3. During evaporation in vacuum silver develops a deposit against the bulb. With increasing thickness the colour of this deposit passes through greenish-yellow, orange, red, violet and blue.

4. The red, the violet and the blue films are distinctly heterogeneous. They consist of a network of very small ultramicrons. The yellow deposit shows a hardly perceptible heterogeneousness and approaches in structure the amorphous-vitreous state.

5. The deposits are not proof against the influence of moist air. The colour changes in the direction yellow → red → blue and the structure becomes coarser. Heating likewise causes a coarsening of structure.

6. Gold forms — in a similar way as silver — coloured deposits, which are ultramicroscopically heterogeneous.

7. Tungsten forms a black deposit, ultramicroscopically it is not soluble.

8. Deposits obtained by cathode-atomizing consist as a rule of coarser particles than the evaporation-deposits.

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Physics. — *"The virtual displacements of the electro-magnetic and of the gravitational field in applications of* HAMILTON'S *variation principle"*, By Dr. A. D. FOKKER. (Communicated by Prof. LORENTZ).

(Communicated in the meeting of January 27, 1917.)

In some papers on EINSTEIN'S theory of gravitation Prof. LORENTZ[1]) recently applied HAMILTON'S principle to the deduction of the principal equations of this theory from one single variation law. Starting from an invariant equation he was able to reach conclusions which again were represented by invariant equations. It was however not necessary to keep the equations invariant during the whole deduction. On the contrary, an artifice, consisting in the choice of a specially defined virtual displacement (without taking into consideration the conditions of invariancy), proved very useful.

Now it is possible to let the invariancy exist continually during

1) H. A. LORENTZ, *On* HAMILTON'S *principle in* EINSTEINS *theory of gravitation, Proceedings,* Kon. Ak. v. Wet. Amsterdam, XIX, p. 751. *Over* EINSTEINS *theorie der zwaartekracht,* I, II, III, Verslagen, Kon. Ak. v. Wet. XXIV, p. 1389, 1759, XXV, p. 468.

the deductions; and that in a way which fully appreciates the fact that the tensor of the ten gravitation potentials and the tensor of the four electrodynamical potentials, being directed quantities, have a geometrical character (§ 12 etc.). Moreover the tensors of stress, momentum and energy appear in a new way from the variation calculation.

In the following paragraphs this will be shown. Thanks to the cited papers and to some others, a short indication will often suffice.

The variation principle.

1. For a material particle, falling under the influence of a force, HAMILTON's principle takes the form:

$$0 = \delta \int_1^2 - m\, ds + \int_1^2 \Sigma(p)\, k_p\, \delta r^p\, ds,$$

where m is the coefficient of mass of the particle, ds the arc-length of the world-line run by the particle in the world referred to a system of four space-time parameters x_1, x_2, x_3, x_4. Further $k_p (p = 1, 2, 3, 4)$ represents the four-vector of the force acting on the particle, while δr^p $(p = 1, 2, 3, 4)$ denote the components of the virtual displacements. In the variation of the motion there corresponds to each point-instant x_m $(m = 1, 2, 3, 4)$ of the unvaried path a point-instant $x_m + \delta r^m$ $(m = 1, 2, 3, 4)$ of the varied path. The final points of the path remain unvaried. As usually we assume

$$ds^2 = \Sigma(ab)\, g_{ab}\, dx_a\, dx_b,$$

where g_{ab} $(a, b = 1, 2, 3, 4, g_{ab} = g_{ba})$ are the gravitation potentials.

If the particle has an electric charge, so that it is influenced by an electro-magnetic field this may be taken into consideration by writing

$$0 = \delta \int_1^2 (- m ds + \lambda \Sigma(l)\, e \varphi_l\, dx_l) + \int_1^2 \Sigma(p)\, k_p\, \delta r^p ds.$$

Here φ_l $(l = 1, 2, 3, 4)$ represent the electro-dynamic potentials, four quantities changing from point to point and determining the field. λ is a constant determined by the choice of the units of mass and charge in which m and e are expressed. Now k_p no longer contains the electric forces.

