## Huygens Institute - Royal Netherlands Academy of Arts and Sciences (KNAW)

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c. in the case of change of solubility and surface-tension through combining with salt- or sugar-solutions.
Divergencies are seen on either side. Some strong-smelling scents, moschus and scatol, set up a sensation even in dilutions, the electrifying power being next to none. With antipyretica on the contrary, the smell-intensity is all but inappreciable, the electrifying power very considerable. For an interpretation we must, in the present stage of our research, look to the five conditions, just mentioned, upon the fulfilment of which the properties depend. Maybe the study of the dielectric-constant will throw some more light on the subject in connection with A . Coern's theory of contact-electricity, according to which the difference of potential, in the case of mutual contact of two substances is proportional to the difference of the dielectric constants ${ }^{1}$ ). The differences between the dielectricconstants of odorous substances and those of water are generally very greal, so that likewise we may look for great charges on numberless droplets, when odorous molecules accumulate on their surfaces in virtue of the fall of the surface-tension. The rapid evaporation from the measureless area makes these differences all the more probable, especially when a slight rise of the temperature increases the evaporation and the sprayer is placed at the proper distance from the intercepting screen.

Chemistry. -- "The equilibrium solid-liquid-gas in binary systems which present mixerl crystals". (Fourth communication). By Prof. H. R. Krcyt. (Coinmunicated by Prof. Ernst Coher).
(Ciommunicated in the meeting of September 30, 1916.)

1. In the former communication ${ }^{2}$ ) has been communicated a research of Dr. W. D. Helderman and myself on the three phase equilibrium solid-liquid-gas ( $\mathrm{S} L \mathrm{G}$ ) in the system bromine-iodine. The three-phase line exhibited in its PT-projection two maxima and one minimum, a peculiarity which is closely connected with the appearance of the compound IBr . For the general knowledge of the binary systems it is not uninteresting to more closely consider these equilibria; all the more so becanse the appearance of two maxima and one minimum on the melting branch of a com-

[^0]pound is also possible when no formation of mixed-crystal takes place whaterer. The mutual proportions, however, are very different there and, as we will notice below, the conditions for the appearance of these remarkable points are also different.
2. The general equation for the three-phase equilibrium in a binary system can, as is well-known ${ }^{1}$ ), be written as follows.
\[

$$
\begin{equation*}
T \frac{d P}{d T}=\frac{\left(x_{S}-x_{L}\right) Q_{G S}-\left(x_{S}-w_{G}\right) Q_{L S}}{\left(x_{S}-x_{L}\right) V_{G S}-\left(w_{S}-x_{G}\right) V_{L S}} . \tag{1}
\end{equation*}
$$

\]

or

$$
\begin{equation*}
T \frac{d \mu}{d T}=\frac{Q_{G S}-\frac{x_{S}-x_{G}}{x_{S}-x_{L}} Q_{L S}}{V_{G S}-\frac{x_{S}-x_{G}}{x_{S}-x_{L}} V_{L S}} \tag{2}
\end{equation*}
$$

in which the symbols with a double phase-index indicate respectively the heat and the change in volume when one gram-molecule of one phase is dissolved in an infintely large quantity of the other, the external conditions being kept constant. QaS has, therefore, the order of a sublimation heat, $Q_{L S}$ that of a melting heat. Hence $Q_{G S}$ is nsually a few times greater than $Q_{L S}$. Likewise $V_{G S}$ is always greater than $V_{L S}$ and this in the order of $10^{+}$times greater.


Fig. 1.

$$
\begin{equation*}
\frac{v_{S}-v_{G}}{v_{S}-v_{L}}=\frac{Q_{G S}}{Q_{L S}} \tag{}
\end{equation*}
$$

[^1]Now it will be evident how the formula describes the course taken by the three-plaseline. For in the $T w$-projection we notice that in $O_{A} r_{S}=x_{L}=x_{G}$, and that startung from $O_{A}$, the fraction $\frac{x_{S}-x_{G}}{x_{S}-x_{L}}$ gradually increases and thas the possibility exists that it attains to a value equal to $\frac{Q_{G S}}{Q_{L S}}$.

