II. No reaction is possible, in which all phases of the invariant point may participate.

When e.g. the phase $F_1$ cannot take part into one single reaction, then in (1) and (2) $a_1$ becomes $= 0$. Then we have an invariant point with $n+1$ phases, for which the considerations sub $I$ are true.

Leiden, *Inorg. Chem. Lab.* (*To be continued*).

**Physics.** — "*Experimental Inquiry into the Laws of the Brownian Movement in a Gas.*" By Miss A. SNETHLAGE. (Communicated by Prof. P. ZEEMAN).

(Communicated in the meeting of Feb. 24, 1917).

1. In a former paper [1]) some objections have been advanced to EINSTEIN's formula for the Brownian movement by Prof. VAN DER WAALS JR. and me. According to this formula:

$$\overline{\Delta^2} = \frac{2RT}{N} B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which $\overline{\Delta^2}$ represents the mean square of the displacement which a "Brownian particle" obtains per second in a definite direction. Equation (1) has been derived on the supposition that the particle meets in its movement with a resistance of friction. Accordingly $B$ is the inverse value of the factor of resistance which is found when the particle travels with constant velocity under influence of an external force. Statistical mechanics, however, teaches that a particle, in equilibrium with the surrounding molecules, does not experience a force dependent on its velocity, hence no ordinary friction. We have written the equation of motion in the form:

$$\ddot{u} = -pu + q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and derived a value for $\overline{\Delta^2}$, which does not lay claim to great accuracy, but leads, at least for the Brownian movement in a gas, to $\overline{\Delta^2}$ being proportional with $\frac{1}{a^2}$, when $a$ represents the radius of the particle.

According to STOKES' formula with CUNNINGHAM's correction:

$$\frac{1}{B} = 6\pi\zeta a k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which $\zeta$ represents the coefficient of friction of the medium and

$$k = \left(1 + A\frac{\lambda}{a}\right)^{-1}.$$

[1]) These Proc. 18, 1916, p. 1322.

$\lambda$ is the mean length of path of the molecules, $A$ is a constant determined at 0,873 in GUYE's laboratory at Geneva [1]). Different investigators have performed measurements of $\overline{\Delta^2}$, however almost always from the mutual differences of the times in which a particle travels a definite distance under the influence of an external force. It is, however, the question whether the thus determined values of $\overline{\Delta^2}$ are the same as those of equation (1). For with the movement under an external force the distribution in space of the molecules of the medium is disturbed, and it moves for a part with the particle. The chance to a Brownian displacement upward or downward will no longer be symmetrical. Only one investigation is known to me, that by FLETCHER [2]), in which no external force acted on the particle. The data obtained in this way, are, however, not numerous.

It seemed therefore not superfluous to me, to start another inquiry into the validity of equation (1). In my experiments, carried out in the Physical Laboratory at Amsterdam (Director Prof. ZEEMAN), the displacement of a particle was measured while the gravity and the electric force were in equilibrium with each other. This can be established with pretty great accuracy; in order, however, not to be disturbed by a small residual force I observed the movement in horizontal instead of vertical direction.

2. I made use of the well-known method of MILLIKAN [3]) and EHRENHAFT [4]).

When $v_v$ represents the velocity of fall of the particle with mass M, $v_v'$ the velocity under influence of gravity and an electric force of equal direction, $v_s$ the velocity of rising, when this electric force is reversed, the following equations hold:

$$Mg = \frac{1}{B} v_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

$$e\mathfrak{E} - Mg = \frac{1}{B} v_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4b)$$

$$e\mathfrak{E} + Mg = \frac{1}{B} v_v' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4c)$$

$e\mathfrak{E}$ is the absolute value of the electric force, e the charge of the particle.

From (4a) and (4b) follows:

1) A. SCHIDLOF et Mlle J. MURZYNOWSKA, Arch. de Genève 4,40,1915, p. 386 and 486.
2) H. FLETCHER, Phys. Rev. 33, 1911, p. 81.
3) R. A. MILLIKAN, Phys. Rev. 29, 1909, p. 560.
4) See for a full description: F. EHRENHAFT. Wien. Sitz.ber IIa, 123, 1914, p. 53.

$$e\mathfrak{E} = \frac{1}{B}(v_v + v_s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If $B$ is known, then $\mathfrak{E}$ can be calculated.

According to (3) we may write (4*a*) as:

$$\tfrac{4}{3}\pi a^3(\varrho - d)\,g = 6\,\pi\zeta\,akv_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$\varrho$ is the density of the particle, $d$ that of the medium, $g$ the acceleration of gravity.

From (6) we can calculate $a$.

SCHRÖDINGER [1]) showed that:

$$v_v = \frac{L}{\overline{t_v}}$$

in which $\overline{t_v}$ is the mean of the times required by the particle to fall over the distance $L$. The Brownian movement namely causes the measured times to differ somewhat inter se.

