growth in experiments lasting $3^1/_2$ hours; that drawn in full represents the growth in experiments of 7 hours. At and above 35° practically no growth takes place in the second period of $3^1/_2$ hours.

In conclusion the temperature coefficient has been calculated for intervals of 10°, relating to the observations during $3^1/_2$ hours.

We see, as has been indicated by COHEN STUART[1]) in his study of the subject, that VAN 'T HOFF's rule at most applies only over a small range; for the rest the coefficient falls with rise of temperature.

Utrecht, April 1916. *Botanical Laboratory of the University.*

Astronomy. — "*Determination of the constant of Precession and of the Systematic Proper motions of the stars, by the comparison of* KÜSTNER'S *catalogue of* 10663 *stars with some zone-catalogues of the "Astronomische Gesellschaft"*". By C. DE JONG. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of April 28, 1916).

1. *Introduction.*

The research, the results of which will here be given in an abbreviated form, originated in a subject for a prize essay which the University of Leiden gave out in 1914, that of determining the constant of precession and the systematic proper motions by a comparison of KÜSTNER'S Catalogue of 10663 stars (Veröff. Bonn N°. 10) with Zone-Catalogues of the "Astronomische Gesellschaft". The essay which I wrote, was awarded the prize by the Faculty of Natural Science in Leiden. Prof. E. F. VAN DE SANDE BAKHUYZEN then suggested to me to continue the research and make it into a complete whole by using all the available material, i.e. that for which the difference of epoch with KÜSTNER is not too small, and reducing it in a strictly systematic manner; this suggestion I followed willingly.

It is indeed of importance to derive the constant of Precession and the elements of the motion of the sun from the above mentioned material; it is the only combination of catalogues with, at all considerable difference of epoch in which, for both, the magnitude-error has been eliminated or determined with sufficient accuracy. For the zone-catalogues of the Astr. Ges. two determinations of the error in

1) C. P. COHEN STUART. A study of temperature coefficients and VAN 'T HOFF'S rule. Proc. Kon. Akad. van Wet. Amsterdam, 1912.

question have been made by AUWERS, while researches also have been carried out at Leiden for the zone observed there. KÜSTNER, by placing gauze screens in front of the objective of the telescope, has eliminated the error in a very satisfactory manner. Moreover I had the privilege of discussing various difficulties with Prof. BAKHUYZEN himself, and of experiencing his continued interest in my work, for which I take this opportunity of thanking him very sincerely.

2. *Material used.*

With KÜSTNER's Catalogue (Aequin 1900) the following zone-catalogues of the Astr. Ges. (Aequin 1875) were compared in R.A. and Decl.:

1. Berlin A, Decl. $+15°$ to $+20°$;
2. Berlin B, „ $+20°$ „ $+25°$;
3. Leipzig I, „ $+10°$ „ $+15°$;
4. Leiden, „ $+30°$ „ $+35°$. [1])

The positions of the latter were reduced from 1875 to 1900 by means of the mean of the precession-values given in the two catalogues (constants according to PETERS-STRUVE). KÜSTNER's catalogue (Kü) proved to have the following numbers of stars in common with the zone-catalogues, with the epochs as given:

	Number of stars	*Epochs*	*Difference of epochs*
Be A	768	1870,5—1896,5	26,0 years
Be B	812	1881,0—1897,0	16,0 „
Lei I	711	1874,3—1896,45	22,15 „
Leiden	926	1873,8—1898,0	24,2 „

Catalogue Berlin BECKER (Be B) has only a small difference of epoch with KÜSTNER, but its great accuracy compensates this to a great extent.

3. *Immediate results of the comparison.*

To the differences $\Delta\alpha$ and $\Delta\delta$ Kü—A.G. found directly by the comparison various corrections had first to be applied, in order to make them suitable for further discussion. These corrections are the following:

1. *The reductions of the A. G. catalogues to the system of* AUWERS's *fundamental Catalogue of the A. G.* These were assumed according

[1]) The comparison with Leiden in R.A. had been made before at the Leiden Observatory; I was able to use the results.

to AUWERS's tables occurring in A.N. 3844, with the exception of the A.R. Leiden, for which the magnitude equation lately deduced by Dr. E. F. v. D. S. B. was adopted; [1])

2. *the reduction of Kü to the Fundamental system of* AUWERS, only needed for the Decl. and applied according to the table in the introduction to KÜSTNER's Catalogue, p. 35;

3. the variation during the difference of epoch of the reductions $\Delta\alpha_\alpha$ and $\Delta\delta_\alpha$ of AUWERS' old Fund. Cat. to the new Cat. of the Berliner Jahrbuch (A.N. 3927);

4. the variation during the difference of epoch of the corresponding reductions $\Delta\alpha_\delta$ and $\Delta\delta_\delta$.

Reductions 1 and 2 were introduced everywhere completely, 3 was not immediately applied to the results of the comparison, while of 4 only parts were added to the coordinate-differences found. Besides 1 and 2 the following corrections were applied on account of 4:

a. to the differences Kü—A.G. in R.A.:

Be A	+ 0ˢ020	instead of	+ 0ˢ0205
Be B	+ 0.012	„ „	+ 0.0128
Lei I	+ 0.018	„ „	+ 0.0185
Leiden	0.000	„ „	+ 0.0202

b. to the differences Kü—A.G. in Decl.

Be A	— 0″25	instead of	— 0″25
Be B	— 0.14	„ „	— 0.11
Lei I	0.00	„ „	— 0.22
Leiden	0.00	„ „	— 0.21

The remaining parts of 4 were brought into account after the solution of the equations. For Be A use is also made of the "Reduction der aus den Zonenbeobachtungen abgeleiteten Deklinationen auf A. G. C." occurring on p. 131 of the Introduction to this Catalogue. The corrections 3 are solved separately.

From the differences in α and δ thus corrected various means were formed. In order to be able to test the influence of the magnitude of the stars used — either on account of remaining magnitude-errors or of cosmic influences — upon the constants derived from them, *groups according to magnitude* were formed, and that for each hour of the right ascension separately. In this it proved preferable, on account of the limited number of stars, to confine ourselves to *two* groups according to magnitude. The division was made according to the magnitudes given by Kü; the magnitude 8.50 was taken as the limit. Thus

[1]) Annalen der Sternwarte in Leiden, **9**, 386.

1. a "bright" group, called group B.
2. a "faint" group, called group F.

were formed.

Further the ever difficult question of the exclusion of stars had now to be weighed. NEWCOMB, in his deduction of the precessional constant from the BRADLEY stars, excluded all stars with proper motion above certain limits depending upon the magnitude and entirely ignored these. In this way he loses more than $^1/_5$ of the whole number of stars. In the reduction of the material of this paper it did not seem advisable to confine oneself to this method. Besides the deduction of the results in NEWCOMB's manner I made a second calculation practically *without exclusion*, that is, only some extremely large differences were excluded. The work was therefore done in two ways:

1. all stars (with the exception of a few very large differences) are used; solution *A*;
2. stars with P. M. above certain limits are excluded: solution *E*.

There were, therefore, four solutions made: BA, BE, FA, FE.

Excluded unconditionally were: (see I):

in Be A all stars with $\Delta\alpha > 0^s5$ or $\Delta\delta > 7''5$;
in Be B „ „ „ $\Delta\alpha > 0.35$ „ $\Delta\delta > 5.0$;
in Lei I „ „ „ $\Delta\alpha > 0.4$ „ $\Delta\delta > 6.0$;
in Leiden „ „ „ $\Delta\alpha > 0.5$ „ $\Delta\delta > 7.5$.

These are altogether only about twenty in number. Moreover, in all four catalogues the double stars are unconditionally excluded, being respectively 18, 31, 15 and 37 in number. It is unnecessary to point out that this last exclusion is certainly justified. As regards the unconditional exclusion on account of too large differences it may be further remarked that in this the two coordinates have also exercised an influence upon each other, as stars with too great a $\Delta\alpha$ are also excluded in δ, and vice versa. In this way the A groups were formed.

For the E groups the following values are accepted as the greatest

Limits for the *E*-groups.