- Then $\frac{d P}{d T}=0$, the maximum is attained $; \frac{d P}{d T}$ then becomes negative and the line on the $P T$-projection falls to $O_{B}{ }^{1}$ ).

The question whether indeed a maximum occurs thus depends on the concentration ratios of the coexisting phases and those of the heat values. By means of another method we previously arrived at a similar result (see our first communication).

The three-phase line proceeds without a maximum or minimum if the component $A$ which at each temperature has the highest vaponr pressure, bas a higher triplepoint temperature than $B$. $a_{S}-a L$ is then continuously negative. which prevents the numerator from becomug zero. The impossibility of a maximum is moreover directly evident by looking at the spacial figure for that case.
3. In Fig. 2 the diagram ( $P T$ and $T a$ ) is drawn for a system in which occurs a compound ${ }^{9}$ ) 'on the melting branch' of which in succession $x_{S}=x_{L}, x_{S}=x_{G}$ and $x_{L}=x_{G}$ is realised. In such case that branch exhbits also two maxima and one minimum. A further remarkable fact is the occurrence of a point towards $\frac{d P}{d T}=\infty$, and moreover the maximum point of sublimation and the

[^2]

Fig. 2.
minimum melting point in which points the lines for the quasi-mary two-phase equilibrium compound $+L$ and compound $+G$ meet the three-phase line, respectively.

The connexion hetween this graplic representation of the three-phase equilibrium and the analytical expression used in § 2 has been worked out in the papers cited. On CTF the concentration fraction $\frac{-x_{S}-x_{G}}{x_{S}-x_{L}}$ grows stcadily greater (compare the Tw-projection). The value for the maximum $T$ is soon attained, then the numerator of equation 2 becomes negative. Just before $c_{s}$ becomes $=x_{L}$ (comp. equation 1)
$\left(a_{S} S-x_{L}\right) \Gamma_{G S}$ will become $=\left(x_{S}-x G\right) V_{L S}$.
and, therefore $\frac{d P}{d T}=\infty$ (point $R$ ) for the concentration fraclion approaches $\propto$ (when $x_{S}=x_{L}$ ) and so a value of about $10^{4}$ is once attained.

The nomeraior then has a great negative value, the denominator starts from $R$, also negative ; hence $\frac{d P}{d T}$ positive. In $F^{\prime} x_{L}=w_{S}$; the concentration fraction in equation (2) which before $F$ is very largely positive, becomes very largely negative beyond $l$. This negative value declines until in $G$ it las become 0 , as in that point $x_{G}=w_{S}$. The concentration fraction now again assumes a rising positive value and in $H$ it becomes 1 as in that point ${ }_{r_{G}}=v_{L}$ and it will thus soon again attain a value causing

$$
\begin{equation*}
\frac{x_{S}-x_{G}}{x_{S}-x_{S}}=\frac{Q_{G S}}{Q_{L S}} \tag{3}
\end{equation*}
$$

we then are in the minimum $T$. For if now we examine the $T w$ figure between $h$ and $d$ we readily notice that after the crossing of the $G$ - and $L$-branch in $h a S-v G$ increases much quicker in (negative) value than $x_{S}-x_{L}$; hence, the concentration fraction thus continues to rise at first, but in $d d^{\prime}, x_{L}$ and $v_{G}$ do not usually differ much ${ }^{1}$ ) as both are situated very close to the $B$-axis. Consequenily the value of the concentration fraction has again begun to fall and once again the relation (3) has been satisfied, so causing the maximum $T$ to appear. For the appearance of the minimum $T_{1}$ and the

[^3]second maximum $T_{2}$ the crossing at $H$ and the appearance of a minimum pressure in the $P v$-sections of the system is, therefore, a necessity. ${ }^{1}$ )
4. Now in the system Br-I also appear two maxima and one minimum, but in this system neither $a_{S}$ becomes $=a_{L}$, nor $v_{S}={ }^{n} G$, nor $x_{L}-v_{G}$ at any temperature, as has been proved by the determinations of the melting and boiling curves by Muerum Termogt ${ }^{2}$ ). Only, the differences $x_{S}-v_{G}$ and $x_{S}-x_{L}$ show and approach to 0 and this is sufficient to cause the
 appearance of the three marked points. For this we have only to examine Fig. 3, which indicates sketchily the proportions in the system Br-I in $P T^{-}$ and $T x$ figure.