The measurement of $\overline{\Delta^2}$ took place in the following way: I observed a great many times the time in which the particle covers a certain distance in horizontal direction, when the gravity is neutralized by an electric force. To find $\overline{\Delta^2}$ from these times of displacement we ask: what is the chance that the particle after a time $t$ crosses for the first time a dividing line at a distance $l$, no matter on which side? We confine ourselves to the $X$-movement. I have made use of the method which SCHRÖDINGER [1]) uses for a similar problem.

When at a time $t=0$ a great number $N$ of particles start from the point $O_{(x=o)}$, the number with coordinates between $x$ and $x+dx$ at the moment $t$ will be:

$$N\sqrt{\frac{\alpha}{\pi t}}\,e^{-\frac{\alpha x^2}{t}}\,dx\,.$$

We see the meaning of $\alpha$ by calculating the mean value of $x^2$, It then appears that:

$$\alpha = \frac{1}{2\,\overline{\Delta^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Calling the points that lie at a distance $l$ on the right and on the left of $O$, $A$ and $B$ (fig. 1), we shall calculate how many particles have passed neither of the points in the time $t$.

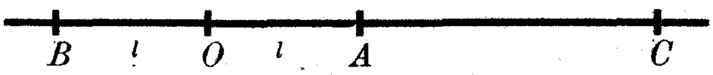

Fig. 1.

[1]) E. SCHRÖDINGER. Phys. Zeitschr. 16, 1915, p. 289.

It is easy to say how many *have* passed over $A$ or $B$. Let us confine ourselves first of all to $A$ alone. When a particle reaches $A$, the chances that it will lie on the leftside or on the rightside of $A$ some time later, will be equally great. Hence the number of particles $N_1$ that reaches $A$, is equal to double the number on the righthandside of $A$.

$$N_1 = 2N\sqrt{\frac{a}{\pi t}}\int_l^\infty e^{-\frac{ax^2}{t}}\,dx$$

The number of particles that has passed $B$ is of course equally great. The required number $M$, which has neither reached $A$ nor $B$, is however not equal to $N - 2N_1$, for we have counted the particles that have passed over the path $OAB$ among the two groups $N_1$, as also the particles $OBA$. (The meaning of this way of writing is clear.) The number $N_2$ that has travelled the path $OAB$ is equally great as the number that has reached a point $C$ at a distance $3l$ from $O$. This is of course again:

$$N_2 = 2N\sqrt{\frac{a}{\pi t}}\int_{3l}^\infty e^{-\frac{ax^2}{t}}\,dx\,.$$

Thus we find:

$$M = N - 2N_1 + 2N_2\,.$$

Now, however, the particles $OABA$ and $OBAB$ have again been counted among each of the groups $N_2$ etc. Continuing in this way we find:

$$M = N - 2N_1 + 2N_2 - 2N_3 + \ldots, ad\ inf. \quad \ldots \quad (8)$$

in which:

$$N_m = 2N\sqrt{\frac{a}{\pi t}}\int_{(2m-1)l}^\infty e^{-\frac{ax^2}{t}}\,dx\,.$$

In order to find the chance $P(t)\,dt$ that a particle for the first time passes one of the points $A$ or $B$ between $t$ and $t + dt$, we must differentiate (8) with respect to $t$:

$$P(t)\,dt = -\frac{1}{N}\frac{dM}{dt}\,dt\,.$$

$$M = N - 4N\sqrt{\frac{a}{\pi t}}\left\{\int_l^\infty e^{-\frac{ax^2}{t}}\,dx - \int_{3l}^\infty e^{-\frac{ax^2}{t}}\,dx + \int_{5l}^\infty \ldots\ ad\ inf.\right\} \quad (9)$$

This series is convergent for all finite values of $t$.

If we now introduce a new variable $y$, so that:

$$y^2 = \frac{ax^2}{t}$$

then (9) is transformed to:

$$M = N - \frac{4N}{\sqrt{\pi}} \left\{ \int_{l\sqrt{\frac{\alpha}{t}}}^{\infty} e^{-y^2}\, dy - \int_{3l\sqrt{\frac{\alpha}{t}}}^{\infty} e^{-y^2}\, dy + \dots \text{ ad inf.} \right\}$$

hence:

$$P(t)\, dt = 2l \sqrt{\frac{\alpha}{\pi}}\, t^{-\frac{3}{2}} \left\{ e^{-\frac{\alpha l^2}{t}} - 3e^{-\frac{9\alpha l^2}{t}} + \dots \text{ ad inf.} \right\} dt \quad . \quad (10)$$

This series is convergent for all values of $t$.

If we now put:

$$\frac{\alpha l^2}{t} = z_1^2 \quad , \quad \frac{9\alpha l^2}{t} = z_2^2 \quad , \quad \frac{25\alpha l^2}{t} = z_3^2 \text{ etc.}$$

then we get:

$$P(t)\, dt = -\frac{4}{\sqrt{\pi}} \{ e^{-z_1^2}\, dz_1 - e^{-z_2^2}\, dz_2 + \dots \text{ ad inf.} \} . \quad . \quad (11)$$

Our purpose is to determine $\alpha$.