Catalogue	4th magnitude		5th—7th magnitude		8th magnitude		9th magnitude	
Be A	0^s20	$3''0$	0^s15	$2''2$	0^s12	$1''8$	0^s10	$1''5$
Be B	0.12	1.5	0.10	1.5	0.08	1.5	0.06	1.5
Lei I	0.20	3.0	0.15	3.0	0.12	3.0	0.10	3.0
Leiden	0.20	2.5	0.20	2.5	0.20	2.5	0.20	2.5

allowable proper motion during the interval between the epochs.

In this the two coordinates have had no influence upon each other; a star which had to be omitted in the computation of the E-mean of the $\Delta\alpha$ on account of too large a value was *not* excluded from the $\Delta\delta$ on that account. In this way in forming the E groups for each catalogue about 100 stars were excluded.

As in the discussion of the results from the comparison between Kü. and Be A a great difference became apparent between the values of the precession-constant m derived from the bright and from the faint groups (that is from BA and BE on the one hand and from FA and FE on the other) it was thought desirable to institute a further research into this point. For this purpose the *fainter group in Berlin A was split into two, one between magnitude* 8.50 and 9.15, the other below 9.15. For both groups, called F_1 and F_2 solutions A and E were made. In the following table these groups are found as F_1A, F_1E, F_2A and F_2E.

As already said, in the formation of the E groups the two coordinates had no influence upon each other. The opposite point of view might also be defended, while it may be said in favour of the method here followed, that it is illogical to exclude stars from one coordinate because of a large accidental error in the other. In any case it seemed desirable to see what would be the influence of the exclusion, also according to the other coordinate. This was done for the R.A. of Berlin A; for this catalogue E′ groups were formed for which the same exclusion-limits were used as in the E groups, but in which exclusion also took place on account of the other coordinate. The groups formed on this principle are called BE′ and FE′.

In the following table I have collected the hourly means formed after the different principles; in this table 0^h means the group from 23^h30^m to 0^h30^m, etc. while under n the number of stars used is given. The unit is in the R.A. $0^s.01$, in the Decl. $0''.1$.

MEAN DIFFERENCES KU—A.G.

1) *Berlin A, Right-Ascensions.*

Hour	*BA* $\Delta\alpha$	n	*BE* $\Delta\alpha$	n	*FA* $\Delta\alpha$	n	*FE* $\Delta\alpha$	n
0	+ 9.15	15	+3.72	11	—1.30	12	+2.42	9
1	+10.00	10	+0.92	4	+4.42	14	—0 19	11
2	+ 5.82	12	+6.07	10	—0.01	14	+0.21	10
3	+ 2.22	9	+2.22	9	+2.46	10	+0.83	9
4	+10.56	14	+6.16	11	+2.85	10	- 0.58	9
5	+ 2.51	15	+2 07	12	+2 69	15	+2.69	15
6	— 1.01	23	—0.04	22	—0.25	23	+0.72	19
7	+ 2.45	11	+1.42	10	+2.51	19	+0.55	17
8	— 3.58	15	—1.67	14	—1.55	24	—0.73	21
9	— 1.36	17	—0.44	15	+0.31	18	—1 79	15
10	— 3.89	12	—3.89	12	—5.66	21	—2.44	17
11	— 5.70	12	-2.82	10	—7.80	20	—6.19	16
12	—12.02	8	—4.68	5	—6 83	17	—4.24	14
13	-- 5.94	11	—5.94	11	—6.03	19	—4.52	17
14	—13.89	13	—8.22	8	—4.73	18	- 6.42	16
15	— 7.00	8	—7.00	8	—4.13	23	—3.06	17
16	— 4.16	11	—2 37	10	—5.43	22	—4.22	20
17	— 4.88	14	—1.92	12	—0.81	22	—1.28	20
18	— 0.26	13	—0.26	13	0 96	19	—1.73	18
19	- 0.82	10	+0.63	9	—1.34	19	—1.34	19
20	-- 2.08	17	—2.08	17	+0 54	21	+0.54	21
21	+ 1.72	13	+0.66	12	+2.65	22	+1.24	20
22	+ 4.62	10	+4.62	10	+0.96	20	—0.22	19
23	+ 5.29	10	—0.04	7	—0.18	17	+0.67	11

Hour	F_1A $\Delta\alpha$	n	F_1E $\Delta\alpha$	n	F_2A $\Delta\alpha$	n	F_2E $\Delta\alpha$	n
0	−2.78	6	+2.62	4	+0.18	6	+2.26	5
1	+5.57	6	−2.34	5	+3.55	8	+1.60	6
2	−1.49	7	+0.13	6	+1.28	8	+0.32	4
3	+4.07	6	+1.46	5	+0.05	4	+0.05	4
4	+3.61	7	−1.40	6	+1.07	3	+1.07	3
5	+2.22	10	+2.22	10	+3 64	5	+3.64	5
6	+1.51	9	+0.79	7	−1.38	14	+0.68	12
7	+1.23	10	+1.23	10	+3.93	9	−0.43	7
8	−2.01	11	−2.01	11	−1.16	13	+0.68	10
9	−0.21	8	−3.70	6	+0.73	10	−0.52	9
10	−5.88	8	−2.62	5	−5.14	14	−2.37	12
11	−6.27	7	−4.98	5	−8.64	13	−6.74	11
12	−7.69	11	−3.49	8	−5.23	6	−5.23	6
13	−5.27	13	−4.13	12	−7.37	6	−5.04	5
14	−6.77	6	−6.77	6	−3 71	12	−6.21	10
15	−4.67	6	−2 36	5	−3.94	17	−3.35	12
16	−6.81	8	−5.66	7	−4.71	14	−3.53	13
17	−3.07	10	−1.53	9	+1 07	12	1 06	11
18	−3.14	10	−3.14	10	+1.46	9	+0.04	8
19	−0.07	12	−0.07	12	−3.53	7	−3.53	[illegible]
20	+0.38	12	+0.38	12	+0.76	9	+0.73	9
21	+3.08	13	+2.12	12	+2.32	9	+ 26	8
22	−1.31	11	−1.31	11	+3.72	9	+1 29	8
23	−2.82	8	+0.67	3	+2.18	9	+0.68	8

Hour	BE′ Δα	n	FE′ Δα	n
0	+3.72	11	+0.98	6
1	+0.92	4	+0.20	9
2	+5.95	9	+0.87	11
3	+2.22	9	+1.83	8
4	+5.97	10	−0.59	8
5	+1.22	11	+2.34	12
6	−0.04	22	+0.79	18
7	+1.42	10	+0.55	17
8	−1.67	14	−1.13	19
9	−0.44	15	−1.81	12
10	−3.89	12	−2.44	17
11	−2.82	10	−6.38	14
12	−4.68	5	−3.14	11
13	−5.94	11	−4.52	17
14	−8.22	8	−5.60	13
15	−7.00	8	−2.61	15
16	−1.63	9	−4.24	17
17	−1.92	12	−0.77	19
18	−0.26	13	−1.73	18
19	+0.63	9	−1.07	17
20	−2.21	16	+0.83	19
21	+0.47	11	+2.18	18
22	+4.62	10	−0.22	19
23	+1.55	6	+0.67	11

