racemic mixed crystals and racemic compounds. If, for instance, in the system of d and l-carvoxim the three-phase tension is to be determined, one might think, in connection with the above that we can perhaps come to a conclusion as to the much discussed question whether we are dealing here merely with a maximum in a series of mixed crystal or whether we are dealing with a racemic compound giving continuous mixed crystal series with the antipodes. Meanwhile it is shown on closer investigation that the resolving of the problem cannot be expected in that manner.

6. The peculiar form of the three-phase line and the correlated spacial figure give rise to different theoretical considerations. We hope to soon revert to the matter, also in connexion with the discussion in the previous paragraph.

Utrecht, June 1916. VAN 'T HOFF-*Laboratory*.

Physics — "*The field of n moving centres in* EINSTEIN's *theory of gravitation*". By J. DROSTE. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of June 24, 1916).

1. If in one or other field of gravitation there is placed a particle, i.e. a body so small that, though influenced by the field, it does not itself exercise any influence on the field, it will move in such a way that the first variation of the time integral of

$$L = (\sum_{ij} g_{ij} \dot{x}_i \dot{x}_j)^{\frac{1}{2}}$$

calculated after some definite way, is zero. Here $x_4 = t$, and so $\dot{x}_4 = 1$. If $\dot{x}_1, \dot{x}_2, \dot{x}_3$ are small with respect tot unity (i.e. nearly the velocity of light), g_{44} will be much larger than say $g_{11}\dot{x}_1^2$. We will call a term one of the first order if, after division by the square of a component of a velocity, it gets a moderate value. Now, as in NEWTON's theory, which accounts for the phenomena very closely, it follows from the equation of energy that a term, multiplied by the constant of gravity $\varkappa$, is of the same order as the square of a velocity, we will call also such a term one of the first order and consequently a term, which contains $\varkappa^2$, of the second order.

Our purpose is the calculation of L up to the terms of the second order inclusive. If there is no body whatever that can produce a field, we shall have

$$g_{11} = g_{22} = g_{33} = -1 \quad , \quad g_{44} = 1 \quad , \quad g_{ij} = 0 \; (i \neq j).$$

In g_{ij} ($i \neq 4$, $j \neq 4$) we now have to go only up to the terms of the first order inclusive, in g_{14}, g_{24}, g_{34} up to the terms of order $1\frac{1}{2}$, in g_{44} up to the terms of order 2. As the quantities g_{14}, g_{24}, g_{34} may be considered to arise from the motions of the bodies that produce the field (viz. by the changes that arise consequently in the field of the first order), we wil suppose that g_{14}, g_{24}, g_{34} only contain terms of order $1\frac{1}{2}$ (or higher). We consequently put

$$g_{ij} = \alpha_{ij} + \varkappa\beta_{ij} (i \neq 4, j \neq 4), \; g_{i4} = \alpha_{i4} + \varkappa^{3/2}\sigma_{i4} (i \neq 4). \; g_{44} = \alpha_{44} + \varkappa\beta_{44} + \varkappa^2\gamma_{44}$$
$$g^{ij} = \alpha^{ij} + \varkappa\beta^{ij}, \qquad g^{i4} = \alpha^{i4} + \varkappa^{3/2}\sigma^{i4}, \qquad g^{44} = \alpha^{44} + \varkappa\beta^{44} + \varkappa^2\gamma^{44}$$

For all values of i and j the quantities α_{ij} and α^{ij} represent the values of g_{ij} and g^{ij} in the case of absence of any gravitating body.

As to the differential coefficients of these quantities a differentiation with respect to x_1, x_2, x_3 will not change their order, a differentiation with respect to x_4, however, will raise their order by $\frac{1}{2}$.