From (10) we can calculate the mean value of $\frac{1}{t}$.

We find for this:

$$\overline{\frac{1}{t}} = \frac{1}{\alpha l^2}(1 - \tfrac{1}{9} + \tfrac{1}{25} - \dots \text{ ad inf.})$$

If we represent the sum of the series between brackets by $f$, and put:

$$\overline{\frac{1}{t_\Delta}} = \frac{1}{t_\Delta^*}$$

then

$$\alpha = \frac{f}{l^2} t_\Delta^*$$

and

$$\overline{\Delta^2} = \frac{l^2}{2f} \frac{1}{t_\Delta^*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The index $\Delta$ annexed to $t$ denotes that we mean the times of displacement. $f$ is to be approximated with an arbitrary degree of accuracy. For our purpose suffices $f = 0.916$.

The observations give $t_\Delta^*$, hence $\overline{\Delta^2}$ can be calculated.

3. I shall briefly describe the apparatus which I used for my observations.[1]) I assume the method of MILLIKAN and EHRENHAFT to be known.

---

[1]) Compare for a full description my thesis for the doctorate, which will shortly appear.

The *condenser* consisted of 2 square brass plates placed horizontally. I have worked with 2 different condensers $C_1$ and $C_2$ of the following dimensions:

| | $C_1$ | $C_2$ |
|---|---|---|
| sides of the plates | 20 mm. | 14 mm. |
| thickness | 2 ,, | 8 ,, |
| distance about | 3 ,, | 2 ,, |

$C_2$ has been used most.

I observed through a *microscope* placed horizontally. A micrometer was adjusted between the two lenses of the eye-piece. It consisted of two sets of lines drawn normal to each other, 0.1 mm. apart. The magnification with respect to this micrometer amounted to from 4 to 5, the total magnification to from 80 to 100.

For the *illumination* I used first an arc lamp of 8 amp., later a so-called reductor lamp. This lamp burns 14 volt and has a very small incandescent body, hence a very great brightness per unit of area. I worked with a lamp of 100 candles.

The *electric circuit* was arranged in such a way that of *one and the same particle* I could successively measure the movement under influence of a constant force and of an alternating force. Fig. 2 schematically represents the course of the electric current. By reversing the double-pole double-throw switch $O_1$, I could successively insert the condenser $C$ into the continuous-current circuit and into the alternate-current circuit. In the fig. the two condenser plates are in connection with the points $M$ and $N$ of the adjustable resistances $W_{1-3}$, which short-circuit the battery $G$. $B_2$ is an alternate-current voltmeter. By opening $S_1$ and throwing over $O_1$, $C$ was brought into connection with the poles of the secondary winding of the transformator $T$, which converted the 110 volt of the municipal electric current supply to $\pm$ 2000 volt. $B_1$ is a voltmeter of BRAUN, $L$ is a lamp resistance. $V_1$ and $V_2$ are lequid resistances, $S_1$ and $S_2$ breakers of the current.

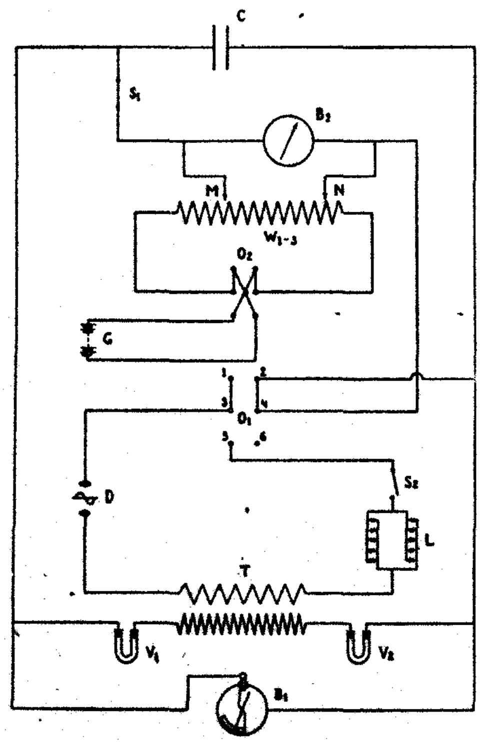


Fig. 2.

For the *registration of the movement* I had at my disposal a MORSE registering instrument, the paper ribbon of which was moved by a three-phase motor of ¼ H.P. A MORSE-key served as signal instrument. This arrangement appeared very accurate when tested by means of a chronometer. [1])

The mercury particles were obtained by EHRENHAFT's method. For the other substances I made use of an oil spray which is sold for medical purposes.