2) *Berlin A, Declination.*

Hour	*BA* $\Delta\delta$	n	*BE* $\Delta\delta$	n	*FA* $\Delta\delta$	n	*FE* $\Delta\delta$	n
0	−0.99	15	−0.99	15	+0.46	12	−1.64	8
1	+0.92	10	−0.97	9	−7.16	14	−6.05	10
2	−5.78	12	−5.78	12	−6.60	14	−7.08	12
3	−4.31	9	−4.31	9	−12.19	10	−7.68	8
4	−6.39	14	−4.98	12	−6.62	10	−0.79	8
5	−7.97	15	−4.92	14	−13.43	15	−10.04	12
6	−5.32	23	−5.32	23	−7.94	23	−5.51	21
7	−6.08	11	−3.11	10	−4.84	19	−5.72	17
8	−5.27	15	−2.88	14	−10.44	23	−6.20	19
9	−6.37	17	−4.39	16	−6.41	18	−5.58	15
10	−7.53	12	−7.53	12	−5.43	21	−6.66	20
11	−0.45	12	−0.45	12	−8.23	20	−5.21	17
12	−2.59	8	−2.59	8	−6.68	17	−4.74	13
13	−3.25	11	−3.25	11	−4.49	19	−3.35	18
14	+0.91	13	−0.60	10	−6.53	18	−5.53	13
15	−9.55	8	−4.64	7	−4.26	23	−4.92	20
16	−3.59	11	−8.01	10	−7.29	22	−6.18	19
17	+0.69	14	+0.69	14	−3.71	22	−2.74	21
18	+0.19	13	+0.19	13	−8.81	19	−8.81	19
19	−6.55	12	−6.55	12	−7.34	19	−5.30	17
20	−2.21	17	−0.25	16	−3.28	21	−3.46	19
21	−1.63	13	−3.21	12	−5.47	23	−4.48	20
22	−5.13	10	−5.13	10	−3.99	20	−3.99	20
23	−6.73	10	−2.35	8	−8.71	17	−4.11	13

3) *Berlin B, Right-Ascension*

Hour	BA $\Delta\alpha$	BA n	BE $\Delta\alpha$	BE n	FA $\Delta\alpha$	FA n	FE $\Delta\alpha$	FE n
0	+2.70	8	+1.10	7	+0.12	20	−0.17	18
1	+2.57	15	+1.16	12	+2.27	19	+0.01	16
2	+2.70	7	−1.94	5	−1.30	20	−0.73	15
3	+2.37	13	+0.27	12	−1.14	19	−1.53	17
4	+3.35	23	+2.46	20	+1.42	13	+0.53	12
5	+3.53	15	+1.72	13	+2.96	15	+2.65	13
6	+0.10	21	+0.91	20	−2.29	19	−0.08	13
7	+0.00	19	+0.00	19	−1.08	18	−0.72	17
8	+1.29	15	−0.67	14	−0.81	15	−0.68	13
9	−3.01	12	− 0.58	10	−1.35	22	−0.41	21
10	-2.93	12	−1.84	9	−2.06	16	−1.59	15
11	−3.27	7	− 3.27	7	−3.29	21	−1.97	16
12	−3.22	9	−3.22	9	− 4.62	24	-1.32	15
13	−4.41	12	−4.41	12	−0.87	19	−0.15	13
14	−2.70	11	−2 70	11	−4.72	20	−3.48	17
15	−2.15	8	- 2.15	8	−1.42	18	−1.77	17
16	−0.79	8	0.79	8	+0.42	20	+0.69	15
17	−1.37	12	−1.74	10	−1.74	30	−1.72	28
18	3.18	20	−2.64	19	−3.07	16	- 3.22	11
19	−0.17	24	−0.19	20	- 1.78	16	−1.30	15
20	+1.79	17	+1.79	17	−0.03	12	−0.03	12
21	+1.27	15	+1.12	13	−0.31	17	+0.95	15
22	+3.72	16	+0.68	14	−1.09	17	−0.93	15
23	+1.21	12	+1.21	12	−0.11	17	−0.13	13

4) *Berlin B, Declination.*

Hour	BA $\Delta\delta$	BA n	BE $\Delta\delta$	BE n	FA $\Delta\delta$	FA n	FE $\Delta\delta$	FE n
0	−4.75	8	−1.15	6	− 5.05	22	−4.2	19
1	−6.05	15	−6.05	15	−10.0	19	−8.0	16
2	−6.75	7	−6.75	7	− 6.3	19	−7.7	15
3	−5.65	13	−4.05	12	− 4.1	19	−3.5	18
4	−8.35	23	−5.95	19	− 8.7	13	−7.7	12
5	−8.45	15	−8.45	15	−10.0	16	−7.1	13
6	−4.75	21	−4.75	21	− 9.8	19	−7.0	17
7	−8.55	19	−5.55	15	− 8.8	18	−7.7	16
8	−5.35	15	−5.35	15	− 6.6	15	−5.3	14
9	−7.35	12	−2.55	9	− 2.9	22	−3.9	21
10	−5.85	12	−2.45	10	− 5.3	16	−4.5	15
11	−8.65	7	−4.95	6	− 9.2	20	−6.8	17
12	−3.45	10	−3.45	10	− 9.4	24	−5.6	19
13	−6.35	12	−5.25	11	−11.4	19	−6.3	13
14	−6.25	11	−6.25	11	− 4.9	20	−4.3	19
15	−5.65	8	−5.65	8	− 5.7	18	−5.7	18
16	− 6.9	8	−3.25	7	− 6.4	20	−5.2	18
17	−4.4	12	−0.05	10	− 2.3	30	−1.3	29
18	−4.85	20	−2.95	17	− 7.1	16	−6.4	15
19	−4.25	24	−3.25	20	− 5.3	16	−5.3	16
20	−0.15	17	−0.15	17	− 4.3	12	− 2.1	11
21	−2.05	15	−0.65	14	− 3.9	17	−3.9	17
22	−4.05	16	−4.05	16	− 4.9	17	−3.9	16
23	−1.55	12	−3.15	11	− 9.5	17	−8.65	16

5) *Leipsig I, Right-Ascension.*

Hour	BA $\Delta\alpha$	BA n	BE $\Delta\alpha$	BE n	FA $\Delta\alpha$	FA n	FE $\Delta\alpha$	FE n
0	− 1.60	15	−1.58	13	−1.73	13	−0.18	11
1	+ 0.37	7	+0.37	7	−1.25	22	+0.25	18
2	+10.70	7	+7 84	5	+1.80	15	−0.19	12
3	+ 3.38	10	−1 20	8	−1 88	12	−0.82	11
4	+ 8.28	13	+3 77	10	+0.22	16	− 0.28	12
5	− 0 06	9	−0.06	9	+5.98	23	+3.44	20
6	− 0.26	17	+2.24	15	+1 06	25	+0.89	23
7	− 1.62	11	1.62	11	−1.68	15	−1.68	15
8	− 7.97	15	−6.06	13	−2.58	16	−1.92	13
9	− 6 02	13	−3.56	11	−0.32	12	−3.94	10
10	− 9 00	10	−6.15	8	−3.32	11	−3.32	11
11	− 6 33	9	4.11	8	−7.28	16	−5.35	14
12	−11.04	5	−9.95	4	−5 94	16	−4.99	15
13	− 4.85	11	−6.34	8	−1.54	15	−2 32	14
14	− 4.06	10	−4.32	8	−4.06	20	−4.21	19
15	− 0.18	8	− 0.18	8	−4.24	13	− 3.05	10
16	− 4.90	10	−4 90	10	−5.40	20	−3.86	18
17	− 3.33	14	−3 33	14	−6.00	14	−3 56	12
18	− 2 92	14	− 2.92	14	−4.33	15	−2.90	14
19	+ 2 16	15	+1.46	14	− 2 40	26	−1.69	23
20	+ 1.08	13	+1.08	13	−0.10	22	+0.10	20
21	+ 2.90	22	+1.80	20	+4 32	17	+2.88	15
22	+ 2.45	12	+4.06	11	−0.73	23	− 0.02	22
23	− 1 13	7	+0 77	6	−1.20	19	−0.57	15