Starting in $O_{A}$ the value of the concentration fraction $\frac{x_{S}-x_{G}}{x_{S}-x_{L}}$ will at first again increase and the ralue mar become eqnal to $\frac{Q_{G S}}{Q_{L S}}$, thus causing the maximum $T$ to appear. As the numerator in equation (2) here becomes negative, the three-phase line will fall. This fall will be very pronounced, for the denominator of the concentration fraction decreases very strongly. For at about $x=0.5$ the narrowing on the melting curve is very strong; it is, however, conceivable that the denominator in the second member of equation (2) will not turn to zero, for then the concentration fraction should assume the value 10000 .

Meanwhile, not only the denominator of the concentration fraction assumes a small value, but the numerator will also decline particularly after the temperature has been passed at which the narrowing on the melting curve occurs. It is well known that vapour pressure ${ }^{3}$ ) or boilingdiagrams of systems with a compound present a narrowing all the stronger when the dissociation is less. The boiling diagram ${ }^{4}$ ) of the system Br - [ also exhibits such a narrowing, even at a temperature about $100^{\circ}$ above that of the three-phase line. This narrowing

[^4]in the $P$. $v$-diagrams is strongest in the part of the figure at the side of the component with the smaller vapour tension ${ }^{1}$ ), hence at the iodine side. Whereas the numerator of the concentration fraction is now declining, its denominator gets larger when we have passed the narrowing in the melling diagram, faromable cooperation for allowing the value of the concentration fraction to rapidly lall again to that of $\frac{Q_{G S}}{Q_{L S}}$. The point $T_{1}$ will, therefore, be rapidly attained. Indeed it appeared in the research of Dr. Hecderman and myself that this point is altained at a temperature about $4^{\circ}$ above the melting interval of the mixture $x_{s}=0.50$ mamely, at $x_{s}=$ about 0.54. From the $T x$ side of figure 3 we can now readily read that the concentration fraction still continues to decrease but afterwads gradually rises again in consequence of the fact that $x^{2} 5-x_{L}$ decreases more rapidy than $a S-n g$. Hence, the possibility of a sccond maximum $T_{2}$ is created.

We will not discuss here any possible intermediate cases where a degeneration of maximum and minimum to points of inflection may take place. If will be quite evident now, that, where the narrowings are decisive for the appearance of the minimum, the above inentioned configuration occurs the more decidedly when the compound is less dissociated.
5. We will now call attention' to the difference in behaviour letween a compound as explained by Fig. 2 and Fig. 3, that is to say between a dissociating compound and one miscible with its dissociation products in the solid condition. If such a compound withont formation of mixed crystals is heated to fusion at a constant pressure the equilibrium will be quasi-unary ${ }^{2}$ ), the compound has a sharp melting point. With the compound which forms mixed crystals such is not the case, for such a compound has a melting interval. In connexion therewith the equilibria at their own rapour tension, as read off from the PT-diagram are also more complicaled.