4. After the condenser had been carefully adjusted horizontal, I proceeded to the measurement of $t_v$, $t_s$, $t'_v$, and $t_\Delta$ of a definite particle. This was brought above in the field of vision, and at the moment that it, falling, crossed one of the 2 horizontal lines, which served as marks, the MORSE-key was pressed down. Then the field was excited, and the time of rising $t_s$ was measured in the same way. This happened several times in succession, the time of falling $t'_v$ also being noted down, when the electric field was reversed. Then gravity was cancelled by an electric force, and the indicator was pressed down when the particle in a horizontal direction passed a following vertical dividing line on the left or on the right of the preceding one. The sense of the displacement was indicated by different signs.

When for a particle the observation was over, the distances between the dots on the paper were measured. These distances, expressed in cm., which are proportional to the times of falling and rising, are indicated by $\tau$. The factor of reduction of $\tau$ to $t$ was determined repeatedly with an accurate chronometer.

Equation (6) only holds for spherical particles. I used the following criterion to test the spherical shape. When the particle has different dimensions in different directions, it will be orientated under influence of an electric force, and experience another resistance than in falling. Now follows from equations (4*a*, *b*, *c*):

$$\frac{2v_v}{v_v'-v_s} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If, however, the factor of resistance in the falling is $\frac{1}{B}$, in the movement under an electric force $\frac{p}{B}$, equation (13) holds no longer, but instead:

[1]) This had already appeared before in experiments by Prof. ZEEMAN in an optical determination of the current velocity in a cylindrical tube. These Proc. **18**, 1916, p. 1240.

$$\frac{2v_v}{v_v' - v_s} = p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13a)$$

Now in measurements with ammonium chloride I actually found values of $p$ departing as much as $50^0/_0$ from unity. In this $p$ was always smaller than 1, which points to a position of the particle with its length in the direction of the electric force.

For the particles with which I performed my experiments, equation (13) was always sufficiently fulfilled.

They were: *a.* electrically sprayed mercury (which will be discussed later),

*b.* ice oil, density 0.87,

*c.* potassium mercury iodide, density 2.56.

I used the last substance in order to get a length of radius lying between that of the mercury and oil particles. The time of falling, namely, depends besides on the radius, also on the density (equation 6). In the observations $t_v$ must not be too small, the measurements becoming too inaccurate in this case, and not too large, because then the deviations owing to the Brownian movement have too much influence and the number of times of falling required to determine $v_v$, then becomes very great. For this reason I could observe particles with smaller radius of a heavier substance than of a lighter one.

For the measurement of $\overline{\Delta^2}$ I used only series for which at least 100 times of displacement were observed. SCHRÖDINGER [1]) calculates the relative accuracy of the results, obtained in such a way, for an analogous problem at $\sqrt{\frac{2}{n}}$, when $n$ represents the number of elements of the series. In our case the accuracy will not differ much from this.

The distribution of the times of displacement that is to be expected follows from equation (11). The chance that $t$ lies between $t_1$ and $t_2$ is :

$$\int_{t_1}^{t_2} P(t)\,dt = -\frac{4}{\sqrt{\pi}} \left\{ \int_{(z_1)_1}^{(z_1)_2} e^{-z_1^2}\,dz_1 - \int_{(z_3)_1}^{(z_3)_2} e^{-z_3^2}\,dz_3 + \ldots \right\} \qquad (14)$$

in which :

$$(z_1)_1^2 = f\frac{t_\Delta^*}{t_1} \quad etc.$$

The distribution of the $\tau$'s is the same.

1) E. SCHRÖDINGER, l.c.

In fig. $3a$ and fig. $3b$ I show for two particles in how far the observed and the calculated distribution agree with each other.

Fig. $3a$ refers to the mercury particle N°. 123, fig. $3b$ to the oil particle N°. 158.

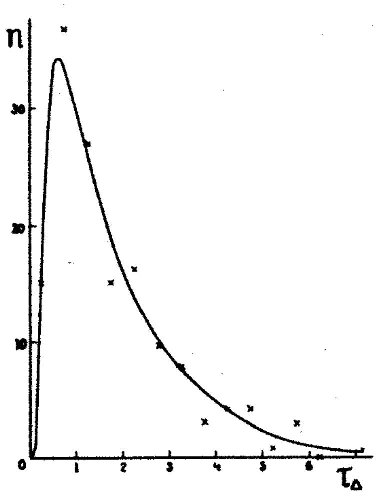

Fig. $3a$.

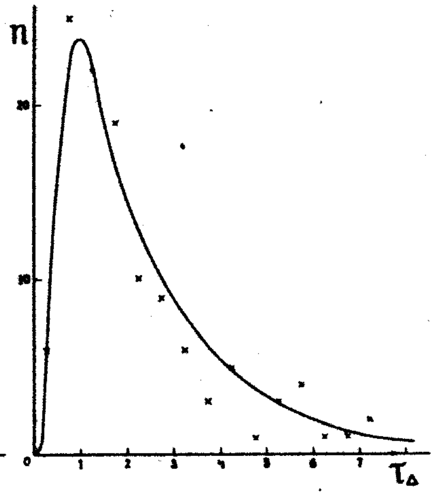

Fig. $3b$.

