5. The above application of the theory of similarity also shows, that in making experiments with various liquids, as described in section 4 of Comm. N°. 151d, by changing K and T_0 in all possible ways a double set of curves would be obtained forming a net-work which would be identically the same for all liquids[1]. By the equations (2) the elements belonging to the curves in one net could be calculated from those belonging to the corresponding curves in the other; conversely having obtained the nets for two liquids the values of μ and η might be composed and in this manner these quantities (more especially η) might be determined for one of the liquids.

Possibly this method may find its practical application some time.

Physics. — "*The viscosity of liquefied gases. IX. Preliminary determination of the viscosity of liquid hydrogen.*" By J. E. VERSCHAFFELT and CH. NICAISE. (Communication N°. 151g from the Physical Laboratory at Leiden). (Communicated by Prof. H. KAMERLINGH ONNES).

(Communicated in the meeting of February 24, 1917).

1. The measurements were made with the same apparatus as was used for the determination of the viscosity of liquid air (see Comm. N°. 149b, IV), into which, however, some improvements were introduced. In the first place, some nickel plated paper screens were placed under the cap to intercept the radiation of the cap; in the second place a small tinned hand-pump was introduced[2] into the liquid, the rod of which passed through a small metal tube in the cap and could be worked up and down by means of an india-rubber tube which made an air-tight connection with the metal tube, by which means the liquid could be stirred previously to each experiment, and any slight differences of temperature or density could be equalized. The thermometer and the syphon tubes were taken away, as being unnecessary in these experiments; the temperature was deduced from the atmospheric pressure (the liquid, which in this case possessed

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[1] As an instance. with the same restriction as in the experiments of section 3, curve II would also be found for water with $K = 807$ and $T = 15.21$.

[2] Of the pattern generally used in the cryostats of the laboratory (see for instance Comm. N°. 123).

a sufficiently high degree of purity, was boiling [1]) under a pressure which was kept constant to a few mms of mercury by the laboraratory pumps, and was only a little higher than the atmospheric pressure [2])) ; the emptying of the apparatus could be very quickly accomplished by the evaporation of the liquid, after removing the external vacuum glass, which, as in the experiments with liquid air, contained liquid air. For the internal vacuum-vessel, which contained the liquid hydrogen, a completely silvered glass was taken, in which only two opposite windows were left open, so as to enable us to observe the height of the liquid when filling the glass ; as the external vessel was silvered with only a transparent strip left open, it was only necessary to turn this outside glass a little way in order to protect the liquid hydrogen practically completely from external radiation.

2. Technical difficulties in connection with the use of liquid hydrogen and the much lower temperature ($\pm$ 20° K.) did not arise; the only thing was, that, as was to be expected, the damping of the oscillations was very small (about 6 times less than in liquid air) in consequence of which the internal friction of the suspension wire acquired a very high and unpleasant degree of importance. This friction, in fact, proved to be not only comparatively large, but to depend to a high degree upon accidental circumstances, difficult to estimate and control; consequently, although for each experiment separately an accurate logarithmic decrement could be deduced, only a very moderate agreement could be found between the various experiments [3]). In the first place it was found that unstretching and re-stretching of the wire (by exchanging the cylinders C) altered the viscosity of the wire very greatly (usually increasing it); in the second place the viscosity was a function of the time, which only decreased slowly, in an approximately exponential manner, and required some days to become constant; in the third place the gas in which the wire was placed proved to have a great influence upon its viscosity; pumping a vacuum, filling with air, replacing

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[1]) The boiling. which was entirely superficial, without the formation of bubbles, took place very slowly, thanks to the screens.

[2]) The small difference of pressure between the vapour pressure in the apparatus and the air pressure outside, was read on the small open manometer Ma (see figure in Comm. N⁰. 149b).

[3]) This is clearly also the cause of the differences which were observed in the experiments with liquid air (see Comm. N⁰. 149b V). This instability in the internal friction in the wire has given similar difficulties to previous investigators: see for instance ZEMPLÉN, Ann. d. Phys. 19, 802, 1906.

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air by hydrogen (this especially) or the reverse, caused a great increase in the friction [1]), which, as we have said, gradually become less again [2]).

Under these unfavourable circumstances, in order to be able to arrive at provisional results with the apparatus as it was constructed, we were obliged to change our method of working to some extent and to demand a much smaller degree of accuracy from the results. The determination of the times of oscillation by registration, in particular (see Comm. N^0. 149b, IV, § 4), was a complication disproportional to the accuracy, and could be quite adequately replaced by a purely chronometric determination, by means of a stop-watch, which showed $^1/_5{}^{th}$ of a second [3]). Further, the sensibility of the wire to changes of condition made it necessary that the apparatus should remain unchanged during a whole series of experiments, that is, that the cylinders should not be exchanged; as this did away with the use of the cylinders altogether (see Comm. N^0. 149b, IV, § 5) they could just as well be left out [4]). We, therefore, continued the work with a constant oscillating system; in consequence of which the friction of the gas upon that part of the apparatus not immersed in liquid had to be eliminated in a different way. We did this in the following manner: besides the experiments in which the sphere oscillated in the liquid, we also made experiments with the sphere just above the liquid oscillating in vapour at a low temperature; from the knowledge of the density and viscosity of this vapour, by means of the formulae (24') and (28) in Comm. N^*. 148b, the retarding couple could be determined which the sphere experienced by the friction in the vapour; this couple could be subtracted from the total moment in the experiment in the vapour; the difference we considered might be taken as giving the couple which the oscillating system experiences by friction in the experiments in the liquid.

