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by the sensory reaction of smell (of trinitrobutyltolnol e.g. 1.10-16 mgrms is distinguishable in one litre of air). In the present experiments we purposely used a rather sensitive electroscope. Our first care was insulation and the aroiding of sources of errors. It is, therefore, not out of the bounds of probabılities that for judiciously selected and very sensitive instruments the electrification-phenomenon and the sense organ will appear to vie with each other in giving the reaction. Apart from experimental conditions, the small quantity of the substance, manifesting itself by virtue of electrification is, as I think, dependent on molecular weight, on volatility and on a lowering effect upon the surface tension. They are the very factors constituting the physical conditions that must be fulfilled by a substance to act biologically as an odorous substance.

Anatomy. - "On the determination of the position of the maculaplanes and the planes of the semicircular canals in the cranium". By Dr. H. M. de Burlet and J. J. J. Koster. (Communicated by Prof. H. Zivaardemaker).
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\$1. In order to be able to give an exact determination of the position of the macula-planes and the planes of the semicircular canals in the cranium, a first requirement is to connect the situation of these planes with fixed data taken from the cranium, which can easily be found again in each specimen. Absolute value cannot be assigned to the most exact determination, because the data taken from the cranium are always liable to variation; such a determination must therefore always be taken as an individual one.

By comparison of found sizes with different indiriduals of one and the same species, an impression may be obtained about the variation and an average position can be approximated.

The following refers to the rabbit, on whose cranium-basis we have fixed a line by two points, which points can easily be traced in each rabbit-cranium; The situation of these points moreover being so, as to enable us to demonstrate them accurately in seriessections. Pl. 1 shows the rabbit's cranium-basis seen from above. An imaginary line $a b$, connects the Incisura intercondyloidea ( $a$ ) with a little spina (b), which regularly appears at the proximal end of the basi-occipitale. This line, which is the starting-point of our determinations, is almost conform with the line of intersection of the

medium plane, with the top-level of the basioccipitale. This last description is to be regarded as an aid to orientation; with the presently following determinations the imaginary line $a, b$ (craniumbase-line) only plays a part.

In order to determine the position of any plane in the cranium (e.g. a maculaplane) two angles must be known, viz.:
$1{ }^{1 \text { st }}$. the angle, which the desired plane forms Pl. 1. The basioccipitale with the medium plane, of the rabbit seen from $2^{\text {nd }}$. the angle between cranium basisline and above.
the line of intersection of the planes mentioned under 1. Thus we can first of all determine the position of the four macula-planes and that of the six planes of the semicircular canals in the cranium.

Moreover it is important to know the mutual position of these planes, i. e. we must try to find the angle:
$\boldsymbol{a}$, between the two planes of the posterior semicircular canals (Plane of posterior canal left and right $P P C L$ and $P P C R$ ),
$\beta$, between the two planes of the superior semicircular canals ( $P S C L$ and $P S C R$ ),
$\gamma$, between the two planes of the exterior semicircular canals ( $P E C L$ and $P E C R$ ).

I $R$, between the planes of the superior and the exterior semicircular canal on the right ( $P S C R$ and $P E C R$ ).

II $R$, between the planes of the posterior and the exterior semicircular canal on the right ( $P P C R$ and PECR).

III $R$, between the planes of the posterior and the superior semicircular canal on the right ( $P P C R$ and $P S C R$ ).

I $L$, between the planes of the superior and the exterior semicircular canal on the left ( $P S C L$ and $P E C L$ ).

II $L$, between the planes of the posterior and the exterior semicircular canal on the left ( $P P C L$ and $P E C L$ ).

III $L$, between the planes of the posterior and the superior semicircular canal on the left ( $P P C L$ and $P S C L$ ).
$U U$, between the Utriculusmacula-planes ( $U L$ and $U R$ ).
$S S$, between the Sacculusmacula-planes ( $S L$ and $S R$ ).
SUR, between the planes of Macula utriculi and Macula sacculi on the right ( $U R$ and $S R$ ).