I have obtained the curved lines by calculating the to be expected number of times of displacement between $\tau = 0$ and $\tau = 0.5$ by means of equation (14), starting from the measured value of $\tau^*_\Delta$, and by drawing this number as ordinate of the point $\tau = 0.25$. In the same way the ordinate of $\tau = 0.75$ gives the number of times of displacement between $\tau = 0.5$ and 1.0 etc.

The crosses give the corresponding values found from observation.

5. I will now proceed to the discussion of the results. I used for this 13 series obtained with oil, 13 with potassium mercury iodide, and 14 with mercury. For some series the time of fall, hence also the radius, proved the same, e.g. for N°. 152 and 153. Such series I have combined. Everything was recalculated to 17° C.

For most experiments $l = 1.87 \; 10^{-3}$, $L = 2.24 \; 10^{-2}$.

Table I (p. 1015) gives the results obtained with oil and mercury iodide, arranged in descending values of $a$.

I will first try to determine whether $\frac{1}{\overline{\Delta^2}}$, hence also $t^*_\Delta$, is proportional to $a^2$, to $ak$ or to $a$.

The circles in fig. 4 (p. 1016) represent observations with oil, the crosses observations with potassium mercury iodide. For the present we shall leave the series with mercury out of consideration.

TABLE I.

| Oil. Number | sign of the charge | number $\tau_\Delta$ | $a \times 10^5$ | $t^2_\Delta$ | Potassium mercury iodide. Number | sign of the charge | number $\tau_\Delta$ | $a \times 10^5$ | $t^2_\Delta$ |
|---|---|---|---|---|---|---|---|---|---|
| 154 | + | 114 | 5.85 | 4.50 | 204 | + | 104 | 5.99 | 4.06 |
| 172 | − | 133 | 5.79 | 4.25 | 200 | − | 167 | 4.13 | 3.42 |
| 170 | − | 199 | 4.88 | 3.61 | 189—190 | + | 207 | 3.91 | 2.73 |
| 163 | | 113 | 4.12 | 2.80 | 198 | − | 127 | 3.41 | 2.79 |
| 156 | − | 106 | 4.03 | 2.32 | 194—205 | − | 272 | 3.31 | 2.12 |
| 157—173 | −+ | 222 | 3.86 | 2.87 | 187 | − | 118 | 2.99 | 1.94 |
| 161 | − | 111 | 3.35 | 2.17 | 192 | + | 242 | 2.89 | 2.07 |
| 152—153 | ++ | 202 | 3.28 | 2.51 | 184 | − | 146 | 2.78 | 1.91 |
| 169 | + | 139 | 3.19 | 2.37 | 191 | − | 185 | 2.56 | 1.77 |
| 171 | − | 202 | 2.97 | 2.35 | 203 | − | 109 | 2.42 | 1.77 |
| 158 | + | 121 | 2.87 | 1.88 | 206 | + | 111 | 2.29 | 1.70 |

We inquire which of the three lines determined by the equations:

$$t^2_\Delta = Ca^2 \quad \text{(15}a\text{)}$$

$$t^2_\Delta = C'ak. \quad \text{(15}b\text{)}$$

$$t^2_\Delta = C''a. \quad \text{(15}c\text{)}$$

agrees best with the observations.

To settle this point it is required that we choose a point through which we can lay those curves.

A curve drawn at sight across the points will run very close along point $P$ with coordinates 4.88 and 3.61. We shall choose this as starting point and lay through it the curves 1, 2, and 3 agreeing with equations (15 $a$, $b$, and $c$).