2. Applied to a limited extension of the four-dimensional world HAMILTON's principle is represented by the equation:

$$0 = \delta \int L \, dx_1 \, dx_2 \, dx_3 \, dx_4 + \int \Sigma(p) \, K_p \, \delta x^p \, dx_1 \, dx_2 \, dx_3 \, dx_4 \quad . \quad . \quad (1)$$

Here K_p denotes the p^{th} component of the force acting on the system per unit of volume. $\sqrt{-g} \, dx_1 \, dx_2 \, dx_3 \, dx_4$ being a scalar (if g is the determinant of the g_{ab}), $K/\sqrt{-g}$ and not K must be a covariant vector, which further will be denoted by k. For the same reason not L, but $L/\sqrt{-g}$ must be a scalar, if the variation law shall be expressed invariantly. We suppose the function of LAGRANGE L to consist of different separate parts for the gravitational field, for the matter, for the electro-magnetic field and for the electric convection-current.

Structure of the function of LAGRANGE.

3. The contribution of the *gravitational field* to L will be denoted by $\sqrt{-g} \, H$. It will be known, that for H must be taken $G/2\varkappa$, where G is a scalar indicating the curvature of the field figure and $\varkappa$ the gravitation constant. By means of RIEMANN's symbol, G may be expressed as follows:

$$G = \Sigma(im) \, g^{im} \, G_{im},$$
$$G_{im} = \Sigma(kl) \, g^{kl} \, (ik, lm),$$
$$(ik, lm) = \tfrac{1}{2} \left(g_{im,kl} + g_{kl,im} - g_{il,km} - g_{km,il} \right) +$$
$$+ \Sigma(ab) \, g^{ab} \left\{ \begin{bmatrix} im \\ a \end{bmatrix} \cdot \begin{bmatrix} kl \\ b \end{bmatrix} - \begin{bmatrix} il \\ a \end{bmatrix} \cdot \begin{bmatrix} km \\ b \end{bmatrix} \right\}.$$

The quantities g^{ab} $(a, b = 1, 2, 3, 4)$ are the algebraic complements of the g_{ab}; $g_{im,kl}$ is written for the second derivative of g_{im} with respect to x_k and x_l; and CHRISTOFFEL's symbols mean:

$$\begin{bmatrix} im \\ a \end{bmatrix} = \tfrac{1}{2} \left(g_{ia,m} + g_{ma,i} - g_{im,a} \right).$$

Further the notation g^{ab}_{c} and g^{ab}_{cd} for the first, respectively second derivative of g^{ab} with respect to x_c and x_d will be used from time to time.

4. The contribution of the *matter* to L will be denoted by $\sqrt{-g} \, R$. In order to find out what has to be put for $\sqrt{-g} \, R$ we must investigate how the element $-m \, ds$, which occurs in the variation law for the motion of a single material particle, can be extended to $\sqrt{-g} \, R \, dx_1 dx_2 dx_3 dx_4$ for the matter we are considering. LORENTZ has indicated [1]) what $\sqrt{-g} \, R$ becomes for a con-

[1]) l. c. XIX p. 754, XXV, p. 478.

tinuously varying current of incoherent material points or for a more general case in which there are acting certain molecular forces between the points.

For an ideal gas $\sqrt{-g}\, R$ will be the sum of the elements of the world-lines described per unit of time by the molecules present in a unit of volume, each element multiplied by $-m$, if m is the mass of the molecule that describes the element.

Now it is known that for a molecule with the mass m the momentum is given by

$$i_a = -m\, \Sigma (b)\, g_{ab} \frac{dx_b}{ds}$$

for $a = 1, 2, 3$, and that the energy is $-i_4$. For an ideal gas the expressions for the stresses, the momentum and the energy per unit of volume and the energy-current can be written down directly. Without entering into details by introducing a distribution function I only give the table of notations

$$\begin{array}{cccccc} \sqrt{-g}\, T_1^1 & \sqrt{-g}\, T_1^2 & \sqrt{-g}\, T_1^3 & \sqrt{-g}\, T_1^4 & & X_x\; X_y\; X_z - I_x \\ \sqrt{-g}\, T_2^1 & \sqrt{-g}\, T_2^2 & \sqrt{-g}\, T_2^3 & \sqrt{-g}\, T_2^4 & & Y_x\; Y_y\; Y_z - I_y \\ \sqrt{-g}\, T_3^1 & \sqrt{-g}\, T_3^2 & \sqrt{-g}\, T_3^3 & \sqrt{-g}\, T_3^4 & (=) & Z_x\; Z_y\; Z_z - I_z \\ \sqrt{-g}\, T_4^1 & \sqrt{-g}\, T_4^2 & \sqrt{-g}\, T_4^3 & \sqrt{-g}\, T_4^4 & & S_x\; S_y\; S_z\; E. \end{array} \qquad (2)$$

Here the coordinates x, y, z and t[1]) are supposed to be used. T_a^m is a mixed tensor. It may be called the *dynamical* tensor. It is not symmetrical. The covariant tensor

$$T_{ab} = \Sigma\, (m)\, g_{mb}\, T_a^m ,$$

on the contrary is symmetrical.