SUL, between the planes of Macula sacculi and Macula utricelli on the left ( $S L$ and $U L$ ).
§2. We now come to the question, how to obtain the knowledge of the desired data. Various attempts have been made to define the angles between the macula-planes and the planes of the semicircular canals, by direct measuring either on casts from the labyrinth, or on enlarged models of it. We shall more fully refer to this elsewhere. Suffice it to say, that according to our opinion, the question put here, first of all lends itself to a mathematical treatment, as it guarantees a great deal of accuracy. Let us first discuss that which refers to the position of the planes of the semicircular canals, later on that which refers to the postion of the Macula-planes.

## A. The planes of the semicircular canals.

The first difficulty which offers is that a plane of a semicircular canal in general does not exactly lie in one level. Apart from the thickness of the tube, the semicircular canals show a divergence from the level plane, which may perhaps best be indicated by the term "swaying". In our determination no account has been taken of this factor however; we have been contented with characterizing a plane of a semicircular canal by taking three fixed points. ${ }^{1}$ )
Two of these points coming into consideration for the determination of the plane of the semicircular canal are obvious, they are:
point $a$, the place where the semicircular canal runs into the utricle,
point $b$, the place where the ampulla narrows into the actual semicircular canal.
The third point (c) must be determined on the circumference of the semicircular canal. In order to get to work systematically, this point was taken at an equal distance from $a$ and $b$, with all semicircular canals, measured on the circumference.

In Pl. 2 a semicircular canal is shown; the plane through the points $a, b$, and $c$ of this semicircular canal is plane $S$. Plane $S$ cuts the interperpendicular planes of projection of $P_{1}, P_{2}$ and $P_{3}$ along the lines $S_{1}, S_{3}$, and $S_{3}$.

The distances between the points $a, b$, and $c$ to the three planes of projection (being the projections from $a, b$ and $c$ ) are known, as will be expounded later on. This enables us to determine the passages of plane $S$, by means of the method usual in descriptive geo-

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Fig. 2.
metry. It would take us too far to expound such a construction here, it can be found in any book on descriptive geometry. For explanation see PI. 3; 3a shows a three dimensional representation conform to Pl .2 in which the projections of the points $a, b$ and $c$ are also indicated, whilst in $\mathrm{Pl} .3 b$ the projections and the passages have been put down in one plane.

$a$
Fig. 3.
$b$
Projections of the points of defiation and of the passages of a semicrrcular canal.
The passages of a second plane of a semicircular canal in the same system of projection having been determined according to this method, the angle between these two planes of semicircular canals can be construed and measured in the way usual in descriptive geometry. Pl. $4 a$ and $4 b$, reproduce the mode of constuction of the angle


Consliuction for the determmation of the angle between two planes cutting one another.
between two planes.
Making use of the same system of projection we are thus able to construe not only two, but all six planes of semicircular canals and to measure the angles $a, \beta, \gamma, I R$, II R, III R, IL, II L, III L (see page 2).

As we now know the angles, which the planes of the semicircular canals form between them, there still remains to be determined, the position of the planes of the semicircular canals in the cranium. As has been expounded this requured the knowledge of:
$1^{\text {st }}$. the angle made by a plane of a semucircular canal with the medium plane,
$2^{\text {nd }}$. the angle between the cranium basishne and the intersecting line of the planes mentioned under 1.

The method we have been following up to now, has made known to us the size of the angles $\alpha, \beta$ and $\gamma$, these are the angles formed by conformable planes of semicircular canals, on the right and on the left. In the case of absolute symmetry, the medium plane must be a plane that cuts the angles $\alpha, \beta$ and $\gamma$ into two ; in other words the half of the values found above for the angles $\alpha, \beta$ and $\gamma$ represents the size of the angle asked under $1^{1 \text { sst. }}{ }^{1}$ )

We know the projections of both points of definition of the cranium base-line, just as we likewise know the projections from the points $a, b$ and $c$ of the semicircular canals, which will be discussed more in detail.
${ }^{1}$ ) The question as to how far we can admit the existence of a perfect symmetry between the right and left labyrmth will be discussed in a delaled article on this subject to be published later on.