It appears that 1 represents the observations very imperfectly,

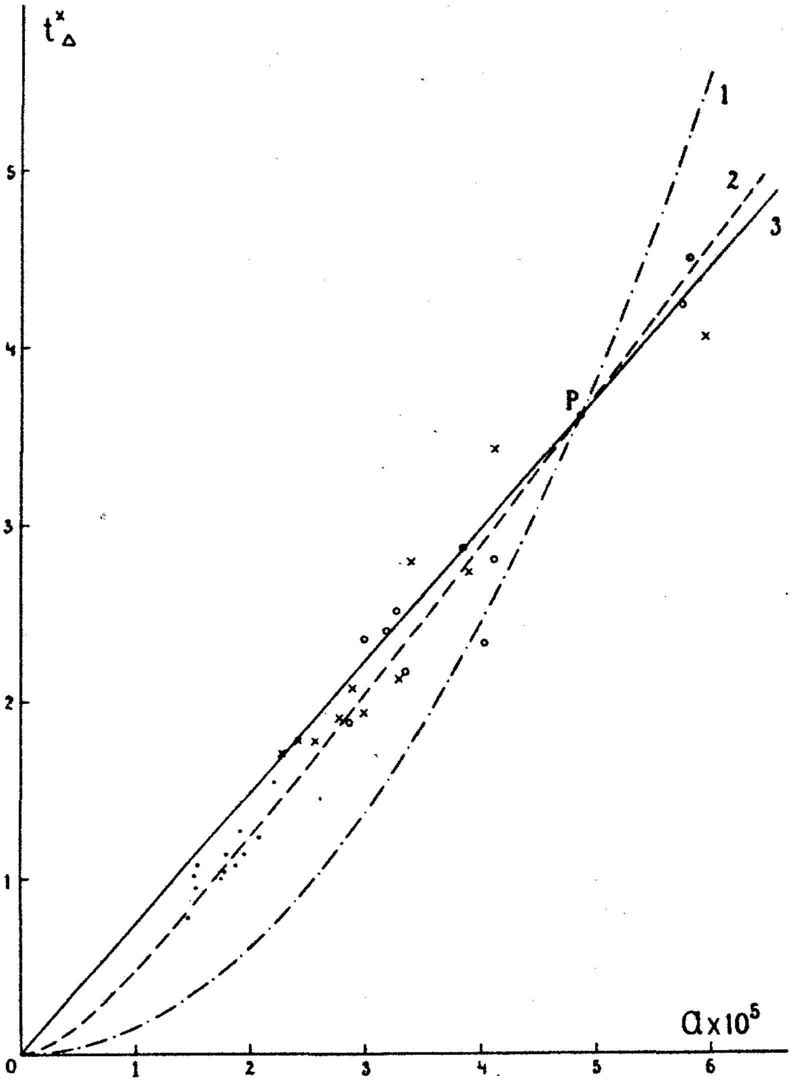


Fig. 4.

and it is no better when we choose another point instead of $P$. The curves 2 and 3 however are in good agreement with the situation of the points, taking into consideration that the deviations are naturally considerable, quite, apart from errors.

It cannot be decided with certainty whether 2 or 3 should be preferred. It seems to me that 2 is slightly more satisfactory. At any rate the supposition that $\overline{\Delta^2}$ should be proportional to $\frac{1}{a^2}$ is contradicted by the experiment, whereas EINSTEIN's formula, at least as far as the connection between $\overline{\Delta^2}$ and $a$ is concerned, is confirmed.

6. In this we have not yet made use of the electrically sprayed mercury particles. These have given rise by their behaviour to the question of the subelectrons. EHRENHAFT [1]) thought he had to conclude from his experiments with this substance: electricity is not divided into quanta, or if it is, the quantum is much smaller than the electron assumed up to now. Among the opponents of this thesis especially TARGONSKI [2]) has tried to give an explanation of the phenomenon by assuming that the particles possess a much slighter density than that of mercury. This would result in a charge that was calculated much too small. TARGONSKI determines the spec. grav. of the grey layer which covers the wall of the vessel and the surface of the mercury after repeated spraying, and finds for it 7.3. He derives from this that the mean density of the sprayed particles is much smaller still. He does not determine, however, the density of the drops themselves. I think I have found a means in my experiments to determine it directly, though it be not with very great accuracy. The particles were sprayed in my experiments in ordinary air, in those by EHRENHAFT and TARGONSKI in dry nitrogen.

With the aid of the known value of $t_\Delta^*$ the corresponding value of $a$ can be read from fig. 4 when we assume that the curve 2 is

TABLE II.

| Number | sign of the charge | $t_V$ | $t_\Delta^*$ | $a$ from fig. | $\rho$ calc. |
|---|---|---|---|---|---|
| 146 | — | 3.22 | 1.55 | 2.35 | 9.7 |
| 94 | + | 3.49 | 1.22 | 1.95 | 12 |
| 116—120 | — | 3.99 | 1.13 | 1.85 | 12 |
| 102—137 | +— | 4.08 | 1.17 | 1.90 | 11 |
| 141 | — | 4.28 | 1.07 | 1.80 | 12 |
| 150 | — | 4.52 | 1.12 | 1.85 | 10 |
| 110 | — | 4.67 | 1.04 | 1.75 | 11 |
| 138 | + | 4.77 | 0.97 | 1.65 | 12 |
| 117 | + | 5.80 | 1.07 | 1.80 | 8.6 |
| 123 | + | 6.01 | 0.93 | 1.60 | 10 |
| 124 | — | 6.02 | 1.01 | 1.70 | 9.1 |
| 149 | — | 6.46 | 0.77 | 1.40 | 12 |

1) F. EHRENHAFT l. c.

2) A. TARGONSKI, Arch. de Genève, 4, 41, 1916, p. 207.

the correct one. Then the density follows from equation (6). I have carried out the calculations, and collected the results in table II. The density appears to be about 11 on an average, much higher, therefore, than according to TARGONSKI. Probably a layer of a lighter substance forms at the surface, but the chief component remains mercury.

