Physics. — *"On the diffraction phenomenon caused by a great number of irregularly distributed apertures or opaque particles"*. By Dr. W. J. DE HAAS. Communicated by Prof. H. A. LORENTZ. (Communication from the physical laboratory of TEYLER's Institute).

(Communicated in the meeting of December 29, 1917 [1]).

1. When before the objective of a telescope focussed on a distant lightpoint, we keep a screen, containing a great number of irregularly distributed equal circular apertures, we observe a well-known diffraction phenomenon. It will be known that the same phenomenon is observed when the investigation is made with a transparent screen, covered with opaque corpuscles which correspond to the above mentioned apertures, e. g. a glass plate strewn with grains of lycopodium. The diffraction image consists of a light central spot and round this the rings which would be found in the case of one single aperture; we now see however fluctuations in the intensity which we can best describe as a fibrous radial structure. VON LAUE [2]) investigated this phenomenon, and came to the conclusion that it could not be explained by classic optics. In this communication the considerations will be discussed which made me doubt the rightness of this conclusion. First I want to stipulate however, that I have not any objection to VON LAUE's theoretical considerations on the diffraction phenomenon by monochromatic light [3]).

2. My different investigations will only shortly be discussed. I confined myself to visual observation. The diffraction phenomena were studied in two different ways. First I worked with turned copper rings over which thin paper impregnated with indian ink had been stretched, when still wet.

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[1]) The essential contents were read in the meeting of Sept. 29, 1917.

[2]) M. v. LAUE, Die Beugungserscheinungen an vielen unregelmässig verteilten Teilchen, Sitzungsber. Akad. Berlin 1914, p. 1144; Mathematische Betrachtungen an vielen unregelmässig verstreuten Teilchen, Mitteilungen. Phys. Gesellsch. Zürich, 1916, p. 90; Ein Versagen der klassischen Optik, Berichte deutsch. physik. Ges. **18**, p. 19, 1917,

[3]) See H. A. LORENTZ, Zittingsverslag Dec. 1917, p 1120.

When dried, this formed a sufficiently opaque tightly stretched screen. With a fine needle small apertures were made in it. In order to vary the magnitude of these apertures the needle was fixed in a round holder. This holder fitted into a cylindrical tube in which it could be slided up and down and be fixed in a definite position. This enabled us to make all apertures equally wide and also to alter their width (by protruding a longer or a shorter part of the needle from the cylinder).

In the second method I worked with glass-plates covered with lycopodium powder or with glass-plates on which small mercury spheres had been precipitated by evaporation. The mercury can easily be precipitated in spheres of $\frac{1}{200}$ mm diameter or less. They are however not all of the same magnitude. Three such plates were in my possession.

The first method offers many advantages. The phenomenon can be continuously followed with the eye while the number of apertures is increased, while in the case that the refracted light has the same direction as the incident beam, the latter does not blind as in the other methods. Visually the details of the phenomenon could be observed much quicker, more accurately and more easily than photographically, while the occurrence of colour-phenomena did not remain unobserved.

The source of light was an arc-lamp at a distance of about 5 m from the objective of the telescope. The lamp had been screened off as perfectly as possible, while the apertures in a screen of lead that could be rotated formed the light-points. In order to vary the experiments there were apertures of 0.5, 0.4, 0.3, 0.25, 0.15 mm diameter.

Beginning with a small number of apertures in the screen of black paper and passing gradually to a greater number, we see how the diffraction image of a few apertures, which agrees with the classic theory, changes into the phenomenon of the fibrous structure. This change is very striking.

Working with natural light we see with two holes a small number of diffraction lines, the middle one of which is uncoloured and passes through the point 0, where the image of the considered light-point is formed. This point will be called the centre. On both sides of the middle and most intense line a small number of lines is observed, which are alternately light and dark, and which show the spectral colours. Let the system of lines round the centre be denoted by A, the uniformly illuminated field on both sides by B.