In the equations of the field

$$2\, G_{ij} = -2\varkappa T_{ij} + \varkappa g_{ij} T \; . \; . \; . \; . \; . \; . \; . \; . \qquad (1)$$

the left hand member is

$$2\, G_{ij} = 2\sum_l \left(\frac{\partial}{\partial x_j} \begin{Bmatrix} il \\ l \end{Bmatrix} - \frac{\partial}{\partial x_l} \begin{Bmatrix} ij \\ l \end{Bmatrix} \right) + 2\sum_{lm} \left(\begin{Bmatrix} il \\ m \end{Bmatrix} \begin{Bmatrix} jm \\ l \end{Bmatrix} - \begin{Bmatrix} ij \\ m \end{Bmatrix} \begin{Bmatrix} ml \\ l \end{Bmatrix} \right),$$

where

$$\begin{Bmatrix} ij \\ l \end{Bmatrix} = \sum_n g^{ln} \begin{bmatrix} ij \\ n \end{bmatrix}, \quad \begin{bmatrix} ij \\ n \end{bmatrix} = \frac{1}{2} \left(\frac{\partial g_{in}}{\partial x_j} + \frac{\partial g_{jn}}{\partial x_i} - \frac{\partial g_{ij}}{\partial x_n} \right).$$

Putting

$$\begin{bmatrix} ij \\ n \end{bmatrix}_\beta = \frac{1}{2} \left(\frac{\partial \beta_{in}}{\partial x_j} + \frac{\partial \beta_{jn}}{\partial x_i} - \frac{\partial \beta_{ij}}{\partial x_n} \right)$$

and giving to

$$\begin{bmatrix} ij \\ n \end{bmatrix}_\sigma \text{ and } \begin{bmatrix} ij \\ n \end{bmatrix}_\gamma$$

a corresponding meaning (putting $g_{ij} = \alpha_{ij} + \varkappa\beta_{ij} + \varkappa^{3/2}\sigma_{ij} + \varkappa^2\gamma_{ij}$ for all values of i and j, so that many of the β_{ij}, σ_{ij} and γ_{ij} become zero), we get

$$\begin{Bmatrix} ij \\ l \end{Bmatrix} = \sum_n (\alpha^{ln} + \varkappa\beta^{ln} + \varkappa^{3/2}\sigma^{ln} + \varkappa^2\gamma^{ln}) \left(\varkappa \begin{bmatrix} ij \\ n \end{bmatrix}_\beta + \varkappa^{3/2} \begin{bmatrix} ij \\ n \end{bmatrix}_\sigma + \varkappa^2 \begin{bmatrix} ij \\ n \end{bmatrix}_\gamma \right),$$

and so, omitting terms of higher than the second order, we obtain

$$\begin{Bmatrix} ij \\ l \end{Bmatrix} = \varkappa\alpha^{ll} \begin{bmatrix} ij \\ l \end{bmatrix}_\beta + \varkappa^{3/2} \alpha^{ll} \begin{bmatrix} ij \\ l \end{bmatrix}_\sigma + \varkappa^2 \sum_n \beta^{ln} \begin{bmatrix} ij \\ n \end{bmatrix}_\beta + \varkappa^2\alpha^{ll} \begin{bmatrix} ij \\ l \end{bmatrix}_\gamma \; . \; . \qquad (2)$$

2. We now proceed to the calculation of the terms of the first

order in (1). The second part of G_{ij} contributes nothing, and in the first part we need only substitute the first term of (2). We so get

$$2\varkappa \sum_l \alpha^{ll}\left(\frac{\partial}{\partial x_j}\begin{bmatrix} il \\ l \end{bmatrix} - \frac{\partial}{\partial x_l}\begin{bmatrix} ij \\ l \end{bmatrix}_\beta\right) =$$

$$= \varkappa \sum_l \alpha^{ll}\left(\frac{\partial^2 \beta_{il}}{\partial x_j \partial x_i} + \frac{\partial^2 \beta_{ij}}{\partial x_l^2} - \frac{\partial^2 \beta_{il}}{\partial x_l \partial x_j} - \frac{\partial^2 \beta_{jl}}{\partial x_l \partial x_i}\right) \quad . \quad . \quad . \quad (3)$$

If we indicate an index, that does not take the value 4, by placing it in parentheses, we can write for this

$$-\varkappa \sum_{(l)} \left(\frac{\partial^2 \beta_{ij}}{\partial x_l^2} - \frac{\partial^2 \beta_{il}}{\partial x_l \partial x_j} - \frac{\partial \beta^2_{jl}}{\partial x_l \partial x_i}\right) +$$

$$+ \varkappa \sum \alpha^{ll} \frac{\partial^2 \beta^{ll}}{\partial x_j \partial x_i} + \varkappa \left(\frac{\partial^2 \beta_{ij}}{\partial x_4^2} - \frac{\partial^2 \beta_{i4}}{\partial x_4 \partial x_j} - \frac{\partial^2 \beta_{j4}}{\partial x_4 \partial x_i}\right), \quad . \quad . \quad . \quad (4)$$

in which the last term is at least of the second order.