The construction of the angles $\alpha, \beta$ and $\gamma$, made us find the lines of intersection of conformable planes of semicircular canals, lying in the mediumplane, on the left and on the right. In order to find the angle mentioned under $2^{\text {nd }}$, the construction muste still be made, leading to the measuring of the angle formed by those two lines; thus making this question a simple problem of descriptive geometry. But now comes the question : how to get our planes of projection. Before entering upon this subject it will be necessary, to mention some technical details about the treatment of the objects.
The material examined consisted of three rabbits, treated in the following way:
The recently killed animal was rinsed from the aorta with Mülulre's" liquid.

The back part of the cranium (including the hypophyse; the cranium-roof remained intact, the brains were removed) was then treated according to Witmanck's method ${ }^{2}$ ), but this exception that the bone was decalcinated after the impregnation in celloidine. The decalcination in the celloidine offers the advantage, that no removal of the labyrinths with regard to each other can take place. In first decalcinating and consequent enclosing, mistakes caused by removal may take place. The celloidine-block was consequently fastened on the microtome, in such way as to make the direction of the cut. an almost frontal one. The sections made at the first beginning do not contain the object yet.

Before the object appears in the sections, a plane has been formed on the celloidine-block, being the plane of the direction of the cut, which remains' the same all througlr the further manipulation. Round canals were bored perpendicularly in the celloidine-block on this plane, which canals appear as round holes in every section. This was obtained by placing a brass block bored by canals, on the plane of the direction of the cut. A hollow needle pierced through these canals into the celloidine-block gets the direction of the canals in the brass block, these canals being made in such a way, as to cause them to stand perpendicularly on the base, in other words the canals made in the celloidine-block stand perpendicularly on the plane of the direction of the cut.
Pl. 5: schematically sketchés six sections lying on top of each other, e.g. $25 \times$ enlarged. $P_{3}$ is a plane through the axis of two bored canals. $P_{2}$ is the plane on which the sections lie (the plane of the direction of the cut) $P_{1}$ is a plane standing perpendicularly on $P_{3}$ and $P_{3}$;
${ }^{\text {I }}$ ) Zeitschrift fär Ohrenheilkunde. Bd. 51. bldz. 148.


PI. 5. Explanation: See text.


Pl. 6. Part" of cut 211. Series Ia a point of definition of a Semicircular canal. U. Utricle. S. Saccule. C. Ductus cochlearis. Cer. Cerebellum.
f. Nervus Facialis. St. Stapes.

Pl. 6 shows part of the top section shown in Pl. 5. The circles $A$ and $B$ are the sections of the bored canals. Line $P_{8} P_{8}$ connecting the centres of these circles is the line of intersection of Plane 3 with the plane of the direction of the cut, while line $P_{1} P_{1}$ shows the line of intersection of Plane 1 , with the plane of the direction of the cut. If point $a$ (pl. 6) is a point of definition of a semicircular canal, the ordinates of this point $a a_{1}$ and $a a_{3}$ can be easily determined, viz. by direct measuring. The ordinate $a a_{2}$ depends on the number of the section in the series, its size is determined by the thickness of the sections and the degree of enlargement. (See Pl.5).

It will be easy to understand now, that in this way, not only from a point $u$, but also from 18 points, being $6 \times 3$ points of definition of six planes of semicircular canals, the ordinates, with regard to the planes $P_{1}, P_{2}$ and $P_{3}$ which are used as planes of projection, can be determined.

By means of the projections of these 18 points 6 planes can be construed, according to the method of descriptive geometry and the angles between those planes can be measured, conformable to the description given above.

## B. The planes of the otoliths.

In order to determine the position of the planes of the semicircular canals, it has proved necessary to neglect one property viz. "the swaying". Similarly when determining the position of the planes of the otoliths, it must be kept in view, that these planes do not exactly answer to the membranes of the otoliths. A factor of curvature is neglected here as well, the sacculus- and the utriculus-otolith-membranes viz. both forming curved planes, which are hard to represent in our calculation. What has been described as the otolith-membrane here, is consequently a simplified "flattened-out" otolith-membrane. The mistake arising out of this is very small as regards the utricule otolith-membrane, as this membrane almost answers to a level plane. It is different with the saccule otolithmembrane of which the greater distal part is almost entirely level as well; the proximal part however is bent fairly strongly to the lateral side.

As is known the otolith-membranes have more or less the form of an ellipse, of which the long axis runs from proximal to distal.