It may be remarked that the sum of the diagonal components is equal to

$$\Sigma\, (a) \sqrt{-g}\, T_a^a = - \sqrt{-g}\, R.$$

5. The contribution to L of the *electric current* and the electromagnetic field may be divided into two parts, $\lambda \sqrt{-g}\, S$ and $\lambda \sqrt{-g}\, M$, λ being the same constant as in § 1.

For $\sqrt{-g}\, S\; dx_1 dx_2 dx_3 dx_4$ we take the extension of the element $\Sigma\, (l)\, e\, \varphi_l dx_l$ that occurred in the variation law for a single charged particle. If the extension is effected in such a way that we pass to a continuous electric convection-current, we find

[1]) X_x, Y_x, Z_z are the forces, exerted in the direction of X, Y, or Z by the surroundings of a unit cube, on a face for which the outwardly directed normal has the direction of the axis indicated by the index.

$$\sqrt{-g}\, S = \Sigma\,(m) \sqrt{-g}\; W^m \varphi_m .$$

$\sqrt{-g}\; W^m$ $(m = 1, 2, 3, 4)$ denotes what is usually indicated by ϱv_x, ϱv_y, ϱv_z and ϱ. Here, as in other places, the factor $\sqrt{-g}$ occurs because we take the different quantities per units of time and volume, expressed in the coordinates and not in natural units. It is to be noted that at a change of the g_{ab}, $\sqrt{-g}\; W^m$ remains unchanged. This corresponds to the fact that for a single charged particle the term $\Sigma\,(m)\, e\varphi_m dx_m$ is independent of the gravitation potentials.

For the *electro-magnetic field* the scalar may be constructed in the following way. From the potentials the covariant field-intensities are derived:

$$f_{pq} = \frac{\partial \varphi_q}{\partial x_p} - \frac{\partial \varphi_p}{\partial x_q}.$$

From these we form the contravariant intensities of the field:

$$F^{ab} = \Sigma\,(mn)\, g^{am}\, g^{bn} f_{mn}.$$

Finally we form the scalar:

$$M = -\tfrac{1}{4}\, \Sigma\,(abmn)\, g^{am}\, g^{bn} f_{ab} f_{mn} ,$$
$$= -\tfrac{1}{4}\, \Sigma\,(ab)\, F^{ab} f_{ab}$$

Further it may be remarked here, that

$$\frac{\partial M}{\partial f_{ab}} = -\tfrac{1}{2} F^{ab}, \text{ and } \frac{\partial M}{\partial g^{am}} = -\tfrac{1}{2}\, \Sigma\,(bn)\, g^{bn} f_{ab} f_{mn}.$$

SCHWARZSCHILD [1]) has already used the integrand $\sqrt{-g}\, S$ in the variation law. Recently TRESLING [2]) has communicated to the Academy of Sciences how this term may be used in HAMILTON's principle.

Except as to the sign, the term $\sqrt{-g} M$ corresponds to the term used by LORENTZ, who writes $\mathfrak{p}_{ab}$ for what has been called here $\sqrt{-g} F^{ab}$ and ψ_{ab} for f_{ab}.

Variations of the field quantities.

6. In the first place we shall consider the variation which is obtained by varying the *electric field* in such a way, that everywhere the potentials φ_m are changed to an amount $\delta\varphi_m$.

The $\delta\varphi_m$ $(m = 1, 2, 3, 4)$ will be infinitesimal continuous functions of the coordinates.

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[1]) K. SCHWARZSCHILD, *Zur Elektrodynamik. I.* K. Ges. Wiss. Göttingen, Math. phys. 1903.

[2]) J. TRESLING, *The equations of the theory of electrons in a gravitation field of* EINSTEIN *deduced from a variation principle. The principal function of the motion of the electrons.* These Proceedings. XIX p. 892.