We must remind that according to § 3 all this is superposed on the diffraction image (spot and ring) of one single aperture. With three apertures which form an arbitrary triangle, we observe three systems of lines A, respectively perpendicular to the sides of this triangle. They cross each other in the neighbourhood of the centre O.

With four or more apertures a similar phenomenon is observed and so on.

We get the impression that always line-systems, each perpendicular to a line connecting two apertures (of which systems the number always increases with the number of apertures) cross each other near the centre. This would be the reason why near the centre no pronounced lines are observed and instead of these a sunflower-like structure. But at a greater distance from the centre the line-systems must diverge. This suggested to me the conception that *these* are the fibres of the phenomenon of LAUE. Working with 50 apertures, the diffraction image has already quite the same aspect as for a glass-plate covered with lycopodium; the only difference is, that in the first case the fibrous structure is coarser than in the second. With a small number of apertures even when they are distributed in an "accidental" way, it is however possible that among all lines of connexion some directions are more represented than others.

3. This conception may be elucidated by a simple mathematical consideration.

Before the objective of a telescope focussed on a light-source at an infinite distance a screen has been placed over which a great number n of equal circular apertures is distributed. The screen is placed perpendicular to the axis of the telescope. Let F be the principal focus of the objective, and let us consider the distribution of the light in the focal plane V passing through F. According to a well-known theorem of the diffraction theory the intensity at a point P of the plane can be represented by the product of two factors. The first of these is the intensity that would be due to one single aperture, while the second is the intensity i that would be observed if instead of the given apertures we had at their centres n equal apertures so small, that they might be considered as points. Both factors are functions of the position of P in V. The first determines the intensity in the diffraction image B of one single aperture, where the intensity changes relatively slowly from point to point. Into this diffraction image B the factor i introduces irregular fluctuations, by which the intensity changes much more quickly from point to point. We may say that these fluctuations are seen on the background formed by the image B. If really there are radially directed light and dark fibres, these fluctuations must necessarily be caused by the second factor i. Therefore we can confine ourselves to the consideration of the diffraction image of n point-like apertures.

4. Let P be a point of the focal plane and let the vibrations (monochromatic light) reaching that point from the different apertures be represented by:

$$\cos(vt + \varphi_1), \quad \cos(vt + \varphi_2) \quad . \quad . \quad . \quad . \quad \cos(vt + \varphi_n),$$

where $\varphi_1, \ldots \varphi_n$ denote the phases and where for the sake of simplicity the amplitude has been put equal to 1. It will be known that we may write for the resulting intensity

$$i = n + \Sigma\, 2 \cos(\varphi_a - \varphi_b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$i = n + \Sigma\, i_{ab} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The summation has to be taken over all combinations of two apertures.

Thus it is evident that the intensity can be obtained by superposing many fields viz. by simply adding their intensities.

The first field is uniform: n.

On this there are superposed $\frac{1}{2} n(n-1)$ other fields, each of them belonging to one pair of apertures.

The intensity of one of these pairs is

$$i_{ab} = 2 \cos(\varphi_a - \varphi_b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which value varies between $+2$ and -2.

The negative intensities in each of the $\frac{1}{2} n(n-1)$ "elementary fields" do not cause trouble, as the intensity n of the uniform field is great enough to neutralize even very many i_{ab}'s. The expression (1) namely, being derived from

$$(\Sigma \cos \varphi_a)^2 + (\Sigma \sin \varphi_a)^2$$

can never be negative.

In each elementary field (3) we have a line-system as has been described in § 2. It can be proved that along every line perpendicular to the line connecting the apertures a and b (3) has a constant value. From one line in that direction to the other i_{ab} fluctuates. On the line through the centre O $\varphi_a = \varphi_b$, $i_{ab} = +2$. There are equidistant maximum and minimum lines, $i_{ab} = +2$, $i_{ab} = -2$.

5. It is interesting to remark the following: if homogeneous light is used the maximum and minimum lines cover the *whole* field of view. It is to be expected that a great number of such line-systems crossing each other in all possible directions give a "granular" structure. This would be the granular structure, which according to v. LAUE's theoretical considerations must be observed with homogeneous light.