The terms of the first order become

in the case $i \neq 4, j \neq 4: -\varkappa \sum_{(l)} \left(\frac{\partial^2 \beta_{ij}}{\partial x_l^2} - \frac{\partial^2 \beta_{il}}{\partial x_l \partial x_j} - \frac{\partial^2 \beta_{jl}}{\partial x_l \partial x_i}\right) + \varkappa \sum_l \alpha^{ll} \frac{\partial^2 \beta_{ll}}{\partial x_j \partial x_i}$,

in the case $i \neq 4, j = 4$: *zero*,

in the case $i = j = 4: -\varkappa \sum_{(l)} \frac{\partial^2 \beta_{44}}{\partial x_l^2} = -\varkappa \Delta \beta_{44}$.

We now suppose the quantity T_{44} to be the only of all T_{ij}, which contains a term ϱ of order 0. Then the term of order 0 in T being also ϱ, we obtain

$$\sum_{(l)} \left(\frac{\partial^2 \beta_{ij}}{\partial x_l^2} - \frac{\partial^2 \beta_{il}}{\partial x_l \partial x_j} - \frac{\partial^2 \beta_{jl}}{\partial x_l \partial x_i}\right) - \sum_l \alpha^{ll} \frac{\partial^2 \beta_{ll}}{\partial x_i \partial x_j} = \delta_{ij}\, \varrho, \; (i \neq 4, j \neq 4)$$

$$\Delta \beta_{44} = \varrho$$

The equations are satisfied by

$$\beta_{ij} = 0 \; (i \neq j) \;\; , \; \beta_{11} = \beta_{22} = \beta_{33} = \beta_{44} = \beta, \quad . \quad . \quad . \quad (5)$$

β being a solution of

$$\Delta \beta = \varrho \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This solution, however, is by no means unique. We may e.g. add to it

$$\beta_{ij} = \frac{\partial^2 \varphi}{\partial x_i \partial x_j};$$

where φ is a function of x_1, x_2, x_3; we will, however, not do so.

3. Substituting the solution (5) into the expression (4) and omitting the terms of order 2, we obtain

$$\delta_{ij}\,\varkappa\,\frac{\partial^2\beta}{\partial x_4{}^2}\ (\textit{in the case } i \neq 4,\ j\neq 4).\quad -2\varkappa\frac{\partial^2\beta}{\partial x_4\partial x_i}\ (\textit{in the case } i\neq 4,\ j=4),$$

$$-3\varkappa\frac{\partial^2\beta}{\partial x_4{}^2}\ (\textit{in the case } i=j=4).$$

We now substitute the second term of (2) into the first part of $2G_{ij}$. This yields an expression similar to (4), the difference being only that $\varkappa^{3/2}$ and σ occur in it instead of $\varkappa$ and β. We easily find

in the case $i\neq 4,\ j\neq 4$: $\quad -\varkappa^{3/2}\left(\frac{\partial^2\sigma_{i4}}{\partial x_4\partial x_j}+\frac{\partial^2\sigma_{j4}}{\partial x_4\partial x_i}\right).$ (*order* 2)

in the case $i\neq 4,\ j=4$: $\quad \varkappa^{3/2}\sum_{(l)}\left(\frac{\partial^2\sigma_{il}}{\partial x_l\partial x_i}-\frac{\partial^2\sigma_{i4}}{\partial x^2_l}\right),$ (*order* $1\frac{1}{2}$)

in the case $i=j=4$: $\quad 2\varkappa^{3/2}\sum_{(l)}\frac{\partial^2\sigma_{4l}}{\partial x_4\partial x_l}.$ (*order* 2)

