Physics. — "*The viscosity of liquefied gases. X. The viscosity of liquid hydrogen.*" By Prof. J. E. VERSCHAFFELT. Communication N°. 153*b* from the Physical Laboratory at Leiden. (Communicated by Prof. H. KAMERLINGH ONNES.)

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A preliminary value having been obtained from the experiments described in Comm. N°. 151*g*, a new series of measurements was undertaken in order to find a more accurate value of the quantity in question. The construction of the apparatus to be used for this purpose was based on the considerations referring to similarity in the oscillatory motion of a sphere in a viscous fluid (comp. Comm. N°. 148*e* and N°. 151*f*). Taking in view the materials available for this construction it appeared to be possible to construct the apparatus in such a manner, that the motion in liquid hydrogen would be approximately similar to the motion in carbon disulphide[1]) which was studied on a former occasion (Comm. N°. 151*d* § 3), using a sphere of the same radius (2 cm.). In this case by equation (2) of Comm. N°. 151*f* (comp. also § 4 and the table), assuming for liquid hydrogen the preliminary value of $\eta = \cdot 00011$, the oscillating system had to be given a moment of inertia of about 30 c.g.s. units and a time of oscillation of about 40 secs.; this would correspond to a logarithmic decrement of $\cdot 07$ and a value of $\psi = \frac{T-T_0}{T_0} = \cdot 011$ [2]) (therefore $T-T_0 = \cdot 44$).

In these new experiments a hollow exhausted glass sphere was used (weight 8.9 grms), blown by Mr. KESSELRING, glass-blower of the laboratory, as nearly accurately round as was possible[3]). The sphere ended in a tube which had served in the blowing and was

[1]) Originally similarity was tried with the motion in liquid air or ether (comp. Comm. N°. 151*f* § 4), but of all the substances included in this study of a possible similarity carbon disulphide was the most advantageous, because it made it possible to reach a comparatively high logarithmic decrement, without its being necessary to use too high a time of oscillation (comp. Comm. N°. 151*f*, § 4); the higher decrement is then obtained by a smaller moment of inertia.

[2]) The value of this quantity given in Comm. N°. 151*f* in the table is too small.

[3]) The diameters measured in different directions did not differ from the mean by more than $^3/_{1000}$.

also made use of for the suspension, the axis of the tube representing the axis of revolution of the sphere. The equatorial radius of the sphere was 2.0049 ± 0.0025 cms. The sphere with its tube was sealed (accurately centred) to an oscillating system, consisting of a glass tube and a copper tube with disc, as in Comm. N°. 149*b* § 2, only thinner and lighter (weight 26.5 grms). This oscillating system to which also, as in the previous experiments, a hollow cylinder belonged which was put round the copper tube and rested on the disc was suspended from a thin manganin wire (·04 mm. thick and 55 cms. long). For the rest the apparatus was the same as in the preliminary experiments with liquid hydrogen.

As in the experiments with liquid air two hollow cylinders were used. The one (C_1) made of brass weighed 33.125 grms ($= m$) and had the dimensions R_e (external radius) $= 1.1915 \pm 0.0017$, R_i (internal radius) $= 0.3970 \pm 0.0019$, h (height) $= 1$ cm. about. Its moment of inertia was therefore

$$K_1 = \tfrac{1}{2} m (R_u^2 + R_i^2) = 26.12 \pm 0.07.$$

The other one (C_2) was made from retort-coal[1]) and covered with a thin layer of varnish; it had very nearly the same dimensions as the former, weighed 7.26 grms. and had a moment of inertia $K_2 = 5.77 \pm 0.01^5$. This value was not as in the previous case derived directly from the dimensions and the mass, but was determined by oscillation-experiments in which the oscillating system, leaving out the sphere, was first loaded with the one and then with the other cylinder. The times of oscillation were measured with a stop-watch with an accuracy of at least $^1/_{1000}$ [2]). These same experiments gave for the moment of inertia of the system without cylinder or sphere $K_0 = 3.95 \pm 0.01$. Similarly by the oscillation-method the moment of inertia of the sphere at room-temperature was found to be $K_b = 20.26 \pm 0.05^5$. [3])

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[1]) This material was selected on the ground of its having the desired specific gravity (see further on).