The question, whether the classic theory can explain the observations of v. LAUE would be answered immediately, if the experiment could be made with really homogeneous light. As to the central spot, this may be done, but because of the small intensities it is already very difficult for the first ring. From our different experiments some will be described:

1st. By the aid of WRATTEN filters the light was first more or less monochromatized. Here as in the following experiments the three different screens were used, that with the apertures, the glass-plate with lycopodium and that with mercury. The fibrous structure at the centre of the central diffraction spot vanished and became granular. At a greater distance from the centre the fibres in the central spot remained. It is interesting to remark that the length of each fibre does not increase proportionally with the radius. The fibres near the periphery are relatively much longer than those nearer the centre of the spot.

2nd. The sodium light-source was used. Now the central spot was beautifully granular even at the periphery. Of the first ring a weak shadow was observed. It is difficult to say whether it contained anything radial. The sodium light was formed by the flame of a BUNSEN burner in which a spoon with sodium was held. Strong light and a high temperature of the flame were avoided, though by means of these a very intense sodium-light can be obtained, as e. g. DU BOIS [1]) described. This was done in order to obtain higher monochromatisy. The light-source being therefore weak the observations were not made with a telescope, but with the naked eye. The glass-plate was fixed close to the eye which instinctively was accomodated to the illuminated aperture.

The observations were made in an absolutely dark room. The flame was placed in a perfectly closed chest of iron with a communication tube to the air. Along a fixed sliding the spoon with sodium could be brought into the flame, so that never any light except through one single fine aperture left the chest.

6. With *natural light* the phenomena are quite different from those observed with homogeneous light. Instead of (2) we then must

[1]) DU BOIS. Zeitschr. f. Instrumentenkunde, 1892, p. 165.

take a summation of similar expressions for the different wavelengths, so that instead of i_{ab} we may write

$$S i_{ab}.$$

The sign of summation S refers to the different wavelengths. The distribution of intensity is now a line system with a limited number of lines, one light line in the middle and on both sides light and dark ones. At some distance $S i_{ab} = 0$, because there where for one wavelength $\cos(\varphi_a - \varphi_b)$ has a positive value, there will be a wavelength which is hardly different from it (and therefore makes the same impression in our eye) for which this expression has a negative value.

With natural light we thus have the superposition of a uniform field and of *narrow* line-systems crossing each other near the centre.

From this we may conclude: *if* the classic theory can explain the phenomena investigated by v. LAUE, *these* line systems are the fibres observed by him. But whether the theory can furnish the explanation remains for the present more or less dubious.

This is evident from the following considerations. In each elementary line-system a, b the intensity varies between -2 and $+2$, while in the uniform field it has the much higher value n. If there are e.g. 10000 grains, the superposition of one line system on the uniform field will give fluctuations from 10002 to 9998, which it is of course impossible to observe.

One single line-system is thus undetectable. But the lines connecting each pair of apertures and therefore also the line-systems perpendicular to those lines have all possible directions determined by probability. Lines visible on the background n can be formed when accidentally a number of line-systems has so nearly the same direction that at a distance from the centre which is not too great, the maxima $(+2)$ of one system coincide with those of another system. The question whether the classic theory can explain the phenomenon may thus be formulated as follows:

I. Is such an accidental accumulation of different line-systems in a definite direction to be expected often enough, according to the theory of probability?

The theoretical treatment of this question will be left aside here. Only the following may be remarked: In reality many line-systems will fall out. We do not work namely with a light-point, but with a source of certain dimensions (aperture in the screen). Each point of this source gives its own diffraction image and all these images are shifted with respect to each other. In this way the finest line-systems are effaced, and not only this, but they even vanish *totally*, the mean value of $cos(\varphi_a - \varphi_b)$ being *zero*. As now the distance between the lines a, b is inversely proportional to the distance a, b every line-system corresponding to apertures or grains at a considerable distance from each other will vanish. It remains however questionable whether this vanishing of a number of line-systems increases the visibility of the others.