Substituting the fourth term of (5) in the first part of $2G_{ij}$ we get

in the case $i\neq 4, j\neq 4$: $\quad -\varkappa^2\sum_{(l)}\left(\frac{\partial^2\gamma_{ij}}{dx_l{}^2}-\frac{\partial^2\gamma_{il}}{\partial x_l\partial x_j}-\frac{\partial^2\gamma_{jl}}{\partial x_l\partial x_i}\right)+$

$$+\varkappa^2\sum_l a_{ll}\frac{\partial^2\gamma_{ll}}{\partial x_i\partial x_j},\ (\textit{order } 2)$$

in the case $i\neq 4,\ j=4$: $\quad$ *zero*,

in the case $i=j=4$: $\quad -\varkappa^2\,\Delta\gamma_{44}.$ (*order* 2)

We further must substitute the third term of (2) in the first part of $2G_{ij}$. Now we have $\beta^{ij}=0\ (i\neq j)$ and $\beta^{11}=\beta^{22}=\beta^{33}=\beta^{44}=-\beta$. So the third term of (2) becomes

$$-\varkappa^2\,\beta\begin{bmatrix} ij \\ l\end{bmatrix}_\beta$$

and the corresponding part of $2G_{ij}$ will be

$$-2\varkappa^2\sum_l\left\{\frac{\partial}{\partial x_j}\left(\beta\begin{bmatrix} il \\ l\end{bmatrix}_\beta\right)-\frac{\partial}{\partial x_l}\left(\beta\begin{bmatrix} ij \\ l\end{bmatrix}_\beta\right)\right\}=$$

$$=-4\varkappa^2\frac{\partial}{\partial x_j}\left(\beta\frac{\partial\beta}{\partial x_i}\right)+\varkappa^2\sum_l\frac{\partial}{\partial x_l}\left(\beta\left[\frac{\partial\beta_{il}}{\partial x_j}+\frac{\partial\beta_{jl}}{\partial x_i}-\frac{\partial\beta_{ij}}{\partial x_l}\right]\right)=$$

$$=-2\varkappa^2\frac{\partial}{\partial x_j}\left(\beta\frac{\partial\beta}{\partial x_i}\right)-\delta_{ij}\,\varkappa^2\sum_{(l)}\frac{\partial}{\partial x_l}\left(\beta\frac{\partial\beta}{\partial x_l}\right),$$

where the term $-\delta_{ij}\,\varkappa^2\left(\beta\frac{\partial\beta}{\partial x_4}\right)$, being of the third order, is omitted. We so obtain

in the case $i\neq 4,\ j\neq 4$: $\quad -2\varkappa^2\frac{\partial\beta}{\partial x_j}\left(\beta\frac{\partial\beta}{\partial x_i}\right)-\delta_{ij}\,\varkappa^2\sum_{(l)}\frac{\partial}{\partial x_l}\left(\beta\frac{\partial\beta}{\partial x_l}\right).$

in the case $i\neq 4,\ j=4$: $\quad$ *zero*,

in the case $i=j=4$: $\quad -\varkappa^2\sum_{(l)}\frac{\partial}{\partial x_l}\left(\beta\frac{\partial\beta}{\partial x_l}\right).$

At last we have to substitute the first term of (2) in the second part of $2G_{ij}$. This yields

$$2\varkappa^2 \sum_{lm} \alpha^{ll}\alpha^{mm} \left(\begin{bmatrix} il \\ m \end{bmatrix}_\beta \begin{bmatrix} jm \\ l \end{bmatrix}_\beta - \begin{bmatrix} ij \\ m \end{bmatrix}_\beta \begin{bmatrix} ml \\ l \end{bmatrix}_\beta \right) =$$

$$= \varkappa^2 \sum_{lm} \alpha^{ll}\alpha^{mm} \left\{ \frac{\partial \beta_{im}}{\partial x_l}\frac{\partial \beta_{jl}}{\partial x_m} - \frac{\partial \beta_{im}}{\partial x_l}\frac{\partial \beta_{jm}}{\partial x_l} + \tfrac{1}{2}\frac{\partial \beta_{lm}}{\partial x_i}\frac{\partial \beta_{lm}}{\partial x_j} - \right.$$

$$\left. - \tfrac{1}{2}\left(\frac{\partial \beta_{im}}{\partial x_j} + \frac{\partial \beta_{jm}}{\partial x_i} - \frac{\partial \beta_{ij}}{\partial x_m}\right)\frac{\partial \beta_{ll}}{\partial x_m} \right\} = \varkappa^2 \frac{\partial \beta}{\partial x_i}\frac{\partial \beta}{\partial x_j} + \varkappa^2 (\alpha_{ij} + \delta_{ij}) \sum_{(l)} \left(\frac{\partial \beta}{\partial x_l}\right)^2$$