[2]) The chronometer had been previously compared with the standard-clock and was found to fully guarantee an accuracy of $^1/_{1000}$ in the time-readings.

[3]) The times of oscillation at 15° C. and in vacuo were as follows;

System	without	sphere	and	without	cylinder		(weight:	26.5 gr.)	$T = 14.46$ sec.
„	„	„	„	with	„	C_1	(„	59.6 „)	40.16 „
„	„	„	„	„	„	C_2	(„	33.8 „)	22.75 „
„	with	„	„	without	„		(„	35.4 „)	35.89 „
„	„	„	„	with	„	C_1	(„	68.5 „)	52.05 „
„	„	„	„	„	„	C_2	(„	42.7 „)	39.97 „

These observations give as previously (comp. Comm. 149*b* IV § 5) a small

The oscillating system with sphere and cylinder C_2 had therefore very nearly the same moment of inertia (and thus also the same time of swing) as the system without the sphere but with cylinder C_1. [1])

The oscillations of the system when immersed in liquid hydrogen were found to be actually strongly damped ($\delta = 0.1$ about): but the regularity of the damping left much to be desired, so much so that for a somewhat accurate calculation of δ only a few small portions selected from the series of observations could be utilized. [2]) The most regular portion obtained, which was exclusively used, was as follows: the elongations α, expressed in radiants, were observed to the right (+) and to the left alternately; in the table are also given the naperian logarithms of the absolute values of the elongations.

$\alpha = +\ 0.05180$	$\log \alpha = -\ 2.9599$
$-\ \ 4925$	3.0108
$+\ \ 4685$	3.0607
$-\ \ 4460$	3.1101
$+\ \ 4230$	3.1632

From these results it follows that

$$\delta = 0.1012 \pm 0.0003.$$

This is, however, not yet the logarithmic decrement δ_0 for infinitely small amplitudes; in order to find it, we utilize equation *c* of Comm. N°. 151*d*, § 3 found for carbon disulphide, which equation shows, in what manner the maximum-elongation depends upon the

diminution of the moment of the torsional couple M with increasing load (the change amounts to $\cdot 00035\ M$ per gramme); M_0 (couple without load $= \cdot 752 \pm \cdot 002$). From a few observations at different temperatures it appeared moreover, that with rising temperature the couple M diminished by $\cdot 0008\ M$ per degree.

[1]) This was the condition laid down in making cylinder C_2 (comp. Comm. 149*b*, IV, § 5); for this reason this cylinder was made of retort-coal (see note on previous page), after cylinder C_1 had been made before. In this case the condition, laid down before, that the system with or without sphere should have about the same weight, was not fulfilled; as a matter of fact it had been found, that the weight of the system has but very little influence on the torsional moment, and this was confirmed by the present experiments (comp. previous note).

[2]) Indeed, with very small elongations no damping could be observed at all: the oscillating system did not come to rest, but continued to oscillate to and fro over a few scale divisions, sometimes more and sometimes less. This must undoubtedly be partly ascribed to not entirely avoidable convection-currents in the liquid, which was found to receive radiation to a not inconsiderable degree: when the apparatus was closed (as before the liquid was under constant pressure), the vapour-pressure rose at the rate of about 2 cms. mercury per minute, corresponding to a temperature-rise of $\cdot 1^{\circ}$.

time, and therefore also how the logarithmic decrement $\delta = -T\frac{d \log a}{dt}$ changes with the amplitude of the oscillations. For $a = 0.05$ approximately we thus find $\delta = 1.052\ \delta_0$, and hence

$$\delta_0 = 0.0962$$

with an accuracy which may be taken at $1\,^0/_0$.[1])