The beginning and the end of this long axis can be determined in our series, by examining the section in which the said otolithmembrane comes first, and that in which it comes last.

In order to determine the otolith-plane, this line in itself does not suffice, other data must still be got from the otolith-membrane. For this purpose we choose a section lying halfway between the two extreme points of the long axis. In such a section the otolith-membrane shows itself as a line; (Pl. 7 representing part of the section in


Pl. 7. Utricle and Saccule from Pl. 6, by greater enlargement, o. m. otolithmembrane.

Pl. 6, by greater enlargement); a line however not cutting the long axis (middle-line). For the otolith-plane we take a plane brought through the long axis, running parallel to the line last mentioned.

For the construction of an otolith-plane we thus must know the projections of four points, taken from the otolith-membrane: first the projections of the extremes of the long axis; secondly the projections of two points of the middle-line. The execution of this construction thus depends on constructing a plane through a given line (the long axis) running parallel to another line (the middle-line) which two lines do not intersect. From the four planes (left and right utricle and saccule otolith-planes) thus obtained, the angles which they form between. them, are similarly determined, as has been described for the planes of the semicircular canals; the same planes of projection serve here.
§3. The information required in $\$ 1$ can be got not only by means of descriptive geometry, but also by another mathematical method.

If of each of two given planes three points are known by their coordinates, with regard to three planes, running perpendicularly, we are able to determine the angle between the two giren planes, with the aid of analytic geometry.

Prof. Ornstein, who has pointed out to us`this possibility was kind enough to give us some formulas, with the help of which we were able to calculate the angles required between the planes of the semicircular canals and those of the otoliths.

We once more want to render thanks to Prof. Onnstein, very much appreciating the trouble he took in introducing this matter to ns.

This method had the great advantage for us, that the results we first obtained by means of descriptive geometry, could be compared with the results got from the formulas. Through this we disposed of a welcome means for controlling the accuracy of our drawings. It lies beyond our reach to enter into details on the derivation of the formulas used here. Suffice it to represent the method in short:

Formula for the calculation of the angle of two planes.
Let the courdinates of the three points in the first plane be

$$
\begin{array}{llll}
x_{1}, y_{1}, z_{1} & 1 & \text { point. } \\
x_{2}, & y_{2}, & z_{2} & 2 \\
x_{3} & y_{3}, & z_{3}, & 3
\end{array}
$$

Secondly calculate

$$
\begin{aligned}
& y_{1}\left(z_{3}-z_{3}\right)+z_{1}\left(y_{3}-y_{2}\right)+\left(y_{2} z_{3}-z_{2} y_{3}\right)=A_{1} \\
& z_{1}\left(x_{2}-v_{3}\right)+v_{1}\left(z_{3}-z_{2}\right)+\left(z_{2} x_{3}-v_{3} z_{3}\right)=A_{2} . \\
& x_{1}\left(y_{2}-y_{3}\right)+y_{1}\left(x_{3}-x_{2}\right)+\left(x_{2} y_{3}-y_{2} x_{3}\right)=A_{3} .
\end{aligned}
$$

Let the coordinates of the three points in the second plane be

$$
\begin{aligned}
& x_{1}^{\prime}, y_{1}^{\prime}, z_{1}^{\prime} 1^{\text {st }} \text { point. } \\
& x_{2}^{\prime}, y_{2}^{\prime}, z_{2}^{\prime} 2^{\mathbb{a}} \text { point. } \\
& x_{3}^{\prime}, y_{3}^{\prime}, z_{3}^{\prime} 3^{14} \text { point. }
\end{aligned}
$$

Thirdly calculate

$$
\begin{aligned}
& y_{1}^{\prime}\left(z_{2}^{\prime}-z_{3}^{\prime}\right)+z_{1}^{\prime}\left(y_{3}^{\prime}-y_{2}^{\prime}\right)+\left(y_{2}^{\prime} z_{3}^{\prime}-z_{2}^{\prime} y_{3}^{\prime}\right)=A_{1}^{\prime} . \\
& z_{1}^{\prime}\left(x_{2}^{\prime}-w_{3}^{\prime}\right)+x_{1}^{\prime}\left(z_{3}^{\prime}-z_{2}^{\prime}\right)+\left(z_{2}^{\prime} x_{3}^{\prime}-v_{2} z_{3}^{\prime}\right)=A_{2}^{\prime} . \\
& x_{1}^{\prime}\left(y_{2}^{\prime}-y_{3}^{\prime}\right)+y_{1}^{\prime}\left(x_{3}^{\prime}-v_{2}^{\prime}\right)+\left(x_{2}^{\prime} y_{3}^{\prime}-y_{2}^{\prime} x_{3}^{\prime}\right)=A_{3}^{\prime} .
\end{aligned}
$$