Omitting terms of the first order, we so find for the left hand member of the equations of the field

in the case $i \neq 4,\ j \neq 4$:

$$- \varkappa^2 \sum_{(l)} \left(\frac{\partial^2 \gamma_{ij}}{\partial x_l^2} - \frac{\partial^2 \gamma_{il}}{\partial x_l \partial x_j} - \frac{\partial^2 \gamma_{jl}}{\partial x_l \partial x_i} \right)$$

$$+ \varkappa^2 \sum_l \alpha^{ll} \frac{\partial^2 \gamma_{ll}}{\partial x_i \partial x_j} + \delta_{ij}\, \varkappa \frac{\partial^2 \beta}{\partial x_4^2} - \varkappa^{3/2} \left(\frac{\partial^2 \sigma_{i4}}{\partial x_4 \partial x_j} + \frac{\partial^2 \sigma_{j4}}{\partial x_4 \partial x_i} \right) -$$

$$- 2\varkappa^2 \frac{\partial}{\partial x_j}\left(\beta \frac{\partial \beta}{\partial x_i}\right) - \delta_{ij}\, \varkappa^2 \sum_{(l)} \left(\beta \frac{\partial \beta}{\partial x_l}\right) + \varkappa^2 \frac{\partial \beta}{\partial x_i}\frac{\partial \beta}{\partial x_j}.$$

in the case $i \neq 4,\ j = 4$:

$$\varkappa^{3/2} \sum_{(l)} \left(\frac{\partial^2 \sigma_{4l}}{\partial x_l \partial x_i} - \frac{\partial^2 \sigma_{i4}}{\partial x_l^2} \right) - 2\varkappa \frac{\partial^2 \beta}{\partial x_4 \partial x_i},$$

in the case $i = j = 4$:

$$- \varkappa^2 \Delta \gamma_{44} - 3\varkappa \frac{\partial^2 \beta}{\partial x_4^2} + 2\varkappa^{3/2} \sum_{(l)} \frac{\partial^2 \sigma_{4l}}{\partial x_4 \partial x_l} -$$

$$- \varkappa^2 \sum_{(l)} \frac{\partial}{\partial x_l}\left(\beta \frac{\partial \beta}{\partial x_l}\right) + 2\varkappa^2 \sum_{(l)} \left(\frac{\partial \beta}{\partial x_l}\right)^2.$$

From these expressions we see that the third must serve for the calculation of γ_{44}, the second for that of σ_{1l}, the first for that of the six quantities γ_{ij} ($i \neq 4$, $j \neq 4$) after substitution of the values, found for σ_{4l} and γ_{4l} in some terms of it. As we want only the terms in L up to the second order inclusive, the case $i \neq 4$, $j \neq 4$ may be omitted.

The last expression can be reduced a little further. First we have

$$\sum_{(l)} \frac{\partial}{\partial x_l}\left(\beta \frac{\partial \beta}{\partial x_l}\right) = \Delta\left(\tfrac{1}{2}\beta^2\right)$$

and further

$$2 \sum_{(l)} \left(\frac{\partial \beta}{\partial x_l}\right)^2 = \Delta(\beta^2) - 2\beta\Delta\beta = \Delta(\beta^2) - 2\varrho\beta,$$

so that we obtain

$$- \varkappa^2 \Delta\left(\gamma_{44} - \tfrac{1}{2}\beta^2\right) - 2\varkappa^2 \varrho\beta - 3\varkappa \frac{\partial^2 \beta}{\partial t^2} + 2\varkappa^{3/2} \sum_{(l)} \frac{\partial^2 \sigma_{l4}}{\partial t \partial x_l}.$$