6) *Leipzig 1, Declination*

Hour	*BA* $\Delta\delta$	*BA* n	*BE* $\Delta\delta$	*BE* n	*FA* $\Delta\delta$	*FA* n	*FE* $\Delta\delta$	*FE* n
0	+1.47	15	+1.47	15	+ 4.31	13	+ 4.31	13
1	+9.29	7	+5.17	6	+ 2.36	22	+ 2.36	22
2	+5.71	7	+5.71	7	+ 0.07	15	+ 0 07	15
3	+1.90	10	+1.90	10	− 6.75	12	− 6.75	12
4	−5.23	13	-5.23	13	− 6 00	16	− 6.50	14
5	−1.89	9	−1 89	9	− 9.26	23	− 8.28	22
6	+1 94	17	−0.62	16	− 2 56	25	− 2.56	25
7	−2.82	11	−2.82	11	− 0.93	15	− 0.93	15
8	−5.67	15	−5.67	15	− 6.69	16	− 6.69	16
9	−7.38	13	−5.42	12	−10.00	12	−10.00	12
10	+2.40	10	+2.40	10	− 7.82	11	− 7.82	11
11	+2.11	9	+2.11	9	− 6.12	16	− 6.12	16
12	+2.00	5	+2.00	5	− 0.62	16	− 0.62	16
13	−4.64	11	+0.50	10	− 6.67	15	− 4.93	14
14	+0.20	10	+0.20	10	− 3.50	20	− 3.50	20
15	+3.75	8	−0.86	7	− 1.69	13	+ 1.00	12
16	+5.60	10	+5.60	10	− 4.35	20	− 2.95	19
17	−6.28	14	−6.28	14	− 6.57	14	− 3.69	13
18	+2.71	14	+2.71	14	− 0.93	15	− 0.93	15
19	+1.00	15	+1.00	15	− 2.73	26	− 1.52	25
20	+5.15	13	+5.15	13	− 1.77	22	− 1 77	22
21	+5.55	22	+4.29	21	+ 1.12	17	+ 1.12	17
22	+0.17	12	+0 17	12	+ 1 09	23	+ 1.09	23
23	−0.29	7	−0.29	7	− 2.74	19	− 2.74	19

7) *Leiden, Right Ascension.*

Hour	*BA* $\Delta\alpha$	*BA* n	*BE* $\Delta\alpha$	*BE* n	*FA* $\Delta\alpha$	*FA* n	*FE* $\Delta\alpha$	*FE* n
0	−3.71	12	−2.33	11	+2.91	21	+1.24	20
1	+4.34	18	+0.28	16	+1.29	19	+3.89	17
2	−7.00	16	−5.00	14	+0.95	16	+0.95	16
3	+1.04	14	+1.17	13	−5.14	23	−4.23	21
4	−3.48	10	−3.48	10	+1.89	18	−0.07	17
5	−0.48	15	−1.15	13	−0.08	24	−0.08	24
6	−1.08	18	−1.12	16	−0.10	17	−0.10	17
7	−2.06	19	−2.06	19	−1.97	22	−1.08	21
8	−2.65	18	−0.34	15	−2.41	24	−1.48	23
9	−6.35	14	−3.42	12	−3.38	25	−2.12	24
10	−7.11	16	−5.07	13	−3.90	23	−2.92	21
11	−11.21	16	−7.94	14	−6.91	22	−4.18	19
12	−12.15	17	−9.37	12	−8.98	18	−7.30	15
13	−6.26	8	−0.78	7	−5.36	31	−3.53	23
14	−0.30	5	−10.07	2	−12.26	34	−7.44	25
15	−12.60	11	−5.13	7	−10.63	27	−9.02	22
16	−9.27	12	−1.96	8	−6.14	28	−2.37	22
17	−3.51	11	−6.23	9	−3.70	25	−4.04	24
18	−2.22	14	−1.82	13	−2.30	23	−2.30	23
19	−0.14	22	−1.79	21	−0.36	17	−0.36	17
20	−3.08	24	−2.77	22	−0.60	19	−0.65	18
21	−2.21	13	−2.21	13	+4.69	20	+4.69	20
22	−0.27	9	−0.27	9	+1.97	20	+1.97	20
23	+0.14	13	−0.40	11	−2.79	17	−2.08	16

8) *Leiden, Declination.*

Hour	BA $\Delta\delta$	BA n	BE $\Delta\delta$	BE n	FA $\Delta\delta$	FA n	FE $\Delta\delta$	FE n
0	−2.50	12	−2.50	12	−2.74	21	+0.31	18
1	+0.17	18	−1.62	17	−2.32	17	−2.76	15
2	−3.56	17	−3.56	17	−0.88	16	−0.88	16
3	−10.92	14	−8.47	13	−6.12	23	−2.22	20
4	−9.75	10	−9.75	10	−9.95	18	−6.28	14
5	−13.51	15	−8.59	12	−5.82	24	−4.78	22
6	−6.34	18	−3.44	16	−5.80	17	−5.80	17
7	−5.37	19	−4.33	18	−2.73	22	−1.95	21
8	−9.55	18	−3.70	15	−2.54	24	−3.34	23
9	−5.53	14	−1.88	12	−1.98	25	−0.98	24
10	−6.06	17	−4.31	16	−6.85	23	−2.68	20
11	−4.28	16	−3.00	15	−2.29	22	−0.36	21
12	−2.24	17	−1.00	14	−4.22	18	+0.86	14
13	−8.00	8	−8.00	8	−5.58	32	−2.43	22
14	−6.57	7	−5.50	2	−0.15	34	−0.07	30
15	+4.82	11	+4.82	11	−2.33	27	−0.83	21
16	−6.86	11	+0.89	9	−0.75	28	−1.09	23
17	+0.90	10	+1.12	8	−4.12	26	−0.86	22
18	+3.36	14	+0.31	13	−1.26	23	−1.43	21
19	−1.27	22	+0.10	21	−2.44	17	−2.44	17
20	−4.75	24	−3.04	23	−1.03	19	−0.25	17
21	−0.41	11	−0.41	11	−0.10	20	+0.08	18
22	−6.28	9	−6.28	9	−1.55	20	−0.82	19
23	+0.14	14	−1.36	11	−3.67	18	−1.35	17

4. *Method of discussion the results. Solution of the equations.*

In order to render the determination of the constants as simple and as systematic as possible, and thereby to be able to conveniently make use of the results obtained in a previous research by Dr. E. F. v. d. Sande Bakhuyzen and myself concerning the influence exercised upon the determination of the constant of Precession and the systematic Proper-motions by the connection between the value of the parallax of the stars and their apparent distance from the galactic plane [1]), the hourly means were represented by formulae of the form:

$$\Delta \alpha = a + b \sin \alpha + c \cos \alpha \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$\Delta \delta = a' + b' \sin \alpha + c' \cos \alpha \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

and the values of the coefficients were deduced from these equations.

The same weight is given to all hourly means everywhere, in spite of the sometimes considerably diverse number of stars upon which they are founded. By this means we gained the very material advantage that all the inequalities depending upon the sines or cosines of multiples of α become eliminated.

Moreover the centennial variations of the reductions $\Delta \alpha_\alpha$ and $\Delta \delta_\alpha$ of Auwers's Old Fund. Cat. to his new one were developed in formulae of the same form. These expressions, as being probably known with sufficient accuracy — which was doubted at first — were added to the corresponding terms of the formulae (1) and (2).

The following table contains the values of the coefficients of both formulae *per* 100 *years*; in this the centennial variations of the reductions $\Delta \alpha_\alpha$ and $\Delta \delta_\alpha$ have been taken into account.

In order to facilitate the further calculations, instead of the coefficients b and c, the quantities $b \cos \delta$ and $c \cos \delta$ are given in the table. The results are all expressed in seconds of arc.

[1]) These Proceedings, 18, 683—695.

COEFFICIENTS OF THE FORMULAE FOR $\Delta\alpha$ AND $\Delta\delta$ PER CENTURY.