So, when $Q$ is the angle of the planes

$$
\begin{equation*}
\text { Cos. } Q=\frac{A_{1} A_{1}^{\prime}+A_{2} A_{2}^{\prime}+A_{3} A_{2}^{\prime}}{V\left(\overline{A_{12}+A_{2}^{2}}+{\left.A_{8}{ }^{2}\right)\left(A_{1}^{\prime}+A_{2}^{\prime}\right.}^{2}+{A_{3}^{\prime}}^{2}\right.} . \tag{1}
\end{equation*}
$$

The coordinates of the points on which these calculations are founded, must be taken from the data given in $\$ 2$, as here also we can use the same planes of projection as interperpendicular planes. (Page 54). For the four otoliths-planes and the six planes of the semicircular canals, the magntudes $A, A^{\prime}, A^{\prime \prime}$ etc. can be determined; by inserting these values to formula (1), we get the size of the angle $Q, Q^{\prime}, Q^{\prime \prime}$ etc. being what we wished to know.

## $\$ 4$. As has been stated the material investigated consisted of

| Size of the angle between |  |  | Series I |  | Series II |  | Series III |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | drawn | calculated | drawn | calculated | drawn | calcu- <br> lated |
| ${ }^{1}$ ) $P P C L$ | PPCR | $\%$ | $961 / 8^{\circ}$ | $96{ }^{\circ}$ | $102{ }^{\circ}$ | $103.36^{\prime}$ | $99^{\circ}$ | $97.25^{\prime}$ |
| 2) $P S C L$ | PSCR | $\beta$ | 78 | $77.54{ }^{\prime}$ | 81 | $81.31^{\prime}$ | 88 | $86.50{ }^{\prime}$ |
| 3) PECL | PECR | 1 | 176 | $176.3^{\prime}$ | 174 | $173.33^{\prime}$ | 1711/2 | $170.43^{\prime}$ |
| PSCR | PECR | $I R$ | 95 | $95.48^{\prime}$ | 91 | $90.44^{\prime}$ | 92 | $91.35^{\prime}$ |
| PSCL | PECL | $I L$ | 95 | 95.35' | 95 | $96.30^{\prime}$ | 941/2 | 94.54' |
| PPCR | PECR | IIR | 921/2 | $91.10^{\prime}$ | 951/2 | $97.4^{\prime}$ | 81 | $80.44{ }^{\prime}$ |
| PPCL | PECL | III. | 931/2 | 93.56' | 91 | $91.7^{\prime}$ | 871/2 | 89.23' |
| PPCR | PSCR | IIIR | 93 | $94.23{ }^{\prime}$ | 87 | 87.2' | 88 | 89.14' |
| PPCL | PSCL | IIIL | 94 | 93.56 ${ }^{\prime}$ | 87 | $87.47^{\prime}$ | 87 | 86.36 ${ }^{\prime}$ |
| ${ }^{\text {1 ) }}$ Mediumplane | PPC |  | $481{ }_{4}$ | 48 | 51 | $51.48{ }^{\prime}$ | 491/2 | $48.42^{\prime}$ |
| $\left.{ }^{2}\right) \quad 1$ | PSC |  | 39 | $38.57^{\prime}$ | 401/2 | 40.45' | 44 | $43.25^{\prime}$ |
| 3) » | PEC |  | 88 | 88.2' | 87 | $86.46{ }^{\prime}$ | 85.45 | $85.21^{\prime}$ |
| 4) Craniumbase-line | Lineof int. PPC |  | 86 |  | 89 |  | 88 |  |
| 5) » | , PSC |  | 78 |  | 82 |  | $851 / 2$ |  |
| ${ }^{6}$ ) $\quad$ | " PEC |  | 151/2 |  | $\left.{ }^{8}\right)$ |  | 1 |  |
| SL | $U L$ | SUL | 107 | 107.18' | 102 | $97.24{ }^{\prime}$ | 98 | $104.51^{\prime}$ |
| SR | $U R$ | SUR | 103 | 103.14' | 99 | $96.33{ }^{\prime}$ | 100 | 96.50' |
| $\left.{ }^{7}\right) S L$ | $S R$ | SS | 461/2 | 47.9' | 54 | 53.31' | 63 | $63.56{ }^{\prime}$ |
| 3) $U R$ | UL | $U U$ | 174 | 174.56' | 173 | $166.17^{\prime}$ | 176 | $172.21^{\prime}$ |
| ${ }^{\text {7) }}$ Mediumplane | $S$ |  | $231 / 4$ | $23.34^{\prime}$ | 27 | $26.45{ }^{\prime}$ | 311/2 | 31.58' |
| 3) $\quad \square$ | $U$ |  | 87 | 87.28 ${ }^{\prime}$ | 861/2 | 83.8' | 88 | $86.11^{\prime}$ |
| 5) Craniumbase-line | Line of int. SS |  | 35 |  | 44 |  | 681/2 |  |
| 4) " | " UU |  | 39 |  | 31 |  | ${ }^{6} 6$ |  |