The quantities T in the right hand member of the equations of the field are to be calculated only up to terms of the first order inclusive. Consequently

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$$T = (1 - \varkappa\beta)\, T_{44} - \sum_{(l)} T_{ll}\,.$$

In the case $i \neq 4$, $j = 4$ we so obtain

$$\varkappa^{1/2} \sum_{(l)} \left(\frac{\partial^2 \sigma_{l4}}{\partial x_l \partial x_i} - \frac{\partial^2 \sigma_{i4}}{\partial x_l^2} \right) = 2 \frac{\partial^2 \beta}{\partial t \partial x_i} - 2 T_{i4} \quad \ldots \quad (7)$$

and in the case $i = j = 4$

$$\varkappa^2 \Delta (\gamma_{44} - \tfrac{1}{2}\beta^2) = -3\varkappa \frac{\partial^2 \beta}{\partial t^2} + 2\varkappa^{3/2} \sum_{(l)} \frac{\partial^2 \sigma_{l4}}{\partial t \partial x_l} - 2\varkappa^2 \varrho\beta + \varkappa(T_{44} - \varrho) + \varkappa \sum_{(l)} T_{ll} \quad (8)$$

4. We now proceed to the calculation of the quantities T_{ij}. They have to satisfy the equations

$$\sum_{lj} \frac{\partial}{\partial x_j} (\sqrt{-g}\, g^{lj}\, T_{il}) = \tfrac{1}{2} \sum_{ljmn} \sqrt{-g}\, g^{lm} g^{jn} \frac{\partial g_{lj}}{\partial x_i} T_{mn}\,.$$

Expressing the equality of the terms of the lowest order on each side, we have

$$\left.\begin{aligned} &\frac{\partial T_{i4}}{\partial t} - \sum_{(l)} \frac{\partial T_{il}}{\partial x_l} = \tfrac{1}{2} \varkappa\varrho \frac{\partial \beta}{\partial x_i}, \quad (i = 1, 2, 3) \\ &\frac{\partial \varrho}{\partial t} - \sum_{(l)} \frac{\partial T_{4l}}{\partial x_l} = 0\,. \end{aligned}\right\} \quad \ldots \quad (9)$$

In order to be able to calculate the quantities T we must make definite suppositions on the elastic properties of the bodies. Suppose them to be perfect liquids and suppose their expansions and compressions to be adiabatic. Then

$$T_i^l = -\delta_i^l\, p + \{p + \varrho\,(1 + P)\} \frac{ds}{dx_l} \sum g_{im} \frac{dx_m}{ds}$$

(vid. EINSTEIN "Formale Grundlage . . ." p. 1062). p represents the pressure and

$$P = -\int_{\varphi_0}^{\varphi} p\, d\varphi\,,$$

if φ represents the volume (in natural measure) at the pressure p and φ_0 the volume at the pressure φ_0, both of a quantity whose mass is 1 at the pressure 0; $\varrho\varphi = 1$. The tensor T_{ij} becomes now

$$T_{ij} = -p g_{ij} + \{p + \varrho\,(1 + P)\} \sum_{lm} g_{im}\, g_{jl}\, \dot{x}_m \dot{x}_l : \sum_{ab} g_{ab}\, \dot{x}_a\, \dot{x}_b$$

Expanding the denominator and omitting all terms of higher than the first order, we find from this

$$\left.\begin{aligned} &T_{ij} = \delta_{ij}\, p + \varrho\, \dot{x}_i \dot{x}_j \; (i = 4, j = 4)\,, \quad T_{i4} = -\varrho \dot{x}_i\,, \\ &T_{44} = \varrho + \varrho \sum \dot{x}_l^2 + \varkappa\varrho\beta + \varrho P\,. \end{aligned}\right\} \quad \ldots \quad (10)$$

The last equation shows that ϱ is the same quantity as the quantity named so before and occurring in (6) and (8).