3. After several unsatisfactory attempts, we succeeded in carrying out in one day (July 12th 1916) a series of reliable, and

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[1]) Presumably a consequence of occlusion of gases by the metal wire. We have not used quartz wires yet, which probably would not possess this unpleasant peculiarity.

[2]) To avoid further trouble from these changes we left the apparatus always filled with hydrogen.

[3]) By determining the duration of ten oscillations T could still be determined to within about ·01 sec.

[4]) The moment of inertia of the oscillating system was thus, at ordinary temperature, $K = 372·5 + 27·8 = 400·3$. Hereby the period of oscillation becomes smaller than before (17·22 sec.) it is true, but that was not a decisive objection.

as we think, mutually comparable observations. The logarithmic decrements δ as observed are given in the following table, which also contains the mean times at which the observations took place (they lasted about 4 minutes; each time 20 full oscillations were observed [1]), as well as the physical state of the substance which surrounded the sphere.

1.	11ᵘ25ᵐ a.m.	vapour	$\delta = 0.00393$
2.	11 40 ,,	,,	379
3.	11 50 ,,	,,	370
4.	12 15 ,,	liquid	682
5.	12 25 ,,	,,	672
6.	12 40 ,,	,,	663
7.	2 15 p.m.	vapour	340
8.	2 35 ,,	,,	317
9.	2 50 ,,	,,	317

We shall now give a short description of the course of the observations. After the apparatus had been filled with hydrogen gas for a few days, we began to cool it in the morning of July 12ᵗʰ at about 10 a.m. and about 11 a.m. liquid was poured in, only so far that the sphere did not yet touch the liquid, when three observations were taken in the vapour; it appears that the decrement decreased rapidly, which seems to indicate a disturbance in the wire caused by the filling of the apparatus. About 12 o'cl. more liquid was poured in until the sphere was entirely immersed; again three observations were made, which gave a much larger decrement, and this in the same way decreased in the course of time, and to about the same degree as in the vapour. About 1 p.m. the external vacuum glass was removed, so that the liquid hydrogen could boil away pretty quickly; at about 2 p.m. the liquid had so far boiled away that the sphere projected completely out of the liquid; then the vacuum glass with liquid air was again put round the internal one, and three more observations were made in the vapour.

From our observations it follows, that the transference of the sphere from the vapour into the liquid involves an increase of the logarithmic decrement by a mean value of 0.00319, while for the

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[1] As we did not expect a greater accuracy than 1 %/0 about, we considered it useless to raise the degree of accuracy for the separate results by lengthening the series.

friction of the sphere in vapour 20° K. with $\mu = 0.0012$[1]) and $\eta = 0.000011$[2]) a decrement $\sigma = 0.00015$ was calculated. We conclude from this that the decrement caused by the friction of the liquid alone was $\sigma_1 = 0.00334$. The liquid was under a mean pressure of 766 mms. mercury; the temperature was thus $20°.36$K.[3]) and the density 0.0708[4]). From this it follows that $\eta = 0.000117$.

This determination was made before the investigation of the suitability of the method was completed (see Comm. N°. 151d). From that investigation it appears probable, that the value found for η was a few percent too high, but the data are wanting by which the necessary correction might be estimated. We therefore give as the approximate value of the viscosity of liquid hydrogen

$$\eta = 0.00011. \text{[5])}$$

Physics. — "*Critical point, critical phenomena and a few conden-sation-constants of air*". By J. P. KUENEN and A. L. CLARK. (Communication N°. 150b from the Physical Laboratory of Leiden). ·

(Communicated at the meeting of February 24, 1917).

The critical temperature and pressure of air have been determined by OLSZEWSKI[6]), WROBLEWSKI[7]), and WITKOWSKI[8]). Their results do not agree amongst each other as well as might be desired:

	OLSZEWSKI	WROBLEWSKI	WITKOWSKI
t_k	— 140°	about — 140.°5	— 141°
p_k	39 atm.	„ 37 à 37.5	39

The main object of our investigation was to obtain reliable values for the critical constants, including the critical density, which involves a detailed study of the condensation-phenomena in the critical region. WROBLEWSKI noticed that air behaves differently near its critical point

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[1]) The vapour still behaves approximately as an ideal gas.

[2]) According to H. KAMERLINGH ONNES, C. DORSMAN and SOPHUS WEBER. Comm. N°. 134a.

[3]) Normal boiling point: $20°,33$ K., $\dfrac{dp}{dt} = 200 \dfrac{\text{m.M}}{\text{degree}}$ (see Comm. N°. 137d).

[4]) See Comm. N°. 137a.

[5]) This value agrees satisfactorily with that calculated in Comm. N°. 148b on the basis of the law of corresponding states.

[6]) K. OLSZEWSKI. C. r. 99, p. 184, 1884.

[7]) S. v. WROBLEWSKI. Wied. 26, p. 134, 1885.

[8]) A. W. WITKOWSKI. Phil. Mag. (5) 41, p. 288, 1896.