First of all let us remember that in the point $F$ of figure 2 $x_{L}=x_{S}$ and that consequently in equation (2) becomes $T \frac{d P}{d T}=\frac{Q_{L S}}{\gamma_{L S}}$. This, however, is the equation for the melting line of the compound;

[^5]that line is tangent therefore in $F$ to the three-phase line and $F$ is justly called the minimum melting point. Also in $G^{\prime} v_{G}=a_{S}$, hence $T \frac{d P}{d T}=\frac{Q_{G S}}{V_{G S}}$ and the snblimation curve of the compound is tangent here to the three-phase line. And in $H{ }^{v}{ }_{L}=x_{G}$, hence
$$
T \frac{d P}{d T}=\frac{Q_{G S}-Q_{L S}}{V_{G S}-V_{L S}}=\frac{V_{G L}}{Q_{G L}}
$$
which involves the meeting of the three-phase line and the "line of the minima on the $G L$ surface".

By these tangential contacts the $P T$-section at the composition of the compound gets the simple form of fig. $4^{1}$ ) $L G F K$ is a continuous curve


Fig. 4. although $L G$ is the quasi-unary sublimation curve, GF part of the threephase line, and $F K$ the quasi-unary melting line of the compound. $F P$ and $G P$ are demarcations for the complete condensation and evaporation, respectively. From a purely unary diagram, this figure is distinguished only by the fact that the triplepoint has grown to a range $G H$ and the phenomena of the condensation and evaporation take place not at a single limit value, but also over an interval.

With a system of the type Br-I the difference with unary behaviour is much greater still. Because two phases nowhere attain the same composition, we miss the tangents at the three-phase line, in


Fig. 5. fart all lines for quasi-mnary equilibrium get donbled to two streak limits. In Fig. 5 we notice the $P T$-section for $x=0.50$. Fig. 5 therefore indicates the PT' condition diagram for the compound IBr and it mas be readily deduced from the spacial figure (compare Fig. 5 of the third communication) or from our Fig. 3. It is only one possible configuration; at the deduction it will be noticed that the mutual relations of the concentrations of coexisting phases decide the form of a figure such as Fig. 5.

When following the section up from lower temperature we intersect first the region $S+G$. In point 1 the section for the first time meets the three-phase region, namely in the line indicating the composition of the mixed crystals. In point 2 the liquid branch

[^6]is reached; the vapour branch is, of course, only attained at a much higher temperature (point 3) when on the threephase line, $v_{G}=0.50$; the three phase line is then almost sure to have passed the minimum $T_{1}$.
From this figure it is readily observed how little the behaviour of the mixture $x=0.50$ conveys the idea of a single substance. For it is evident from this figure how not only all the sharp demarcations have vanished but how peculiar retroyrade phenomena (between $T_{1}$ and 3 ) complicate the behaviour. Direct stoechiometric criteria no longer exist for such a compound; it is only recognisable from the general connexion of the phenomena ${ }^{1}$ ).
6. It is not our intention to discuss the course of the three-plase_ line in all possible systems with formation of mixed crystal, but it is still of importance to devote some attention to the case that'a maximum or a minimum appears in the sections of the spacial figure.

In Fig. 6 is drawn the case where in the $7 w$-sections appears a maximum for the equilibrium $S-L$, not, however, for the equilibrium $L-G$. Such a maximum often gives rise to doubt: does it indicate a compound or not? A further discussion of this question will be postponed to a future communication, but here it is important to know the three-phase line of this type of system.


In Fig. 6 we notice how, starting in $O_{A}$, the three-phase line rises, for the concentration fraction in (2) has a positive value, this rises and can attain the value where equation (3) is satisfied; the maximum $T$ then appears. The numerator of equation (2) now obtains a negative value. As presently $x_{L}$ will become $=w$ sthe concentration fraction increases very strongly and before $x_{L}$ becomes $=x_{S}$ the denominator in equation (2) becomes zero, so that $\frac{d P}{d T}$ becomes $=\infty$ (point $R$ ); $\frac{d P}{d T}$, then again becomes positive. Soon $n_{L}$ now becomes $=x_{S}$ as represented by point $F$. This point

[^7]$F$ is in many respects comparable with point $F$ in Fig. 2, also here a line $F K$ for quasi-unary melting phenomena will appear ("line of the maxima" in the melting diagrams at different pressure values), hence we might call point $F$ the "minimum quasi unary melting point" of the srstem. As starting from $F a s-x_{L}$ has a negative value whereas $x S-x ;$ remains positive; both numerator and denominator in equation (1) will also be negative; a further appearance of a maximum or a minimum is, therefore, excluded (compare $\$ 2$ last lines). Only when equalisation of phases takes place, (which involves a minimum in the $P x$-figure from $S-G$ or $G-L$ equilibria) further remarkable points would be possible, such as a "maximum quasi-unary sublimation point" a minimum and a maximum on the three-phase line etc.