Substituting (10) in (7) and (8) and returning to the quantities g_{ij} themselves, we find

$$\sum_{(l)}\left(\frac{\partial g_{l4}}{\partial x_l \partial x_i}-\frac{\partial^2 g_{i4}}{\partial x_l^2}\right)=2\varkappa\frac{\partial^2\beta}{\partial x_i \partial t}+2\varkappa\varrho\dot{x}_i \quad . \quad , \quad . \quad . \quad . \quad (11)$$

and

$$\Delta(g_{44}-\tfrac{1}{2}\varkappa^2\beta^2)=\varkappa(\varrho+3p+\varrho P)-\varkappa^2\varrho\beta+2\varkappa\varrho\sum_{(l)}\dot{x}_l^2+2\sum_{(l)}\frac{\partial^2 g_{l4}}{\partial x_l \partial x_l}-3\varkappa\frac{\partial^2\beta}{\partial t^2} \quad (12)$$

5. We now proceed to the solution of (11) and (12). From (6) it follows that

$$\beta=-\int\frac{\varrho dS}{4\pi r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where r means the distance from dS to the point where β is to be calculated. (11) is satisfied by

$$g_{i4}=2\varkappa\int\frac{\varrho\dot{x}_i dS}{4\pi r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

from which we get

$$\sum_{(l)}\frac{\partial^2 g_{l4}}{\partial x_l \partial t}=2\varkappa\frac{\partial^2\beta}{\partial t^2}$$

and consequently (12) becomes

$$\Delta(g_{44}-\tfrac{1}{2}\varkappa^2\beta^2)=\varkappa(\varrho+3p+\varrho P)-\varkappa^2\varrho\beta+2\varkappa\varrho\sum_{(l)}\dot{x}_l^2+\varkappa\frac{\partial^2\beta}{\partial t^2}.$$

Putting now

$$A=\varkappa\int\frac{\varrho(\sum_{(l)}\dot{x}_l^2)dS}{4\pi r}, \quad B=\varkappa\int\frac{\varrho r dS}{4\pi}, \quad . \quad . \quad . \quad . \quad (15)$$

we have

$$\Delta A=-\varkappa\varrho\sum_{(l)}\dot{x}_l^2 \quad , \quad \Delta B=-2\varkappa\beta$$

and so

$$\Delta\left(g_{44}-\tfrac{1}{2}\varkappa^2\beta^2+2A+\tfrac{1}{2}\frac{\partial^2 B}{\partial t^2}\right)=\varkappa(\varrho+3p+\varrho P)-\varkappa^2\varrho\beta.$$

We now consider the case of a number of separate bodies; more in particular we consider an astronomical system. In each body p and P may then be considered to be only dependent of the field of the first order, produced by the body itself, and consequently do not change. The term $-\varkappa^2\varrho\beta$ can be divided into as many terms as there are bodies, in such a way that say in the first body

$$-\varkappa\varrho\beta=-\varkappa\varrho\sum_a\beta_a$$

In this sum the term produced by the first body does not change. It constitutes, together with $3p + \varrho P$ a nearly invariable quantity of the first order. If we put

$$\varrho + 3p + \varrho P - \varkappa\varrho\beta_1 = \varrho'$$

in the first body (and a similar expression in the others), we may consider ϱ' to be the density; in (13), (14), (15) we then may replace ϱ by ϱ'. Indicating now again ϱ' by ϱ, we find

$$\Delta\left(g_{44} - \tfrac{1}{2}\varkappa^2\beta^2 + 2A + \tfrac{1}{2}\frac{\partial^2 B}{\partial t^2}\right) = \varkappa\varrho - \varkappa^2\varrho(\beta)\,, \quad . \quad . \quad (16)$$

where the parentheses about β mean that in each body the part, relating to the body itself, is to be omitted.

The solution of (16) is

$$g_{44} = 1 + \varkappa\beta + \tfrac{1}{2}\varkappa^2\beta^2 - 2A - \tfrac{1}{2}\frac{\partial^2 B}{\partial t^2} + \varkappa^2\int\frac{\varrho(\beta)dS}{4\pi r} \quad . \quad (17)$$