The variation becomes[1]):

$$\delta \int L\, dx_1\, dx_2\, dx_3\, dx_4 = \lambda \int dx_1\, dx_2\, dx_3\, dx_4\, \Sigma\,(mq) \left[\frac{\partial}{\partial x_q} (\sqrt{-g}\, F^{mq}\, \delta\varphi_m) + \right.$$

$$\left. + \,\delta\varphi_m \left\{ \sqrt{-g}\, W^m - \frac{\partial}{\partial x_q} (\sqrt{-g}\, F^{mq}) \right\} \right].$$

If at the boundaries of the four-dimensional extension the $\delta\varphi_m$ are chosen equal to 0, while within this extension they have arbitrary values, then HAMILTON's principle demands that

$$\sqrt{-g}\, W^m = \Sigma\,(q) \frac{\partial}{\partial x_q} (\sqrt{-g}\, F^{mq}), \qquad (m = 1, 2, 3, 4) \quad . \quad (3)$$

These are the four *equations of the field* in an invariant form.

7. The second variation to be considered is a variation of the *gravitational field*. At each point-instant of the extension it may be determined by the changes δg^{ab} of the quantities g^{ab}.

If we have to do with an ideal gas, we may deduce directly that now the variation of $\sqrt{-g}\, R$ is:

$$\delta (\sqrt{-g}\, R) = \Sigma\,(abm)\, \tfrac{1}{2} \sqrt{-g}\, g_{ma}\, T^m_b\, \delta g^{ab}\ ^{2)} \quad . \quad . \quad . \quad (4a)$$

Taking into consideration that,

$$\delta \sqrt{-g} = -\, \Sigma\,(ab)\, \tfrac{1}{2} \sqrt{-g}\, g_{ab}\, \delta g^{ab},$$

$$\delta M = -\, \Sigma\,(abdn)\, \tfrac{1}{2}\, g^{dn} f_{ad} f_{bn}\, \delta g^{ab} = -\, \tfrac{1}{2}\, \Sigma (abcdmn)\, g_{am}\, g^{cm}\, g^{dn} f_{cd} f_{bn}\, \delta g^{ab},$$

$$\delta M = -\, \Sigma\,(abmn)\, \tfrac{1}{2}\, g_{am}\, F^{mn} f_{bn}\, \delta g^{ab},$$

we easily find for the variation of $\sqrt{-g}\, M$

$$\lambda\, \delta (\sqrt{-g}\, M) = \Sigma\,(abm)\, \tfrac{1}{2} \sqrt{-g}\, g_{ma}\, E^m_b\, \delta g^{ab} \quad . \quad . \quad . \quad (4b)$$

where we have put

$$E^m_b = -\,\lambda \Sigma\,(n)\, F^{mn} f_{bn} - \lambda \delta^m_b\, M.$$

δ^m_b is a mixed tensor, the components of which are 1 or 0 according as $m = b$ or $m \neq b$. We shall also introduce the notation $E_{ab} = \Sigma (m)\, g_{am}\, E^m_b$.

We shall see further on that $\sqrt{-g}\, E^m_b$ are the stresses etc. in the electro-magnetic field in the same way as $\sqrt{-g} T^m_b$ are those in the matter.

For the above mentioned reason the variation of $\sqrt{-g}\, S$ will be zero.

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1) It should be kept in mind that $\delta f_{mq} = \dfrac{\partial \delta\varphi_q}{\partial x_m} - \dfrac{\partial \delta\varphi_m}{\partial x_q}$ and that $F^{mq} = -\, F^{qm}$

Comp. for the deduction TRESLING, l.c.

2) Comp. LORENTZ, l.c. XXV, p. 476, form. (63).

8. When g^{ab} changes by δg^{ab}, then g_c^{ab} and g_{cd}^{ab} change by $\frac{\partial \delta g^{ab}}{\partial x_c}$ and $\frac{\partial^2 \delta g^{ab}}{\partial x_c \partial x_d}$. If we consider $\sqrt{-g}\,H$ as a function of the g^{ab} and their derivatives, the variation of $\sqrt{-g}\,H$ becomes