The time of swing of the oscillating system with the sphere immersed in liquid hydrogen (temperature of the liquid about 20° K, the temperature of the room and thus also of the wire being 8.7° C) was

$$T = 40.20 \pm 0.10\ ^{2})$$

so that (see above)

$$T_0 = 40.20 - 0.44 = 39.8\ ^{3}).$$

The hydrogen was under a pressure of 76.9 cms mercury; thus according to the latest data about the vapour pressure of hydrogen [4]), the temperature of the liquid was: $20.39 + \frac{9}{200} = 20°.43$ K. At that temperature the sphere (according to the data supplied in Comm. N°. 85) had a radius of 2.002 cms, and therefore a moment of inertia of $20.26 \times \left(\frac{2.002}{2.005}\right)^2 = 20.23$, so that the total moment of inertia of the oscillating system in the experiment in liquid hydrogen was

$$K = 3.95 + 5.77 + 20.23 = 29.95.$$

For the moment of the couple exerted on the sphere by the viscosity of the liquid we now find (by equation 28′ of Comm. N°. 148*b*).

$$L'_1 = \frac{2\delta_0 K}{T_0} = 0.1448.$$

In order to determine the couple of the frictional forces on the not-immersed part of the oscillating system the sphere was, as previously, removed and cylinder C_2 was replaced by cylinder C_1.

1) This logarithmic decrement is considerably larger than what was expected from the similarity with carbon disulphide (see above). But it should be borne in mind that on account of the complicated structure of the oscillating system there can only be question of similarity in a very rough sense; for as regards the part which is not immersed in the liquid there is no question of similarity at all. For this reason the reduction of δ to δ_0 is not so accurate as might have been the case in the case of perfect similarity.

2) As the decrement of the swings, this time of swing was subject to irregular variations (of a few tenths of a second).

3) This value agrees closely with that which is to be deduced from those mentioned above taking into account the changes in the temperature of sphere and wire.

4) Comp. Comm. N°. 152*a*.

Under these conditions the result was

$$\delta_2 = 0.0091 \text{ }^{1)}$$

which gives (as $T = 40.06$ and $K = 3.95 + 26.12 = 30.07$)

$$L'_2 = 0{,}0137,$$

so that the torsional moment of the friction on the sphere alone amounts to:

$$L' = L'_1 - L'_2 = 0.1311.$$

The density of liquid hydrogen at the temperature of observation (20°.43 K) being 0.0708 [2]), it follows, according to equation (*a*) of Comm. N°. 148*b*, that

$$\eta = 0.000130$$

with a degree of accuracy which may be placed at about 1°/₀.

A further experiment was made from which a preliminary value could be derived for the viscosity of the hydrogen vapour. The oscillations of the system carrying the sphere and cylinder C_2 were observed, while only a little liquid hydrogen was left on the bottom of the vessel, the sphere thus being just above the liquid surface in the vapour. In these circumstances a damping was found with a decrement of 0.0128, whereas the system without the sphere and with cylinder C_1 gave $\delta_2 = 0.0093$ (at the same room-temperature of 18° C and the same pressure of 76.9 cms). The decrement due to the friction on the sphere alone is therefore $\delta_1 = 0.0035$; hence with $T = 40$, $K = 30$ and $\mu = 0.00119$.

$$\eta = 0.000010 \text{ }^{3)}.$$

[1]) This result was obtained from a few experiments at different room-temperatures and different pressures, so that a reduction could be made to the same temperature (8.7° C.) and pressure (76 9 cms.) as in the experiment with the sphere in the liquid. These experiments indicated a small increase of δ with the temperature, the change with pressure being insensibly small

[2]) Comp. Comm. 137*a*.

[3]) This preliminary value agrees well with that found by H. KAMERLINGH ONNES, C. DORSMAN and S. WEBER by the method of transpiration (Comm. N°. 134*a*). The accuracy of this result was further tested by a few observations made in air (at 10° C). These gave for the system with cylinder C_2 and sphere (the latter suspended in a large vessel): $\delta = .0448$ and for the system without sphere, but with cylinder C_1: $\delta = .0154$, so that for the sphere by itself: $\delta = .0294$, giving ($K = 30$, $T = 40$, $\mu = .00126$) $\eta = .000177$, in good agreement with known data (comp. Phys. Rev., 8, p. 738, 1916).