Catalogue	BA	BE	BE'	FA	FE	FE'	F_1A	F_1E	F_2A	F_2E
I) COEFFICIENT a										
Be A	- 0″25	—0″27	—0″24	—0″62	—0″66	—0″55	—0″88	—0″77	—0″41	—0″54
Be B	+0.06	—0.45		—0.92	—0.58					
Lei I	0.93	—0 90		—1.15	—1.00					
Leiden	—1.14	—0.65		—0.39	+0.16					
II) COEFFICIENT $b \cos \delta$										
Be A	+1.33	+0.87	+0.79	+0.58	+0.41	+0.33				
Be B	+0 68	+0.40		+0.34	+0.36					
Lei I	+0.10	+0.09		+0.90	+0 32					
Leiden	+0.18	+0.22		+0.39	+0.22					
III) COEFFICIENT $c \cos \delta$										
Be A	+4.23	+2.38	+2.41	+2.13	+1.62	+1.59				
Be B	+2.80	+1.80		+1.37	+0.71					
Lei I	+3 66	+2.96		+1.73	+1.72					
Leiden	+2.19	+1.40		+2.56	+1.92					
IV) COEFFICIENT a'										
Be A	—1.52	—1.29		—2.55	—2.01					
Be B	—3.39	—2.49		—4.22	—3.44					
Lei I	—1.10	—1.20		—3.23	—3.03					
Leiden	—2.09	—1.54		—1.40	—0 78					
V) COEFFICIENT b'										
Be A	-0.68	—0.38		-0.67	—0.29					
Be B	—1.10	—1.05		—0.88	—0.77					
Lei I	—0.84	—0.82		—0.82	—1.09					
Leiden	—1.69	—1.27		—0.81	—0.67					
VI) COEFFICIENT c'										
Be A	+0.02	+0.03		+0 03	+0.11					
Be B	+0.67	+0.19		+0.21	—0.23					
Lei I	+0.84	+0.58		+1.22	+1.05					
Leiden	+0 18	—0.31		+0.06	0.12					

6

5. *Correction-terms.*

The following relations now apply [1]:

$$a = \Delta m - 0.04\,X \sin\delta + 0.22\,Y \sin\delta$$
$$b\cos\delta = \Delta n \sin\delta + 0.93\,\lambda - 0.04\,Y\cos^2\delta$$
$$c\cos\delta = -(0.93\,Y + 0.20\,Y\cos^2\delta - 0.04\,X\cos^2\delta)$$
$$a' = -0.93\,Z\cos\delta - 0.10\,Z\cos^3\delta - 0.21\,X\cos\delta\sin^2\delta - 0.03\,Y\cos\delta\sin^2\delta$$
$$b' = 0.93\,Y\sin\delta + 0.04\,X\cos^2\delta\sin\delta + 0.08\,Z\cos^2\delta\sin\delta$$
$$c' = \Delta n + 0.93\,X\sin\delta + 0.20\,X\cos^2\delta\sin\delta + 0.04\,Y\cos^2\delta\sin\delta + 0.43\,Z\cos^2\delta\sin\delta$$

where Δm and Δn represent the corrections of the constants of precession m and n and X, Y, Z the components of the motion of the sun.

The following are considered as correction-terms:

in a	: the terms that do not depend upon	Δm
„ $b\cos\delta$	: „ „ „ „ „ „ „	Δn and X
„ $c\cos\delta$	: „ „ „ „ „ „ „	Y
„ a'	: „ „ „ „ „ „ „	Z
„ b'	: „ „ „ „ „ „ „	Y
„ c'	: „ „ „ „ „ „ „	Δn and X

These correction-terms are calculated by means of values for the constants deduced from a preliminary solution:

	B-groups	*F-groups*
X =	+ 0″43	+ 0″43
Y =	− 2″4	− 1″6
Z =	+ 1″9	+ 2″5

They are then subtracted from the immediate results of the equations. The following table contains the results thus corrected. (See p. 83).

The figures in this table will now serve for the determination of the constants of precession and solar motion Δm, Δn, X, Y, Z, the actual unknown quantities of our problem. For this purpose, however, the relative weights of the differences in α and δ between Küstner and the four zone-catalogues must first be deduced.

6. *Relative accuracy of the differences formed. Weights to be attributed to them.*

Auwers [2]) gives a table of the mean errors of the various zone-catalogues of the A. G. deduced from a comparison with Romberg. There are also values for the mean errors given in the zone-catalogues themselves. Both are given below p. 84.

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1) These Proc. **18**, 684, 693

2) Astron. Nachr. 3842—44.

Corrected values of the coefficients.

Catalogue	BA	BE	BE'	FA	FE	FE'	F_1A	F_1E	F_2A	F_2E
I) Coefficient a										
Be A	-0″10	—0″12	—0″09	—0″51	—0″55	—0″44	—0″77	—0″66	—0″30	-0″43
Be B	+0.25	—0.26		—0.79	—0.45					
Lei I	—0.82	—0.79		—1.07	—0.92					
Leiden	-0.86	—0.37		—0.20	+0.35					
II) Coefficient $b \cos \delta$										
Be A	+1.24	+0.78	+0 70	+0 52	+0 35	+0.27				
Be B	+0.60	+0 32		+0.29	+0.31					
Lei I	+0.01	+0.00		+0.84	+0 26					
Leiden	+0.11	+0.15		+0.34	+0.17					
III) Coefficient $c \cos \delta$										
Be A	+4.21	+2.36	+2.39	+2.11	+1 60	+1.57				
Be B	+2.79	+1.79		+1.36	+0.70					
Lei I	+3.64	+2.94		+1.71	+1.70					
Leiden	+2.18	+1.39		+2.55	+1.91					
IV) Coefficient a'										
Be A	—1.52	—1.29		—2.55	-2.01					
Be B	—3.39	—2.49		—4.22	—3.44					
Lei I	—1.10	—1.20		—3.23	—3.03					
Leiden	2.09	—1.54		—1.40	—0.78					
V) Coefficient b'										
Be A	—0.73	—0.43		—0.73	—0.35					
Be B	—1.14	—1.09		—0.95	—0.84					
Lei I	—0.88	—0.86		—0.87	—1.14					
Leiden	—1.75	—1.33		—0.89	—0.75					
VI) Coefficient c'										
Be A	—0.17	-0.16		—0.25	—0.17					
Be B	+0.45	—0.03		—0 12	—0.56					
Lei I	+0.69	+0.43		+1.00	+0.83					
Leiden	—0.09	—0 58		—0.33	—0.51					

6*

MEAN ERRORS OF THE CATALOGUES AND OF THE DIFFERENCES.

Catalogue	m.e. position zonecat		m.e. position Küstner		m. e. difference		m.e. diff. per annum		Weights of	
							$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$
a) according to AUWERS:										
Be A	0ˢ034	0″47	0ˢ021	0″29	0ˢ040	0″55	0″022	0″021	1.99	2.18
Be B	025	30	.020	.27	.032	.40	.028	.025	1.22	1.54
Lei I	.043	.55	.022	.31	.048	.63	.031	.028	1.00	1.22
Leiden	.050	.52	.020	.25	.054	.58	.029	.025	1.07	1 54
b) according to data in the catalogues:										
Be A	0ˢ034	0″45	0ˢ021	0″29	0ˢ040	0″54	0″022	0″021	1.62	1.78
Be B	.027	.38	.020	.27	.034	47	.029	.029	0.93	0.93
Lei I	.037	.47	.022	.31	.043	.56	.028	.025	1.00	1.26
Leiden	.044	.49	.020	25	.048	.55	026	.023	1 16	1.48

As weights the means of those according to *a* and *b* are taken, namely:

Catalogue	Weight of	
	$\Delta\alpha\cos\delta$	$\Delta\delta$
Be A	1.8	2.0
Be B	1 1	1 2
Lei I	1.0	1.2
Leiden	1.1	1 5

7. *Determination of the actual unknown quantities.*

The correction-terms having been applied, the equation

$$a = \Delta m$$

now holds.

In the further calculations I have kept the B groups and the F groups apart, but for the rest I have simply taken the means of the results of the different methods. After this, means were formed from the four catalogues with the weights given in § 6. I thus obtained, always placing the 4 catalogues under one another in the same order, the following results for Δm.

	Groups B	Groups F
	— 0″10	— 0″52
	0.00	— 0.62
	— 0.80	— 1.00
	— 0.62	+ 0.08
Mean:	— 0″33	— 0″51

These are corrections which should be applied to m of STRUVE-PETERS. To obtain the corrections for NEWCOMB's m the difference between the values for m PETERS- NEWCOMB should be added. NEWCOMB [1]) gives the following values:

Centennial precessional motion for 1850,

	m	n
PETERS-STRUVE	4607″63	2005″64
NEWCOMB 1896 (final value)	4607.11	2005.11

From the differences PETERS—N_{96}:

in m + 0″52,
in n + 0.53,

it becomes evident that the values m and n of STRUVE-PETERS do not correspond to one another, when we adopt the most probable value for the planetary precession. I reduce my results therefore to NEWCOMB's values:

Corrections to NEWCOMB's m

	Groups B	Groups F
	+ 0″42	+ 0″00
	+ 0.52	— 0.10
	— 0.28	— 0.48
	— 0.10	+ 0.60
Mean:	+ 0″19	+ 0″01

Finally attributing equal weights to the B and F groups.