The progressive change of these lines strongly reminds us of that of a compound without formation of mixed crystals but the configuration of Fig. 6 does not give us, however, an indication for the existence of a compound. On the contrary, an eventual decrease of the value $r_{G}-x_{L}$ which had such important consequences in $\$ 4$ does not affect any peculiarity on the three-phase line, not even a reduction of $x_{a}-x_{L}$, to zero in a maximom pressure would have that effect; only $x_{G}-x_{L}$ in a minimum would modify the line, but even then the doubt would remain whether we had to do with a chemical compound or not.
7. We must still refer in a few words to a system of the type $d$ and $l$-carvoxim discussed in the previous communication. After the statement in $\oint 6$ it will be evident that the investigation of the three-phase line does not lead to a decision when the compound melts at a higher temperature than that of the components. But there is still another complication in a system of optic antipodes. It happens there that $x_{G}-x_{L}$ is zero at all concentrations. This is the case in the system $d$ - and $l a$-pipecolin ${ }^{t}$ ) albeit there occurs undoubtedly a solid racemic componnd ${ }^{2}$ ) in that system ${ }^{7}$ ).

[^8]In the system of the carvoxims it is sure to be the same case and then the equation of the three-phase line becomes

$$
T \frac{d P}{d T}=\frac{Q_{G S}-Q_{L S}}{V_{G S}-V_{L S}}=\frac{Q_{G L}}{V_{G L}}
$$

The three-phase line simply coincides with the vapour tension line for liquid carvoxim (whether $l, l, r$ or a mixture) between the melting temperature of the components and that of the pseudo-racemic mixed crystal. This curve is then a double line and in a certain sense we might then say that it has one maximum which of course, is plainly noticed in the Px-projection.
But as to the decision between racemic compounds and pseudo-racemic mixed crystals this course of the three-phase line yields no criterion. Only, van Laar ${ }^{2}$ ) bas pleaded for the conception. that the form of the melting line of the carvoxims solely points to a compound, whereas Tammann ${ }^{2}$ ) on account of caloric values exactly arrives at the opposite result.

Utrecht, August 1916. van 't Hofr-Laboratory.

Chemistry. - "On Nitro-derivatives of Alkyltoluidines and the relation between their molecular refractions and those of similar compounds'. By Dr. J. D. Jansen. (Communicated by Prof. P. van Romburgh).
(Ciommunicated in the meeting of June 24, 1916).
Some years ago appeared a communication from Hantzsoh ${ }^{3}$ ) on chromoisomerism and homochromoisomerism of nitroanilines. In the conclusion of this remarkable publication, Hantzsch says that yellow and orange di- and tri-nitroanilines, when their molecular refractions are abnormal, probably contain chromophores of different constitution. The yellow 3.4. dinitro-dimethyl-aniline and the orange 3.4. dinitro-diethyl-anilire, according to his opinion, are not real-, but pseudohomologues, which manifested itself in the molecular refractions,
or

$$
{ }_{n R T l} \frac{x_{G}}{1-v_{G}}=m R T l \frac{x_{L}}{1-x_{L}}
$$

$$
w_{G}=w_{I}
$$

The fact that $a_{12}=a_{1}$ certainly points out that in the liquid and vapour occur no (or at least very few) racemic moleculps.
${ }^{1}$ ) Zeitschr. f. physik. Chcm. 64, 289 (1908).
9) Götlinger Nachrichten 1913.
${ }^{3}$ ) В 43, 1662, (1910).