$$\delta(\sqrt{-g}\,H) = \Sigma(abcd)\left[\delta g^{ab}\left\{-\tfrac{1}{2}\sqrt{-g}\,g_{ab}H+\sqrt{-g}\frac{\partial H}{\partial g^{ab}}-\frac{\partial}{\partial x_c}\left(\frac{\sqrt{-g}\,\partial H}{\partial g_c^{ab}}\right)+\right.\right.$$
$$\left.+\frac{\partial^2}{\partial x_c\partial x_d}\left(\frac{\sqrt{-g}\,\partial H}{\partial g_{cd}^{ab}}\right)\right\}+\frac{\partial}{\partial x_c}\left(\sqrt{-g}\,\frac{\partial H}{\partial g_c^{ab}}\delta g^{ab}\right)+$$
$$\left.+\frac{\partial}{\partial x_d}\left(\frac{\sqrt{-g}\,\partial H}{\partial g_{cd}^{ab}}\delta g_c^{ab}\right)-\frac{\partial}{\partial x_c}\left\{\delta g^{ab}\frac{\partial}{\partial x_d}\left(\frac{\sqrt{-g}\,\partial H}{\partial g_{cd}^{ab}}\right)\right\}\right]. \quad (4c)$$

If $H = G/2\varkappa$, it can be proved[1]) that

$$\frac{\partial H}{\partial g^{ab}}-\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x_c}\left(\frac{\sqrt{-g}\,\partial H}{\partial g_c^{ab}}\right)+\frac{1}{\sqrt{-g}}\frac{\partial^2}{\partial x_c\partial x_d}\left(\frac{\sqrt{-g}\,\partial H}{\partial g_{cd}^{ab}}\right)=\frac{1}{2\varkappa}G_{ab} \quad (4d)$$

Summarizing and choosing the variations δg^{ab} arbitrarily with only this condition that both they and their first derivatives vanish at the boundaries of the extension, HAMILTON's principle requires that

$$0=\int dx_1\,dx_2\,dx_3\,dx_4\,\Sigma(abm)\left[\left\{\tfrac{1}{2}\sqrt{-g}\,g_{am}(T_b^m+E_b^m)+\right.\right.$$
$$\left.\left.+\frac{1}{2\varkappa}\sqrt{-g}\,(G_{ab}-\tfrac{1}{2}g_{ab}G)\right\}\delta g^{ab}\right]. \quad (5)$$

Hence we find the well-known *equations for the gravitational field*

$$G_{ab}-\tfrac{1}{2}g_{ab}G=-\varkappa(T_{ab}+E_{ab}) \quad . \; . \; . \; . \; . \; . \quad (6)$$

The origin of the second term of the left hand side is apparent; it appears by the variation of $\sqrt{-g}$ in the principal function.

Virtual displacement of the matter.

9. The third variation we shall consider will be caused by giving to the *molecules* of our gas *virtual displacements*. We do not choose these displacements different for each individual molecule, but to all molecules which at a certain moment are present in a definite element of volume we give the same virtual displacement, characterized by the infinitesimal vector δr^p (comp. § 1), which may be an arbitrary function of the coordinates. The variation gives directly

$$\int dx_1\,dx_2\,dx_3\,dx_4\,\Sigma(almp)\left[\frac{\partial}{\partial x_m}\left\{\sqrt{-g}\,(-\delta_a^m R-T_a^m)\,\delta r^a\right\}+\right.$$
$$\left.+\delta r^p\left\{\sqrt{-g}\,k_p+\frac{\partial}{\partial x_m}(\sqrt{-g}\,T_p^m)-\tfrac{1}{2}\sqrt{-g}\,g^{al}\frac{\partial g_{am}}{\partial x_p}T_l^m\right\}\right] \quad . \quad (7)$$

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[1]) Comp. LORENTZ, l.c. XXV, p. 472.

If the δr^p are zero at the boundary, then HAMILTON's principle demands that the integral always vanishes, so that

$$\sqrt{-g}\, k_p + \Sigma\,(aml) \left\{ \frac{\partial}{\partial x_m} (\sqrt{-g}\, T_p^m) - \tfrac{1}{2} \sqrt{-g}\, g^{al} \frac{\partial g_{am}}{\partial x_p} T_l^m \right\} = 0 \quad . \quad (8)$$

These are the *equations of motion of the matter* in an invariant form. k_p is a covariant vector and the form between brackets is $\sqrt{-g}$ times the covariant divergency of the mixed tensor T_p^m.