(1) This angle is open to the back.
${ }^{2}$ ) " " " " "front.
" " " " " top.
${ }^{4}$ ) " " " " " back, it is measured above the craniumbase-line.
5) " " " " " "front, " " " "
${ }^{6}$ ) " " " " " " back, "n " ${ }^{7}$ " " "
${ }^{8)}$ The line of intersection of the two horizontal semicircular canals does not come in the mediumplane in this preparation; therefore the value cannot be given here.

- three serres. For each series the data raquired were first determined by way of descriptive, then by way of analytic geometry. The results thus obtained, were collected in a table (p. 59).

With the manipulation of the $3^{\text {id }}$ series, Mr. H. Oort (Med. Cand.) gave us his greatly appreciated help; the results, referring to this series have been determined by him.

In the detailed article on this subject, to be published elsewhere,

- we shall enter more deeply into the discussion, to which this table gives rise; some man points can be stated concisely:

1. It appears first of all, that the angles made by the planes of the semicrrcular canals, lying on one side, do not differ greatly from $90^{\circ}$.

The average ${ }^{1}$ ) value, calculated from the table, amounts to $94^{\circ}$ for the angle between the superior plane and the exterior plane, to $90^{\circ}$ for the posterior plane and the exterior plane and to $89^{1} / 2^{\circ}$. for the posterior plane and the superior plane.
2. The -planes of the exterior semicircular canals on both sides, make between them an average angle of $173{ }^{1} \stackrel{2}{2}^{\circ}$, their line of intersection forms a small angle (once $15^{1} \%_{2}^{\circ}$, once $1^{\circ}$ ) with the cranium base-line (which is almost conform to the clivusline).

The planes of the two superior semicircular canals form between them an average angle of $82,5^{\circ}$, their line of intersection forms an angle of $82^{\circ}$, with the cranium base-line. The planes of the two posterior canals form between them an• average angle of $99^{\circ}$; ther line of intersection forms an angle of $88^{\circ}$ with the cramumbase-line.
3. The otoliths-membranes which are situated on one side form between them an average angle of $101^{\circ}$.
4. The planes of the two utricle-otoliths-membranes form between them an average angle of $173^{\circ}$; their line of intersection forms with the craniumbase-line an angle of $44^{\circ}$.
The planes of the two sacculus-otoliths-membranes form between them an average angle of $54^{1} / 2$, their line of intersection forms with the craniumbase-line an angle of $49^{\circ}$.

[^1]
[^0]:    1) Greater accuracy may be obtaned in this respect. With the aid of analytic geometry one can attain the determination of a plane, which takes into account the swaying of the semicircular canal and consequently gives us the position of it more accurately.
[^1]:    ${ }^{1}$ ) To these average values calculated from a small number of observations, no great importance should be attached; therefore these numbers have not been mentioned in the table.