B and F groups together

Δm (centennial) NEWCOMB $= + 0''10 \pm 0''13$ (m e).

The mean error is deduced from the comparison of the above 8 values for Δm with their mean.

From the equation

$$b \cos \delta = \Delta n \sin \delta + 0.93\, X$$

X can be determined, if $\Delta n = - 0''16$ is substituted as deduced from the preliminary solution. The results for 0.93 X then are·

[1]) The Precessional constant, p. 10.

	Groups B	Groups F
$0.93\,X =$	$+0''96$	$+0''43$
	$+0.52$	$+0.36$
	$+0.04$	$+0.59$
	$+0.22$	$+0.35$
Mean:	$+0''52$	$+0''43$
$X =$	$+0''56$	$+0''46$

From $$c \cos \delta = -(0.93 + 0.20 \cos^2 \delta)\, Y$$
Y may be determined. The following values were found:

	Groups B	Groups F
$Y =$	$-2''69$	$-1''59$
	-2.08	-0.94
	-2.94	-1.52
	-1.66	-2.08
Mean:	$-2''38$	$-1''54$

The equation
$$a' = -(0.93 + 0.10 \cos^2 \delta)\, Z$$
gives the unknown Z. The results are as follows:

	Groups B	Groups F
$Z =$	$+1''45$	$+2''35$
	$+3.16$	$+4.10$
	$+1.14$	$+3.10$
	$+2.17$	$+1.30$
Mean:	$+1''92$	$+2''59$

From $$b' = 0.93\, Y \sin \delta$$
a second value for Y may be deduced.

I find the following values, in which the fact has been taken into account that the weights of the values for Y obtained from the 4 catalogues are very divergent in consequence of the factor $\sin \delta$:

	Groups B	Groups F
$0.54\, Y =$	$-1''16$	$-1''08$
$0.42\, Y =$	-1.35	-1.08
$0.24\, Y =$	-1.04	-1.20
$0.75\, Y =$	-2.31	-1.23
Mean: $Y =$	$-3''01$	$-2''36$

Finally from the equation
$$c' = \Delta n + (0.93 + 0.20 \cos^2 \delta)\, X \sin \delta$$
Δn can be deduced, if the value for X found from the R.A. is substituted in it.

I find thus for Δn (Struve-Peters):

	Groups B	Groups F
Δn Str.-Pet. =	$-0''34$	$-0''36$
	-0.03	-0.53
	$+0.42$	$+0.81$
	-0.66	-0.69
Mean:	$-0''.20$	$-0''.24$

or, deducing corrections to Newcomb's n:

	Groups B	Groups F
$\Delta n\ N_{96}$ =	$+0''19$	$+0''17$
	$+0.50$	$+0.00$
	$+0.95$	$+1.34$
	-0.13	-0.16
Mean:	$+0''.33$	$+0''.29$

B and F together: Δn Newcomb (centennial) $= +0''.31 \pm 0''.18$.

Another method of determining Δn and X consists in solving both unknowns at the same time from sets of two equations with two unknowns, that is from:

$$b \cos\delta = \Delta n \sin \delta + 0.93\, X$$

and $$c' = \Delta n + (0.93 + 0.20 \cos^2 \delta)\, X \sin \delta.$$

In this way I find the following values for X and Δn (Struve-Peters):

Groups B		Groups F		B and F
X	Δn	X	Δn	Δn
$+1''17$	$-0''54$	$+0''53$	$-0''39$	$-0''46$
$+0.49$	-0.00	$+0.56$	-0.57	-0.28
-0.14	$+0.60$	$+0.40$	$+0.82$	$+0.71$
$+0.51$	-0.64	$+0.79$	-0.88	$-0.76.$

In connection with the weights given to the R.A. and the Decl. I attribute to the results from the 4 catalogues the weights 1.9, 1.2, 1.1 and 1.3. I then find as mean values:

	Groups B	Groups F	B and F
$X =$	$+0''60$	$+0''57$	
Δn Str.-Pet. =	-0.21	-0.30	$-0''26$

therefore as correction Δn to Newcomb:

	Groups B	Groups F	B and F
Δn (Newcomb) =	$+0''.32$	$+0''.23$	$+0''.27.$

By substituting the final mean value of Δn (STRUVE-PETERS) $= -0''.24$ for the preliminary value $-0''.16$ in the equation

$$b \cos \delta = \Delta n \sin \delta + 0.93\, X$$

a second approximation for X is obtained from the R.A. only. In this way I find as the mean value from the four catalogues

	Groups B	Groups F
$X =$	$+0''.58$	$+0''.48.$

The result for Δn from the Decl. only does not change perceptibly, if we substitute for X these final values in place of the approximate ones.

Finally I accept the means of both determinations of Δn and X as my final result.

8. *Conclusions.*

In the foregoing the following final values are found for the unknowns:

	Groups B	Groups F	B and F
Δm (NEWCOMB)	$+0''19$	$+0''01$	$+0''10$
Δn (NEWCOMB)	$+0.32$	$+0.26$	$+0.29$
X	$+0.59$	$+0.52$	
Y from the R.A.	-2.38	-1.54	
Y from the Decl.	-3.01	-2.36	
Mean (weights 2 and 1)	-2.59	-1.81	
Z	$+1.92$	$+2.59$	

Let us first consider the value of Δm, $+0''10 \pm 0''13$, and of Δn, $+0''29 \pm 0''18$. From both it is possible to deduce a correction of the luni-solar precession accepted by NEWCOMB; I find:

$$\text{from } \Delta m: \quad \Delta p = \frac{\Delta m}{\cos \varepsilon} = +0''11 \pm 0''14,$$

$$\text{from } \Delta n: \quad \Delta p = \frac{\Delta n}{\sin \varepsilon} = +0''72 \pm 0''45.$$

These values clearly show a difference, in the same direction as remains in the results found by NEWCOMB, even after correction for the systematic differences of distance. We now find.

$$\Delta p\,(\text{Decl.}) - \Delta p\,(\text{R.A.}) = +0''61 \pm 0''47.$$

However, it is not very surprising to find such a difference occurring here. It is only 1.3 times as large as its mean error and may for the greater part be accounted for by the influence which the accidental errors must have in the comparison of the zone-catalogues with KUSTNER's, in consequence of the small difference of epoch. With regard to the possible systematic errors.

a. errors due to magnitude-equations

b. an error in the adopted motion of the equinox

c. systematic errors in the fund. system dependent upon α and δ on the other hand, the research here detailed is certainly not behind other determinations of the constants of precession from faint stars.

As regards *a*, the errors due to magnitude have been eliminated in a very satisfactory way, undoubtedly better than has been possible in any other similar research, while the effect of the errors *b* and *c* does not depend upon the difference of epoch of the catalogues themselves.

The question as to whether to the system N_1, also adopted by AUWERS, must be applied an appreciable correction of the form

$$\Delta E = \Delta E_0 + \Delta E' \times T,$$

in which $\Delta E'$ represents a correction to the centennial motion of the equinox, is discussed by NEWCOMB[1]). If a correction of this form is introduced, a corresponding one must be applied to the Δp from the A.R. namely:

$$\text{Corr. } \Delta p = + 1.09 \ \Delta E'.$$

Of the probable value of $\Delta E'$ NEWCOMB makes an estimate. He comes to the conclusion that we may assume $\Delta E' = + 0''30$. If we do that here also, our results become:

$$\Delta p \,(\text{R A.}) = + 0''44 \pm 0''14,$$

$$\Delta p \,(\text{Decl.}) = + 0.72 \pm 0.45,$$

which values agree very satisfactorily with one another.