In fig. 4 the points indicate the places that the observations occupy in the whole of the experiments, when I start from the supposition that the density is 11. In fig. 5 I have enlarged the first part of

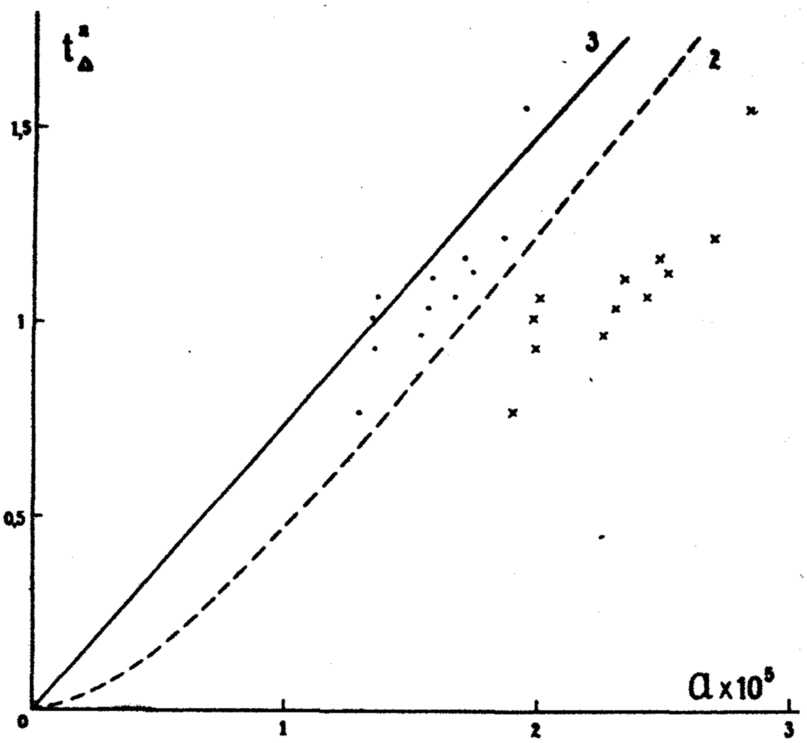


Fig. 5.

the preceding figure in order to show where the points would lie when we had to do with pure mercury. This has been indicated by points. The crosses give the place for $\rho = 7$, which is still too high according to TARGONSKI.

7. To examine whether the formula of EINSTEIN-CUNNINGHAM holds also numerically, we can calculate the value of $N$ from curve 2 of fig. 4 with the formula:

$$\overline{\Delta^2} = \frac{1}{3\pi\zeta a k}\,\frac{RT}{N}.$$

I find for it $N = 6.38\ 10^{23}$.

Among the different calculations of $N$ that of SOMMERFELD, from the theory of quanta of the spectral lines, may be considered as the most accurate one. SOMMERFELD finds $N = 6.08\ 10^{23}$, a value that differs 5 % from that calculated by me.

8. Now also e can be calculated from equation (5). The measurement of this is, however, not so very accurate, the reading of the voltmeter being uncertain to some percentages. In table III I record the results for a number of particles. Let us assign to every particle, the number of electrons given in the third column, then this, with the total charge of the second column, for the electron gives

TABLE III.

| Oil | | | | | |
|---|---|---|---|---|---|
| Number | charge $\times 10^{10}$ | number of electrons | $e \times 10^{10}$ | $m_{calc.}$ | $m_{meas.}$ |
| 156 | 5.4 | 1 | 5.4 | 0.51 | 1.0 |
| 157 | 9.7 | 2 | 4.85 | | |
| 158 | { 4.1<br>{ 11.7 | { 1<br>{ 2 | { 4.1<br>{ 5.85 | { 0.58<br>{ 1.6 | { 1.0<br>{ 2.0 |
| 161 | 4.7 | 1 | 4.7 | 0.53 | 0.8 |
| 170 | 14.7 | 3 | 4.9 | | |
| 171 | 4.1 | 1 | 4.1 | 0.61 | 1.1 |
| 172 | 19.6 | 4 | 4.9 | 1.3 | 1.9 |
| 173 | 9.6 | 2 | 4.8 | 1.1 | 1.3 |

| potassium mercury iodide | | | | | |
|---|---|---|---|---|---|
| Number | charge $\times 10^{10}$ | number of electrons | $e \times 10^{10}$ | $m_{calc.}$ | $m_{meas.}$ |
| 177 | 9.1 | 2 | 4.55 | 1.3 | 1.7 |
| 178 | 17.4 | 4 | 4.35 | 1.8 | 2.2 |
| 183 | 27.8 | 6 | 4.6 | 3.0 | 4 |
| 184 | 47.2 | 10 | 4.7 | | |
| 187 | 11.0 | 2 | 5.5 | | |
| 190 | 39.6 | 9 | 4.4 | | |
| 191 | 11.6 | 2 | 5.8 | 1.7 | 2 |
| 192 | 21.7 | 5 | 4.3 | | |
| 194 | 10.1 | 2 | 5.05 | | |