7. The phenomena may also be treated in a different way (though of course equivalent with the former). We may namely calculate first the total resulting distribution of intensity i for one wavelength, and then superpose all these distributions for the different wavelengths (Si). Doing this, we must keep in mind that the same intensity which for a wavelength λ occurs at a point P, is found for another λ' at a point P' which lies with O and P on a straight line in such a way that $OP : OP' = \lambda : \lambda'$.

According to the classic theory the distribution of light i is "granular" for a definite λ. If we pass to another wave-length λ', the light and dark spots are shifted in radial direction. This will be called "spectral shift".

It is evident that if we pass from monochromatic light to homogeneous light the grains will in this way be changed into fibres.

Exner [1]) thought that the observed fibres could be sufficiently explained in this way. Von Laue combatted this and drew the attention to one of his photographs of the diffraction figure of a plate covered with lycopodium powder on which the fibres in the first diffraction ring are longer than would correspond to the spectral shift.

(See also the end of § 5).

8. In my view v. Laue has paid here no attention to the fact that by the running into one another of the light spots and also of the dark ones, there may be formed fibres, longer than would correspond to the elongation of each separate spot. Let us consider different lines starting from O. Because of the accidental distribution of the light and dark spots (in consequence of the accidental distribution of the lycopodium grains) the distances between the dark spots will be somewhat smaller on one radius than on another. At a certain distance from the centre we shall sooner see a somewhat dark line along one radius than along another. In this way the fibres would be formed. The question is now:

[1]) Sitzungsber. Akad. Wien **76** (1877), p. 522; Ann. d. Phys. **9** (1880), p. 239.

II. This accidental predisposition to run into one another, which according to the laws of probability may be expected along some lines (and which according to the ordinary laws of optics is only a consequence of the irregular distribution of the refracting particles) will it suffice to explain the existence of fibres of the observed length?. Evidently the questions I and II come essentially to the same.

Only when these questions had to be answered in the negative we should to have to conclude, that classic optics were not sufficient to explain the observations. We should then be compelled to assume that already with homogeneous light radial fibres would be found. And as was remarked above it is very difficult to decide this in a direct way. The experiments made with sodium light however do not point in that direction. Another possibility would be that except the above mentioned accidental predisposition there existed still *another* along some radii. Both this and the former possibility would compel us to alter the fundaments of the diffraction theory.

As to the running into one another of the fibres formed by the spectral shift, this will evidently occur the sooner the longer the fibres are. Now the fibres formed by the spectral shift have a length proportional to the distance to the centre 0, so that the running into one another will occur more at a certain distance from the centre than in its immediate neighbourhood. With this the fact is in agreement that, as was mentioned in § 5, the length of the observed fibres increases towards the periphery more than would correspond to the distance from the centre.

8. Finally some experiments will be described, which (together with the experiments with the sodium light) support my conception of the phenomenon.

Investigating the diffraction phenomenon of a screen with about fifty apertures we can prove in a very direct way that in fact the diffraction figure is built up of interference lines. To do this experimentally the ocular of the telescope had been fixed on a small car, which could be moved to and fro over a pair of rails. It is evident, that by a displacement of the car the structure in the diffraction image formed in the focal plane of the objective is effaced in a direction perpendicular to that of the motion. Only when an interference fringe has the direction of the motion it remains unchanged.

In fact, when in the indicated way the ocular is quickly moved to and fro, we firstly see how the fibrous structure of the diffraction image is changed into a homogeneous white spot on which secondly interference fringes in the direction of the motion are seen very distinctly.

If the ocular is moved in an arbitrary way, but so that continually it remains sharply focussed on the light-point in the leaden screen, interference fringes are seen rotating on a homogeneous white spot.

This experiment cannot be made with a screen with a very great number of apertures. Probably this is caused by the fact that in this case the number of interference fringes which have the same direction becomes so great. The distance between the parallel fringes is generally very different, by the superposition only the central fringe will therefore remain and a set of parallel interference fringes in the direction of motion will not be seen.