[^0]:    ${ }^{1}$ ) A. Cosen u. U. Raydt. Göttinger Nachrichte 1909 p. 263.
    ${ }^{2}$ ) Proc. Royal Ac. A'dam, Meeting of June 1916.

[^1]:    1) Compare van der Waals-Kohnstama, Lehibucl der Thermodynamik Il p. 521 et seq. (Leipzig 1912).
    ${ }^{2}$ ) Comp. my communications I and II in these Proc. 1909, 537 and 1910, 206 also Zeitschr. f. physik Chem. 79, 657 (1912).
[^2]:    ${ }^{1}$ ) In the system $p \mathrm{Cil}_{2} \mathrm{C}_{6} \mathrm{H}_{4}-p \mathrm{Br}_{2} \mathrm{C}_{6} \mathrm{H}_{4}$ the maximum lies at about $76^{\circ}$. If by interpolation we calculate the composition of the coexisting phases at that temperatue we find appooximately -

    $$
    \begin{aligned}
    & x_{S}=0.94 \\
    & x_{L}=0.76 \\
    & n_{G}=0.27 .
    \end{aligned}
    $$

    fiom which follows

    $$
    \frac{x_{S}-x_{G}}{x_{S}-x_{L}}=\frac{Q_{G S}}{Q_{L S}}=3,7
    $$

    which is a rather low value for that ratio.
    ${ }^{2}$ ) For a detaled desciption of this class of three-phase lincs compare v. d. WAALS Kohnstamm le; H. W. Bakhuis Roozeboom, These Phoceedings 1905; G. II. Leopond, Dissertation. Amsterdam 1906; A. Snirs, Zeitschr. f. physik. Chem. 78, 70S (1912).

[^3]:    ${ }^{1}$ ) Much less than drawn in Fig. 2.

[^4]:    ${ }^{1}$ ) It is meanwhile evident that this relation is not necessarily decisive. For beyond $H$ the concentration fraction must also attaintheratio of the caloric values (Consult the papers cited).
    ${ }^{2}$ ) Diss. A'dam 1904 and Zeilschr. f. anorgan. Chem. 47, 203 (1905).
    ${ }^{3}$ ) In this connexion see J. J. van LaAR, Zeitschr f. physik Chem. 47, 129 (1904).
    ${ }^{\text {y }}$ ) P. C. E. Meerom Terwogt, loc. cit.

[^5]:    ${ }^{1}$ ) Van LaAr, loc. cit.
    ${ }^{2}$ ) The expression quasi unary, quasi binary elc seems very appropriate for characterising the condition in which a system behaves as if it possesses one variable (or two, three ete.) less than indicated by the phase rule. (As to these equilibria see Barbuis Roozrboom, Ieterogene Gleichgewichte I pg. 34 and following).

[^6]:    ${ }^{1}$ ) Wurre, Disselt Amsterdam 1909 p. 19.

[^7]:    ${ }^{1}$ ) Vgl. H. R. Kroyr, Algeneene Theorie en bizondere Ervaring (Amsterdam 1916).

[^8]:    1). A. Ladennburg, Ber. deutsch. chem. Ges. 44. 676 (1911); H. R. Kruyt, ibidem pg. 995.
    ${ }^{2}$ ) A. Ladenburg and Sobecki, ibidem 43, 2374 (1910).
    $\left.{ }^{3}\right)^{-}$The fact that, in such a system, all G-L equilibria are quast-unary is comected with the peculiar small differences of the components; $\alpha_{1}$ is there undoubtedly $=a_{2}$, also $b_{1}=b_{2} ;$ also $a_{12}$ is evidently equal to $a$, for then $\frac{d a_{x}}{d x}=0$ and also $\frac{d b_{x}}{d x}=0$ and for

    $$
    \left(\left.\frac{\delta \psi}{\partial x^{\prime}}\right|^{\prime} T\right)_{G}=\left(\frac{\delta \psi}{d x \psi V T}\right)_{L}
    $$

    we cau simply write then