| electrically sprayed mercury $v = 11$ | | | |
|---|---|---|---|
| Number | charge $\times 10^{10}$ | number of electrons | $e \times 10^{10}$ |
| 116 | 19.6 | 4 | 4.9 |
| 117 | 15.7 | 3 | 5.2 |
| 124 | 15.7 | 3 | 5.2 |
| 137 | 22.5 | 5 | 4.5 |
| 138 | 21.0 | 4 | 5.25 |
| 146 | 14.4 | 3 | 4.8 |
| 149 | 16.2 | 4 | 4.05 |

the values of the fourth column. These hood lie around MILLIKAN's value 4.77 $10^{-10}$.

I shall discuss the meaning of $m$ in the following §.

9. I have stated in § 3 that I also observed the particle in an alternate field. It then executed a vibrating movement, and made the impression of a luminous line, clearest at the extremity where the velocity was smallest, so that as a whole it resembled a dumb-bell. Accordingly I shall speak of the dumb-bell movement. I have tried to measure the length of this dumb-bell by comparing the falling luminous line with the distance of the dividing lines. This was very difficult, particularly because I had only a few seconds time. Then the constant field had again to be excited by quick throwing over of a number of switches, so that I could make the particle rise again before it disappeared out of the field of vision. Hence the measurements are only estimations with a considerable mean error. I wanted to try and get an answer to the following question: does STOKES-CUNNINGHAM's formula sufficiently express the resistance also for this rapid movement?

Then the movement must satisfy the equation:

$$M\ddot{x} = e\mathfrak{E}_0 \sin 2\pi \frac{t}{T} - 6\pi\zeta a k \dot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$\mathfrak{E}_0$ is the maximum intensity of field, T the period of the alternate current[1]). It is easy to calculate that we find for the length of the dumb-bell from this equation:

$$2A = 2\frac{e\mathfrak{E}_0}{M}\left(\frac{T}{2\pi}\right)^2 \frac{1}{\sqrt{1+\frac{K^2T^2}{4\pi^2}}}$$

in which $K = \frac{6\pi\zeta ak}{M}$.

Now $K\frac{T}{2\pi}$ is large with respect to 1, so that we may write:

$$2A = e\mathfrak{E}_0 \frac{T}{6\pi^2\zeta ak} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

In table III $m_{calc.}$ gives the value of $2A$ calculated from equation (17) expressed in multiples of the distance of 2 lines; $m_{meas.}$ gives the measured lengths. It appears that $m_{calc.}$ is always smaller than $m_{meas}$. This suggests that the resistance for the vibrating movement

[1]) Only in approximation is the intensity of the alternate field represented by a sinus function.

would be smaller than for the movement under constant force. The observations are too rough for quantitative calculations, but the differences of $m_{calc.}$ and $m_{meas.}$ are too great and too much in one direction to be attributed to errors of observation.

*Amsterdam.* *Physical Laboratory.*

**Chemistry**. — *"Current Potentials of Electrolyte solutions."* (Second Communication). By Prof. H. R. KRUYT. (Communicated by Prof. ERNST COHEN).

(Communicated in the meeting of January 27, 1917.)

1. In a former paper [1]) I communicated a series of measurements with respect to the influence of dissolved salts on the current potential, after having made investigations with solutions of the chlorides of potassium, barium and aluminium. These salts were chosen, because they are electrolytes with resp. a monovalent, a bivalent and a trivalent cation. In Tables 2 and 3 similar results are given for investigations made with hydrochloric acid and the chloride of *p*-chloro-anilene. A standard solution of HCl was prepared by conducting gaseous hydrochloric acid in "conductivity water"; to get the solution of $p\, ClC_6H_4NH_2 . HCl$ KAHLBAUM's $p\, ClC_6H_4NH_2$ was dissolved in water containing the equivalent quantity of HCl from the solution first mentioned.

The results given in Tables 2 and 3 show the decrease of the current potential to be here much larger than in the case of potassiumchloride (cf. Table 1, columns 1 and 2). This result is in perfect agreement with the investigations on electric endosmosis (for literature, see my first communication), and it can be easily understood when we suppose, as FREUNDLICH does, that these phenomena are in close relation with the adsorption of the ions: the H-ion, and also the organic ions (especially aromatic ones) are adsorbed in a greater amount than those of the light metals. A comparison of Tables 2 and 4 shows, that the monovalent H-ion and the bivalent Ba-ion bring about nearly the same lowering of the current potential.

2. A comparison between the electric charges of the capillary tube is still of more importance than that of the current potentials especially with regard to the problems of colloid-chemistry [2]).

[1]) These Proceedings 17, 615 (1914).
[2]) See H. R. KRUYT, These Proceedings 17, 623 (1914).