In order that I might form some opinion upon the question in how far systematic errors depending upon α and δ in the p.m. of the New Fundamental Catalogue of AUWERS could have exercised an influence upon the results, a comparison was made between the N. F. K. of AUWERS and the Fund. Cat. of NEWCOMB. On the basis of the data occurring in the N. F. K. of the Berliner Jahrbuch[2]) a table was drawn up of the differences in μ_α and μ_δ (N F. K.—NEWCOMB) for four groups of stars, corresponding in declination to the four zone-catalogues. Excluding a few very large differences I found as means:

[1]) The Precessional Constant, p. 69 a f.

[2]) Veröffentlichungen des Königl. Astron Rechen-Instituts No. 33, p. 100 et seq.

Differences in centenn. proper motion N. F. K. –Newcomb 1900

	$\Delta\mu_\alpha$	$\Delta\mu_\delta$
$\delta+10^\circ-+15^\circ$	-0^s0057	$-0''105$
$\delta+15^\circ-+20^\circ$	$+0.0154$	$+0.060$
$\delta+20^\circ-+25^\circ$	$+0.0003$	$+0.254$
$\delta+30^\circ-+35^\circ$	$+0.0028$	$+0.210$

The values of $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ were smoothed by formulae of the form:

$$\Delta\mu_\alpha = a + b\delta$$
$$\Delta\mu_\delta = a' + b'\delta$$

this is advisable as the zone-positions are based upon fundamental stars some of which lie outside the zones. I found:

$$a = +0^s0010 \qquad b = +0^s0001$$
$$a' = -0''216 \qquad b' = +0''015$$

and with this the smoothed values

$\Delta\mu_\alpha$	$\Delta\mu_\delta$
$+0^s002$	$-0''03$
$+0.003$	$+0.05$
$+0.003$	$+0.12$
$+0.004$	$+0.27$

These values are subtracted from the differences for each star. From the residual values 2-hour-groups according to R.A. were formed for the four declination-groups together.

In the following table are collected the mean values for the two-hour-groups, where 0^h represents the group from R.A. 23^h to 1^h etc.

Differences N.F.K. — Newcomb dependent upon the A.R.

α	$\Delta\mu_\alpha$	$\Delta\mu_\delta$	Stars
0^h	$+0^s.004$	$-0''.11$	10
2	$-\ .032$	$+\ .12$	7
4	$-\ .004$	$+\ .03$	16
6	$-\ .000$	$+\ .01$	10
8	$+\ .011$	$+\ .11$	9
10	$+\ .027$	$+\ .01$	11

α	$\Delta\mu_\alpha$	$\Delta\mu_\delta$	Stars
12	+ .009	+ .04	8
14	+ .019	— .12	6
16	+ .013	+ .02	10
18	+ .001	+ .23	8
20	— .016	— .14	10
22	— .035	— .00	9

These values I have represented by formulae of the form:

$$\Delta\mu_\alpha = x \sin\alpha + y \cos\alpha$$
$$\Delta\mu_\delta = x' \sin\alpha + y' \cos\alpha$$

attributing equal weight to each 2-hour group. I found:

$$x = +0^s.002 \qquad y = -0^s.021$$
$$x' = +0''.023 \qquad y' = -0''.012,$$

so that

$\Delta\mu_\alpha$ (N. F. K.—NEWCOMB) = smoothed value $+ x \sin\alpha + y \cos\alpha$
$\Delta\mu_\delta$ (N. F. K.—NEWCOMB) = smoothed value $+ x' \sin\alpha + y' \cos\alpha$.

If, therefore, my results which hold good for the system of the N. F. K. of the Berliner Jahrbuch, are to be reduced to the system of NEWCOMB's catalogue, the above quantities must be added to the differences in α and δ per 100 years, with reversed sign.

These systematic corrections are therefore:

$$\text{in } \Delta\alpha \text{ per 100 years: } \left\{\begin{matrix} -0''.03 \\ -0\ .04 \\ -0\ .04 \\ -0\ .06 \end{matrix}\right\} - 0''.033 \sin\alpha + 0''.314 \cos\alpha \left\{\begin{matrix} \text{Lei I} \\ \text{Be A} \\ \text{Be B} \\ \text{Leiden} \end{matrix}\right.$$

$$\text{in } \Delta\delta \text{ per 100 years: } \left\{\begin{matrix} +0''.03 \\ -0\ .05 \\ -0\ .12 \\ -0\ .27 \end{matrix}\right\} - 0''.023 \sin\alpha + 0''.012 \cos\alpha \left\{\begin{matrix} \text{Lei I} \\ \text{Be A} \\ \text{Be B} \\ \text{Leiden} \end{matrix}\right.$$

The addition of these corrections changes my values for the unknowns in the following way

corr. Δm	per century		— 0″045
„ X	„	„	— 0.03
„ Y(R.A.)	„	„	— 0.26
„ Z	„	„	+ 0.11
„ Y(Decl.)	„	„	— 0.07
„ Δn	„	„	+ 0.01

The results then become:

Δm + 0″055
Δn + 0.30

	Gr. B	Gr. F
X	$+0''55$	$+0''49$
Y (R.A.)	-2.64	-1.80
Y (Decl.)	-3.08	-2.43
mean (weight 2 and 1)	-2.79	-2.01
Z	$+2.03$	$+2.70$

Let us now examine first what is found for the luni-solar precession p.

I find from Δm · $\Delta p = +0''.06$

from Δn : $\Delta p = +0.75$

or, if we adopt the correction to the motion of the equinox,

from Δm : $\Delta p = +0''.39$

from Δn : $\Delta p = +0.75$

from which, again combining with weights 2 and 1:

from R.A. and Decl.: $\Delta p = +0''.51$.

From the two values for Δp, found if we adopt AUWERS's New System, follows in the same way

from R.A. and Decl. $\Delta p = +0''.53$.

The agreement of the two results makes it probable that systematic errors dependent upon α and δ cannot have a great influence upon our results.

Taking the mean, therefore, I get as final result

$$\Delta p \text{ NEWCOMB} = +0''.52.$$

As NEWCOMB's final result of 1896 remains quite unchanged by taking the systematic differences of distance of the stars into account (These Proc. **18**. 692) my result is 0''.5 greater than his.

From my value for Δp follows

$$\Delta m \text{ NEWCOMB} = +0''.48$$

$$\Delta n \text{ NEWCOMB} = +0.21$$

The values for the yearly precessional motions which follow from the research here detailed are therefore:

Yearly precessional motions for 1850:

p = lunisolar precession = 50''3736

p' = general precession = 50''2505

m = precession in R. A. = 46''0759

n = precession in Decl. = 20''0532

P = NEWCOMB's constant = 54''9124.

In the second place we will discuss the results obtained for the parallactic motion, if we successively adopt the two systems.

For the components of the parallactic motion the following values are found :

	In AUWERS's system:		In NEWCOMB's system:	
	Groups B	Groups F	Groups B	Groups F
X	+0″59	+0″52	+0″55	+0″49
Y	−2.59	−1.81	−2.79	−2.01
Z	+1.92	+2.59	+2.03	+2.70

From the values in this table I deduce the following values for the coordinates of the apex, A and D, and for the total solar motion and its projection upon the plane of the equator.

	In AUWERS's system:		In NEWCOMB's system:	
	Groups B	Groups F	Groups B	Groups F
A	282°5	286°0	281°1	283°7
	$18^h\ 50^m$	$19^h\ 4^m$	$18^h\ 44^m$	$18^h\ 55^m$
$\sqrt{X^2+Y^2}$	2″65	1″88	2″84	2″07
D	+35°9	+54°0	+35°5	+52°5
$\sqrt{X^2+Y^2+Z^2}$	3″28	3″20	3″49	3″40

Let us first consider the results obtained for the R.A. and Decl. of the apex. It is not possible here to institute a critical comparison of my results with those of others which would in itself form a research. For the sake of orientation in the problem I will merely quote some results obtained by previous investigators along the same lines.

The results deduced for *A* and *D* from the Bradley stars by NEWCOMB and corrected for the systematic difference of distance, according to the research published in these Proceedings, by — 1° and + 2° respectively were 273° and + 33°; in the same way the results of L. STRUVE (corrected by NEWCOMB) become 272° and + 37°. From the comparison of the whole material of his Albany-zone with LALANDE and BESSEL, Boss found for stars of a mean magnitude $8^m.7$: 264° and + 54° [1]), while later on [2]) he accepted as final result

[1]) A. J. **9**, 28

[2]) A. J. **21**, 168.

from various researches for stars $8^m.5$ with small P.M.: 279° and + 45°.

