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the same direction. The result here found is, however, more striking, for, as the declinations were determined in exactly the same way for faint and for bright stars, the greater value for Z (the constant term in $\Delta\delta$) which the former give, cannot be ascribed to constant errors of the declinations of the catalogues used. If systematic errors of the catalogues are to be made responsible for our result, it can only be the consequence of residual magnitude-errors in declination.

This point certainly deserves further investigation. Another point that has not been investigated so far is the possible presence in the differences KÜSTNER—Zonecatalogues of terms dependent upon multiples of α.

Mathematics. — *"Pencils of twisted cubics on a cubic surface".* By Prof. JAN DE VRIES.

(Communicated in the meeting of March 25, 1916).

1. The straight lines of a bisextupel of a cubic surface Φ^3 will be indicated in the usual way by a_k and b_k; the remaining straight lines by c_{kl}. In order to arrive at the wellknown representation of Φ^3 on a plane τ, we lay τ through the straight line c_{12} and consider b_1, b_2 as directrices of a bilinear congruence of rays. Any point P of Φ^3 is then represented by the intersection P', on τ, of the ray passing through P. The intersections A_1, A_2 of b_1, b_2 represent a_1, a_2, whereas a_3, a_4, a_5, a_6 are represented by their intersections A_3, A_4, A_5, A_6. The representation of the straight line b_k is the conic β_k, which is determined by the five *cardinal points* A_l $(l \neq k)$; the straight line c_{kl} is represented by $A_k A_l$. From this representation it may be deduced that any twisted cubic ϱ^3 lying on Φ^3 has a sextuple as chords and is not intersected by the associated sextuple.

2. A ϱ^3 having the sextuple b_k as bisecants is represented by a straight line of τ; a plane pencil with vertex C' is therefore the image of a system of ϱ^3 all passing through the point C. Such a system we shall call a *pencil*; C we call the *singular point* of the pencil (ϱ^3). All ϱ^3 rest on the 15 straight lines c_{kl} and have the straight lines b_k as chords [1]).

To (ϱ^3) belong *six degenerated figures.* For the straight line $C'A_k$

[1]) In my paper "A simply infinite system of twisted cubics" (These Proceedings Vol. XVIII p. 1464) I arrived at the consideration of such a pencil in an entirely different way

is the image of a figure consisting of the straight line a_k and a conic ϱ^2_k in the plane $(C''b_k)$, which is intersected by a_k.

On the curve ψ^3, along which Φ^3 is intersected by a plane ψ, the pencil (ϱ^3) determines an involution I^3.

If a tangent plane is taken for ψ, ψ^3 becomes rational, the involution I^3 has in that case four pairs in common with a central I^3. To it belongs, however, the pair of points lying in the node of ψ^3 and arising from the ϱ^3, which touches at ψ there. So there are three pairs of points that send their connectors through an arbitrary point. From this it ensues that the *bisecants of the curves* ϱ^3 will form *a cubic complex of rays*, Γ^3. C is evidently *cardinal point* of Γ^3, for that point bears ∞^2 rays.

The planes of the *six* conics ϱ^2_k are *cardinal planes*.

3. The rays of the complex passing through a point T form a *rational cubic cone*, which has the straight line TC as *nodal edge*; for it intersects Φ^3 moreover in two points, so that it is chord of two ϱ^3.

The ends U, U' of the chords forming this cone lie on a twisted curve τ^6, which has a node in C; for any plane passing through TC contains apart from that edge only two more points U.

If TC becomes tangent of Φ^3, the nodal edge passes into a *cuspidal edge*. The locus for the vertices of *complex cones with a cuspidal edge* is therefore the *enveloping cone* of Φ^3, which has C as vertex, consequently a *cone of order four*.

For a point N on Φ^3 the complex cone degenerates into the quadratic cone that projects the ϱ^3 determined by N, and a plane pencil of which the plane ν passes through C.

If N lies on one of the conics ϱ^2_k, the complex cone consists of three plane pencils, of which one lies in the plane of the conic, one in the plane (Na_k).

If N is taken on one of the singular bisecants b_k the plane pencil (N, ν) consists of chords of ϱ^2_k.

In a plane ν the *complex curve* degenerates into the *plane pencil* with vertex N and the twice to be counted *plane pencil* with vertex C; for a straight line passing through C is chord of two ϱ^3.

4. The tangents out of N at the cubic ν^3, which ν has in common with Φ^3, are at the same time tangents t at curves ϱ^3. This holds also for the straight line that touches r^3 in C; but the latter, as ray of the congruence $[t]$ is to be counted twice.

From this we conclude that the *class* of $[t]$ is *six*.

Also the tangent in N at the ϱ^3, which passes through N, must be counted for two rays of $[t]$; consequently the *order* too is equal to *six*.

This may also be proved as follows. The pairs of points U, U' of the curve τ^6 are projected out of a straight line l by a pencil of planes in involutorial correspondence (6,6), in which the plane (lT) represents a sextuple coincidence. As the remaining coincidences arise on account of the coincidence of U' with U, T bears *six* tangents of curves ϱ^3.

C is evidently a *singular point* of *order one* for the congruence $[t]$; the tangent plane in C at Φ^3 is the *singular plane* belonging to it. The planes of the six conics ϱ^2_k are *singular planes* of *order two*. The six straight lines b are *double rays*.

5. Analogous considerations hold for pencils (ϱ^3) on a *nodal* cubic surface. The representation is then simply brought about by central projection out of the conical point. The curves ϱ^3 now have one of the six straight lines $a(b)$ passing through the conical point and four straight lines c as chords, or they pass through the conical point and have three straight lines a and three straight lines c as chords.

Physiology. — *"A new group of antagonizing atoms."* I. By T. P. FEENSTRA. (Communicated by Prof. Dr. H. ZWAARDEMAKER).

(Communicated in the meeting of April 28, 1916).

It is a matter of common knowledge that a sodium chloride solution in the concentration of RINGER's mixture arrests the action of the heart some time after the circulating fluid has been administered, and also that contraction can be restored by the addition of potassium chloride and by calcium chloride.

These two salts remove the toxic effect of sodium chloride.[1]) A normal action of the heart is obtained only if the three salts together with sodium bicarbonate are present in the circulating fluid in a definite concentration as in bloodserum. Augmentation or diminution of the amount of one of the constituents of the fluid induces an abnormal action of the heart, which will slow down to a standstill, when the difference becomes too great. The relative apportionments of the three salts must, therefore, be definite and fairly constant.

[1]) Journal of Physiol. Vol. III p. 380, Vol. IV pp. 29 and 222, Vol. V p. 247.

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