10. Consider now the *virtual variation* of the *electric current.* If each electric particle undergoes a displacement δr^p, then the variation of the intensity of the current at a definite point-instant, is

$$\delta (\sqrt{-g}\, W^m) = \Sigma\,(a) \frac{\partial}{\partial x_a} (\sqrt{-g}\, W^a\, \delta r^m - \sqrt{-g}\, W^m\, \delta r^a)\,, \text{[1]}$$

so that the integral is varied by:

$$\int dx_1\, dx_2\, dx_3\, dx_4\, \Sigma(map) \left[\frac{\partial}{\partial x_m} \lambda \left\{ \sqrt{-g}\, (W^m \varphi_a - \delta_a^m S)\, \delta r^a \right\} + \right.$$

$$\left. + \,\delta r^p \left\{ \sqrt{-g}\, k_p + \lambda \sqrt{-g}\, W^m \left(\left(\frac{\partial \varphi_m}{\partial x_p} \right) - \frac{\partial \varphi_p}{\partial x_m} \right) \right\} \right] \quad . \quad (9)$$

If δr^p vanishes at the boundary of our extension, we must have therefore

$$\sqrt{-g}\, k_p + \Sigma_{(m)} \sqrt{-g}\, W^m \left(\frac{\partial \varphi_m}{\partial x_p} - \frac{\partial \varphi_p}{\partial x_m} \right) = 0. \quad . \quad . \quad . \quad (10)$$

This may be called the "*equation of motion*" *for the electric current.* The second term may be said to represent the force exerted by the electric field on the carrier of the charge.

Virtual displacements of the fields.

11. Before calculating the variation which is obtained by a virtual displacement of the electro-magnetic field or of the gravitational field, we have to state what will be meant by this.

Doubtlessly we can say: to give a virtual displacement to the electro-magnetic field means to assume that the four potentials which originally occur at the point-instant x_p $(p = 1, 2, 3, 4)$ will be found after the displacement at the point-instant $x_p + \delta r^p$ $(p = 1, 2, 3, 4)$. From this follows that there will be at one and the same point-instant a variation $\delta \varphi_m$

$$\delta \varphi_m = - \Sigma(p) \frac{\partial \varphi_m}{\partial x_p} \delta r^p \,.$$

[1]) Comp. LORENTZ, l.c. XXIII, p. 1077.

It is, however, immediately evident that $\delta\varphi_m$ is no covariant vector though φ_m is one, so that we should compare with HAMILTON's invariant integral another, which is no longer invariant.

The same difficulty arises if a virtual displacement of the gravitational field is defined as the shift of a set of values g_{ab} from the point-instant x_p to another next to it $x_p + \delta r^p$. By so doing we do not obtain a covariant variation

$$\delta g_{ab} = - \Sigma(p) \frac{\partial g_{ab}}{\partial x_p} \delta r^p .$$

12. A closer examination of the geometrical meaning of the tensor components g_{ab} teaches us that in virtue of the equation $\varepsilon^2 = \Sigma(ab)\, g_{ab}\, dx_a\, dx_b$ they form together an infinitesimal quadratic three-dimensional extension, the "indicatrix" around each point-instant of the field figure.

The whole gravitational field may be said to be represented by the totality of the indicatrices described around the different point-instants, in the same way as in elementary considerations an electric field is described by FARADAY's lines of force. A virtual displacement of the gravitation field must therefore mean a displacement of all these indicatrices, in such a way, that it does not disturb the configuration and intersections of the indicatrices.

Let us consider two neighbouring indicatrices h and j, which intersect in the figure i. We may give the displacements to the indicatrix h and the indicatrix j separately and also to the figure i. We then demand that the shifted figure i shall again be the intersection of the shifted indicatrices h' and j'.

This cannot be managed by the variation specified in the preceding paragraph. There all point-instants of an indicatrix were supposed to undergo one and the same virtual displacement, equal to that which belongs to the centre. Now on the contrary we require that the virtual displacements of the point-instants of an indicatrix be defined by the values of δr^p at the different point-instants themselves.

If the δr^p are not constant, the virtual displacement will generally consist not only in a certain translation, but also in a rotation of the indicatrices. Analogous considerations may be applied to the virtual displacement of the electro-magnetic field. The potentials which together form a covariant tensor of the first order, represent at each point-instant a trivector multiplied by $\sqrt{-g}$, i. e. (in infinitesimal dimensions) a certain linear three-dimensional extension.