Suppose now the bodies to be spheres in case they rest (radii $R_1, R_2, \ldots, R_n$); as they are moving they will have a somewhat other form in consequence of the contraction in the direction of the motion, but the values furnishes by (14), (15) and the last term of (16) will vary in consequence of this only in terms of the third order; this variation we must neglect. Also in (13) we may do so for the calculation of the β which occurs in the third and the sixth term of (17). As to the second term of (17), however, we must consider the contraction in the proportion $(1 - \tfrac{1}{2}\sum_{(i)}\dot{x}_i^2) : 1$. Putting

$$\int_{(i)}\varrho dS = 4\pi m_i,$$

we get from (13)

$$\beta_i = -\frac{m_i}{r_i}(1 - \tfrac{1}{2}\sum_{(i)}\dot{x}_i^2)$$

r_i representing the distance from the i-th body, and the dimensions of the body being small with respect to r_i. Calling the coordinates x_1, x_2, x_3 henceforth x, y, z, so that $\dot{x}_i, \dot{y}_i, \dot{z}_i$ represent the components of the velocity of the i-th body, and supposing $(\dot{x}_i, \dot{y}_i, \dot{z}_i)$ to be almost equal in each point of the i-th body, we obtain from (14)

$$g_{14} = 2\varkappa\sum_i\frac{m_i\dot{x}_i}{r_i},\; g_{24} = 2\varkappa\sum_i\frac{m_i\dot{y}_i}{r_i}.\; g_{34} = 2\varkappa\sum_i\frac{m_i\dot{z}_i}{r_i}.$$

Representing the velocity of the i-th body by v_i, we get from (15), as the dimensions of the body are small with respect to r_i,

$$A = \varkappa\sum_i\frac{m_i v_i^2}{r_i},\; B = \varkappa\sum_i m_i r_i.$$

In the last term of (17) we may consider (β) to be constant and so we have

$$-\varkappa^2 \sum_i \frac{m_i}{r_i} \sum_{j \neq i} \frac{m_j}{r_{ij}},$$

where r_{ij} represents the distance of the i-th body from the j-th.

We so obtain

$$g_{44} = 1 - \varkappa \sum_i \frac{m_i}{r_i} + \tfrac{1}{2}\varkappa^2 \left(\sum_i \frac{m_i}{r_i}\right)^2 - 2\varkappa \sum_i \frac{m_i v_i^2}{r_i} - \tfrac{1}{2}\varkappa \sum_i m_i \ddot{r}_i - \varkappa^2 \sum_i \frac{m_i}{r_i} \sum_{j \neq i} \frac{m_j}{r_{ij}}$$

Putting

$$\varkappa m_i = 2 k_i,$$

we have

$$\left.\begin{array}{l} ds^2 = \left(1 - 2\sum_i \frac{k_i}{r_i} + 2\left(\sum_i \frac{k_i}{r_i}\right)^2 - 4\sum_i \frac{k_i v_i^2}{r_i} - \sum k_i \ddot{r} - 4\sum_i \frac{k_i}{r_i} \sum_j \frac{k_j}{r_{ij}}\right) dt^2 \\ + 8\sum_i \frac{k_i}{r_i}(\dot{x}_i dx + \dot{y}_i dy + \dot{z}_i dz) dt - \left(1 + 2\sum_i \frac{k_i}{r_i}\right)(dx^2 + dy^2 + dz^2) \end{array}\right\} \quad (18)$$

where j in the last term of g_{44} does not take the value i.

This is the field required.

Physiology. — "*Researches on the function of the sinus venosus of the frog's heart*". By E. BROUWER. (Communicated by Prof. HAMBURGER).

(Communicated in the meeting of May 27, 1916).

I. *Effect of $CaCl_2$, KCl, NaCl and osmotic pressure.*

Introduction.

As we know the myogenous theory of the heart supposes the impulse to the automatic motion of the heart to originate in the muscular substance of the sinus venosus. There must be a centre there whence the rhytmic stimulus takes its origin, which stimulus is transmitted through auricle and ventricle. From a chemical point of view there is no longer anything mysterious about the occurrence of such periodical stimuli, since BREDIG has made us acquainted with the periodical contact-catalysis.

The reader will remember his experiment: a mercury-surface is covered with a solution of hydrogenperoxyde. A red layer of HgO is formed, but after a short time it disappears and O_2 being set free, a pure mercury-surface is the result. This phenomenon is repeated rhythmically.

Now it must be esteemed of the greatest importance to get acquainted with the chemism of the stimulus originating in the sinus venosus.