The second experiment that will be described was suggested to me by Prof. H. A. Lorentz.

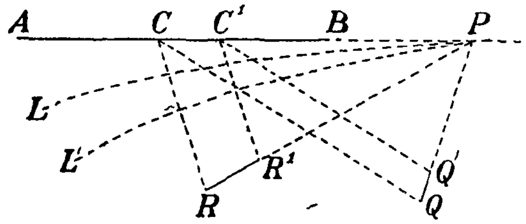
The reasoning leading to it was the following: when with purely monochromatic light the distribution of the intensity is granular, the fibrous character of the diffraction image will be due to the spectral shift of the grains from the violet towards the red. But when now this spectral shift, which forms part of the nature of the phenomenon, is the only cause of the formation of the fibres, we may expect that by a second artificial spectral shift the fibres will be no longer radially directed. We have really succeeded in producing fibres in other directions. When before the objective of the telescope, between this lens and the diffracting plate we place a prism[1]), all diffraction images from the violet towards the red will be shifted in a definite direction, the red image over the greatest distance. The light-point at the centre of the circular diffraction figure, the image of the aperture in the leaden screen, is changed into a light-strip AB, red at the end A and blue at the other end B. If we consider the thus formed diffraction figure on the production of AB on the side of B, we shall reach a point P where the original spectral shift belonging to the diffraction image is neutralized by the superposed spectral shift due to the prism. In the neighbourhood of this point the structure must be granular, while the newly formed fibres at some distance from this point must be directed towards it. These phenomena can easily be observed. So this experiment too proves that the fibres are due to a spectral shift.

They are most easily observed when a well-chosen Wratten-filter is kept before the eye; with such a plate, through which of the

1) Also a small prism may be held between the ocular and the eye.

line-spectrum AB only a part in the green of the length $\frac{1}{6}$ AB could pass, a granular structure was observed in the immediate neighbourhood of the point P. This structure was quite of the same nature as that which under ordinary circumstances was seen near the point O. At a greater distance from P small stripes were observed along lines directed towards P; the length of these stripes increased with the distance from P.

The question may be illustrated by a figure. Let CC' be the part of the linear spectrum transmitted by the filter and let us consider



the two extreme wavelengths for which the image of the light-point lies at C and C'. Let the wavelengths belonging to C and C' be λ and λ, so that, A being the red end of AB, $\lambda > \lambda'$.

Suppose that, working only with the wavelength λ, we saw a light grain at the point Q. It is evident that, working with the wavelength λ' alone, we should see the corresponding spot at a point Q', which is found by drawing $C'Q'$ parallel to CQ and by determining the length of $C'Q'$ by the equation:

$$CQ : C'Q' = \lambda : \lambda'.$$

For intermediate wavelengths the light-spots fall between Q and Q' viz., as CC' is relatively short on the straight line QQ'. The production of this line cuts that of AB at the point P, which is determined by the equation

$$CP : C'P' = \lambda : \lambda', \quad \ldots \ldots \quad (4)$$

and which has therefore always the same position, *whichever* strip of light QQ' we may consider. When further, RR' is a second strip of light, it can be proved, that the lengths QQ' and RR' have the same ratio as the distances $Q'P$ and $R'P$.

Near P the lines were so short, that they could not be distinguished from "grains". When the wavelength λ gave a light spot just at P, the corresponding light spot of the wavelength λ' would coincide with it, as is evident from (4). The existence of a light-spot at P involves of course that without prism we should observe a radially directed line of the length CC'.

We must observe, that the absence of fibres in the immediate neighbourhood of P *proves* that no fibres will be seen with homogeneous light and that those observed with non-homogeneous light are only due to the spectral shift considered in § 7. It is namely evident, that a line at P directed along CP which existed already with monochromatic light, could never be reduced to a point by means of a prism.

Finally we must remark the following. At great distances from P it is no longer true that the fibres are directed along *radii* with P as a centre. In reality the lines along which the fibres are directed have approximately the course indicated in the figure by LL'. To explain this we should have to enter into too many details.