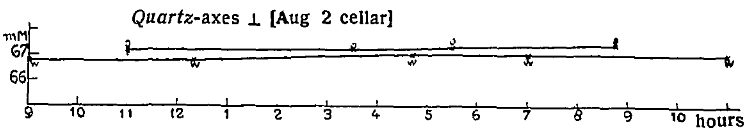
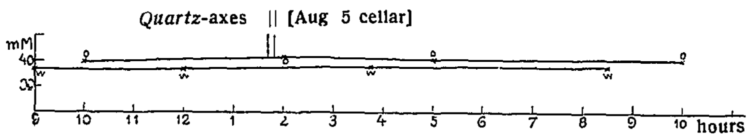


Fig. 4.



These observations exhibit a difference of the readings for the position of the torsion rod after rotation of the apparatus from the E-W to the W-E position. Moreover it appears *that it makes no difference whether the crystallographic axes of the quartz cylinders are parallel or crossed.* At least this difference is only 0,36—0,35=0.01 mm.

The constant difference of 0,35 mm. was traced to be probably due to the asymmetrical, magnetic action of the heavy iron beams of the cellar vault on the weakly magnetic torsion rod of hardened copper of the balance. A magnet placed in the neighbourhood of the apparatus caused a small deviation.

The balance was then transferred to *the vestibule,* a place of rather constant magnetic potential. The difference between the E-W and

W-E observations had disappeared. As an example I give again a table of results, also plotted in fig. 5 and fig. 6.

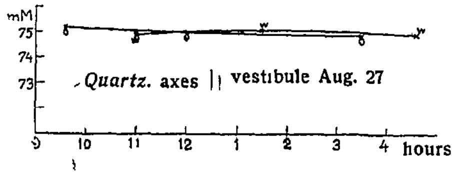


Fig. 5.

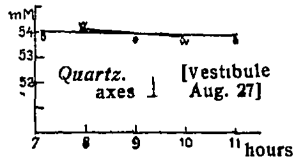


Fig. 6.

Quartz cylinders: *axes* //. Date: August 27.

W.	O.
74.9	75.2
75.1	75.0
74.9	74.9
74.97	75.03, hence E > W 0,06.

Quartz cylinders: *axes* ⊥. Date: August 27.

W.	O.
54.1	54.0
53.8	53.9
	53.8
53.95	53.90, hence E < W 0.05

The above mentioned and further observations justify us in concluding that *an influence of the orientations of a quartz crystal on the ratio of mass to weight is less than* 1 *in thirty millions.*

Subsequent observations were made with two *calc spar cylinders* with the same results as obtained for quartz.

These observations were also made in the vestibule.

The results obtained are in agreement with the conclusion in § 2.

8. The torsion rod was next charged with a small glass tube with yellow *lead oxide* at one end, and one of the quartz cylinders at the other. A difference of the ratio of mass to weight for these substances was certainly less than 1 in twenty millions.

9. *Radioactive substances.* Observations were begun with *uranium oxide,* included in a thin cylindrical glass tube. The results were rather puzzling. A first series of observations in the cellar gave, in the above used nomenclature, E > W 1,2 mm. In this series lead oxide was compared with uranium oxide.

On August 24 I began observations with a *second* glass tube charged with uranium oxide. I resolved to compare this second tube with the first in the torsion balance, expecting to find a deviation of, at the utmost, a few tenths of a mm. when observing in the cellar, and practically zero in the vestibule. An observation of August 24 in the cellar however gave, using the torsion rod, charged with the two uranium oxide tubes, a difference of $W > E$ 2,1 mm.; in the vestibule an observation of August 25 gave $W > E$ 2,2 mm.

The first tube was incidentally broken and part of the contents lost. The further experiments were made with the second tube (balanced against quartz) and gave on August 28, 29, 31 and September 2 the results $W > E$ 1, 1, 2.5, 3 mm.

The deviations found widely exceed the errors of observation. They prove (observations of August 24 and 25) that the two samples of uranium oxide are not identical. Probably both or at least one of the uranium oxides are contaminated by iron. [1]) By this hypothesis we may understand that the magnitude of the deviation changes with time (observations of August 28 and following days), and even exhibits contrary signs (first observations in the cellar compared with the later observations in the vestibule).

I had not yet an opportunity to test quantitatively the suggestion as to the influence of a contamination by traces of iron. This must be reserved to another occasion. Meanwhile, I have now to record observations with *uranyl nitrate*, which go far to prove that radioactive substances also follow the law of proportionality between mass and weight with great accuracy. The uranyl nitrate was included in a thin cylindrical tube and balanced against quartz. The results of observations of September 10 and 11, made in the vestibule, are plotted in figs. 7 and 8.

The curves are not quite parallel to the axis of abscissae, a case

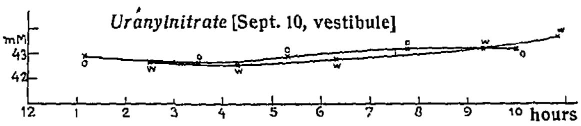


Fig 7.

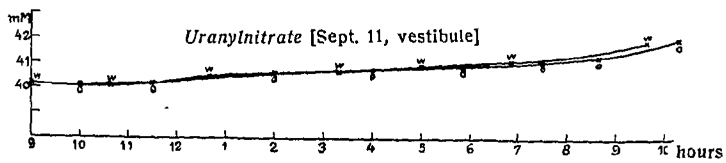


Fig. 8.

[1]) OWEN (Ann. d. Phys. **37**, 686, 1912) finds that the three preparations of uranium, which were used in his magnetochemical experiments, contained much iron. See also there (p. 672) some remarks on the omnipresence of iron.

realized in the cellar observations Figs. 3 and 4, but exhibit a small slope due to the rise of temperature in the vestibule. This slope is however the same for the E. and W.-curves. The lines are even practically coincident for the greater part of the observations of September 11, and the small residual differences exhibit contrary signs in the observations of September 10 and 11. The observations justify us in concluding that *for uranyl nitrate a deviation of the law of the constancy of the ratio of mass and weight is less than 1 in twenty million.*

It seems extremely improbable that the behaviour of uranium should be otherwise, as far as a so fundamental property as mass is concerned, in an oxide than in a nitrate. The deviation found in the case of an oxide, is therefore most probably due to a magnetic contamination. If the deviation found in the case of the oxide were really due to a change of mass, than the nitrate should exhibit about half the effect of the oxide or about 1,5 mm., allowance being made for the quantity of uranium in the two combinations. But the effect is most certainly less than 0.2 mm.

Perhaps I may be allowed to add that electric disturbances during the observations were excluded sometimes by a short exposure of the inner case, opened to this purpose, of the balance to radium rays, sometimes by placing a few scrapings taken from the leaden box containing the radium preparation.

I have projected recently some improvements of the apparatus and the method of observation, by which I hope to be able to increase the accuracy, according to an estimation at the safe side, ten times.

I hope to return to this subject on another occasion.