My results, which apply to mean magnitudes (photom. magnitudes of KÜSTNER), of $7^m.25$ for the bright group and of $9^m.19$, for the fainter group, remain almost the same, whether we take AUWERS's system or NEWCOMB's as basis. Comparing them with the above, it is seen that my values for A belong to the greatest so far obtained, while those for D for my bright group agree very well with those from the BRADLEY stars and the results for my faint group do not differ much from the corresponding ones of Boss. The large difference between the value for D in my two groups, which is the result of the abnormal relation of the two values found for the Z-component, is a striking result, to which I shall return further on.

In the second place, my results for the amount of the parallactic motion must be further considered, both those for the projection of this motion upon the plane of the equator and those for the total motion. We observe first that, for both motions, the reduction to NEWCOMB's system gives somewhat larger values than that to AUWERS's system. Naturally both for the bright and for the faint group.

For the equatorial motion the ratio of group B to group F is according to the two systems 1 : 0.71 and 1 : 0.73, while the ratio between the mean distances of the groups, according to the later researches (Comp. KAPTEYN and WEERSMA Publ. Gron. **24**, 15), should be 1 : 0.63. Here the agreement is, therefore, fairly good, but the result is totally different, if we consider the total motion. For this we find for the faint group results which are only 3 °/₀ smaller than those for the bright group.

However, before endeavouring to draw any conclusion from this, we must consider the significance of my results, in connection with the methods used concerning the exclusion of stars of large proper motion. As mentioned above I made one set of solutions (method A) in which practically only the double stars were excluded (besides the double stars for the four catalogues together only 21 stars) and another set (method E) in which a considerable number of stars with a somewhat large P.M. were excluded. Finally the mean results from these methods were accepted as the final result.

This method was certainly justifiable, where a determination of the precessional motion was aimed at, and perhaps is so still, when we only desire to derive the coordinates of the apex of the parallactic motion. If we wish, however, to determine the amount of this motion, it will be seen that the significance of the results becomes uncertain by the method used. It is necessary, therefore, to consider the results according to the methods A and E separately.

For this purpose I calculated, from the values of the 6 coefficients, after correction for the subsidiary terms, the components X, Y, Z of the solar motion for the A and E groups of each of the catalogues separately, and combined these results with the previously adopted weights.

In this way I found adopting the system of AUWERS's N. F. K.

	BA	BE	FA	FE
X	+0″74	+0″42	+0″62	+0″34
Y (R. A.)	−3.02	−1.74	−1.79	−1.29
Y (Decl.)	−3.34	−2.68	−2.55	−2.17
Y (mean; weights 2 and 1)	−3.13	−2.05	−2.04	−1.58
Z	+2.13	+1.70	+2 89	+2.29

and for the further constants derived from these

	BA	BE	FA	FE
A	283°3	281°6	286°9	282°1
	18h 53m	18h 46m	19h 8m	18h 48m
$\sqrt{X^2+Y^2}$	3″22	2″09	2″13	1″62
D	+33°4	+39°1	+53°6	+54°7
$\sqrt{X^2+Y^2+Z^2}$	3″86	2″70	3″59	2″80

It may be regarded as a satisfactory result of the last calculation that the coordinates of the apex, gained by the *A* and *E* methods, do not differ greatly, and also differ only slightly from my previous results. On the other hand, the result that the Z-component is found larger for the faint than for the bright stars, becomes even more striking, now that it proves to hold good for the results deduced separately by the *A* and *E* method. Further, as was to be expected from former results, the *amount* of the parall. motion gained by the two methods differs considerably, which again shows that the parall. motion increases greatly with the total P. M. The results of the method A are the only ones that have a sharply defined meaning. They give us the parall. motions for the mean of the stars of magnitudes 7.25 and 9.19.

We will, therefore, consider these only, and we will deduce, beside the results above obtained for AUWERS's system, those which are found, if we adopt NEWCOMB's, which can be done at once by applying to the 3 components the differences deduced above.

I then find, repeating the first mentioned values in order to facilitate the comparison:

	In AUWERS's System		In NEWCOMB's System	
	Group BA	Group FA	Group BA	Group FA
X	+ 0″74	+ 0″62	+ 0″70	+ 0″58
Y	— 3.13	— 2.04	— 3.33	— 2.24
Z	+ 2.13	+ 2.89	+ 2.24	+ 3.00
A	283°3	286°9	281°8	284°5
$\sqrt{X^2+Y^2}$	3″22	2″13	3″40	2″31
D	+ 33°4	+ 53°6	+ 33°4	+ 52°4
$\sqrt{X^2+Y^2+Z^2}$	3″86	3″59	4″07	3″79

Here we see again that all the essential features of our results are independent of the choice of the fundamental system.

For the ratio between the equatorial motions for the bright and the faint group we now find 1 : 0.66 or 1 : 0.68, or for a difference of one magnitude 1 : 0.81 or 1 : 0.82, which agrees very satisfactorily with the ratio of the distances given by KAPTEYN and WEERSMA 1 : 0.63 or for one magnitude 1 . 0.79. All agreement, however, disappears again, when we consider the total motion, and thus include the Z-components. In my last results also, the motion in the Z-direction is found to be much greater for the faint stars than for the bright ones and even if we take into consideration that of the centennial motions here derived, barely a fifth part has actually been observed, our result still remains very striking. If we consult the results yielded by the four catalogues separately, we find that the Leiden zone gives a normal result, namely faint : bright = 0.67 : 1, while from the 3 others we derive a very abnormal ratio. Other investigations, which gave greater values for D deduced from faint stars, than when bright stars were used, might point to abnormal circumstances in the same direction. The result here found is, however, more striking, for, as the declinations were determined in exactly the same way for faint and for bright stars, the greater value for Z (the constant term in $\Delta\delta$) which the former give, cannot be ascribed to constant errors of the declinations of the catalogues used. If systematic errors of the catalogues are to be made responsible for our result, it can only be the consequence of residual magnitude-errors in declination.

This point certainly deserves further investigation. Another point that has not been investigated so far is the possible presence in the differences KÜSTNER—Zonecatalogues of terms dependent upon multiples of α.

Mathematics. — "*Pencils of twisted cubics on a cubic surface*". By Prof. JAN DE VRIES.

(Communicated in the meeting of March 25, 1916).

1. The straight lines of a bisextupel of a cubic surface Φ^3 will be indicated in the usual way by a_k and b_k; the remaining straight lines by c_{kl}. In order to arrive at the wellknown representation of Φ^3 on a plane τ, we lay τ through the straight line c_{12} and consider b_1, b_2 as directrices of a bilinear congruence of rays. Any point P of Φ^3 is then represented by the intersection P', on τ, of the ray passing through P. The intersections A_1, A_2 of b_1, b_2 represent a_1, a_2, whereas a_3, a_4, a_5, a_6 are represented by their intersections A_3, A_4, A_5, A_6. The representation of the straight line b_k is the conic β_k, which is determined by the five *cardinal points* A_l $(l \neq k)$; the straight line c_{kl} is represented by $A_k A_l$. From this representation it may be deduced that any twisted cubic ϱ^3 lying on Φ^3 has a sextuple as chords and is not intersected by the associated sextuple.

2. A ϱ^3 having the sextuple b_k as bisecants is represented by a straight line of τ; a plane pencil with vertex C' is therefore the image of a system of ϱ^3 all passing through the point C. Such a system we shall call a *pencil*; C we call the *singular point* of the pencil (ϱ^3). All ϱ^3 rest on the 15 straight lines c_{kl} and have the straight lines b_k as chords [1]).

To (ϱ^3) belong *six degenerated figures.* For the straight line $C'A_k$

[1]) In my paper "A simply infinite system of twisted cubics" (These Proceedings Vol. XVIII p. 1464) I arrived at the consideration of such a pencil in an entirely different way