13. In order to find what has to be put for δg_{ab} and $\delta \varphi_m$, if they are to represent a virtual displacement of the fields in agreement with the geometrical character of the potentials g_{ab} and φ_m just discussed, we shall proceed in the following way. First we shall describe the world by means of somewhat altered coordinates. We introduce the transformation

$$x_m = x'_m - \delta r^m \, (m = 1, 2, 3, 4),$$

where δr^m represent the infinitesimal components of the displacement, the squares of which will be neglected, so that in differentiating a quantity which contains this δr^m or is to be multiplied by it, we need make no difference between partial differentiations with respect to x_m and to x'_m.

After the transformation of the coordinates we shall deform the net of coordinates together with the field in such a way that the surfaces $x'_m = a_m$ come at the place where originally were found the surfaces $x_m = a_m$. This is evidently reached by a virtual displacement of the field characterized everywhere by δr^m. In order to find what we have after the displacement we have only to omit the accents.

For the indicated transformation we have

$$dx_m = dx'_m - \Sigma(p) \frac{\partial \delta r^m}{\partial x'_p} dx'_p .$$

The geometrical character of the g_{ab} implies that the form

$$\Sigma\,(ab)\, g'_{ab}\, dx'_a\, dx'_b = \Sigma\,(ab)\, g_{ab}\, dx_a\, dx_b =$$

$$= \Sigma\,(ab)\, g_{ab} \left(dx_a' - \Sigma\,(p) \frac{\partial \delta r^a}{\partial x'_p} dx'_p \right) \left(dx'_b - \Sigma\,(p) \frac{\partial \delta r^b}{\partial x'_p} dx'_p \right)$$

is invariant.

Hence we deduce easily that

$$g'_{ab} = g_{ab} - \Sigma\,(p)\, g_{pb} \frac{\partial \delta r^p}{\partial x'_a} - \Sigma\,(p)\, g_{ap} \frac{\partial \delta r^p}{\partial x'_b} .$$

Here g_{ab} is the same function of $(x'_m - \delta r^m)$ which g_{ab} was of x_m. Therefore

$$g'_{ab} = g_{ab}\,(x'_m) - \Sigma\,(p) \left\{ \frac{\partial g_{ab}}{\partial x_p} \delta r^p + g_{pb} \frac{\partial \delta r^p}{\partial x'_a} + g_{ap} \frac{\partial \delta r^p}{\partial x'_b} \right\} .$$

If, omitting the accents, we now express that the new net of coordinates can be made to coincide with the original one, when the field is displaced at the same time, we find for the variation at a definite point-instant:

$$\delta g_{ab} = - \Sigma\,(p) \left\{ \frac{\partial g_{ab}}{\partial x_p} \delta r^p + g_{pb} \frac{\partial \delta r^p}{\partial x_a} + g_{ap} \frac{\partial \delta r^p}{\partial x_b} \right\} . \quad . \quad . \quad (11)$$

In the same way we find by a virtual displacement of the electromagnetic field

$$\delta\varphi_m = -\Sigma(p)\left\{\frac{\partial\varphi_m}{\partial x_p}\delta r^p + \varphi_p\frac{\partial\delta r^p}{\partial x_m}\right\} \quad . \quad . \quad . \quad . \quad (12)$$

These variations δg_{ab} and $\delta\varphi_m$ are really covariant tensors. Tensors formed in an analogous way are mentioned without commentary in a paper by HILBERT.[1])

14. For the case of a *virtual displacement of the electric field* we have

$$\delta f_{mn} = -\Sigma(p)\left\{\frac{\partial f_{mn}}{\partial x_p}\delta r^p + f_{pn}\frac{\partial\delta r^p}{\partial x_m} + f_{mp}\frac{\partial\delta r^p}{\partial x_n}\right\}. \quad . \quad . \quad (13)$$

We can now easily calculate the variation of HAMILTON's integral. We find

$$\lambda\int dx_1\,dx_2\,dx_3\,dx_4\,\Sigma(mnp)\left[\frac{\partial}{\partial x_m}(-\sqrt{-g}\,W^m\varphi_p\delta r^p + \sqrt{-g}\,F^{mn}f_{pn}\,\delta r^p) + \right.$$
$$+\,\delta r^p\left\{\frac{1}{\lambda}\sqrt{-g}\,k_p + \frac{\partial}{\partial x_m}(\sqrt{-g}\,W^m\varphi_p) - \sqrt{-g}\,W^m\frac{\partial\varphi_m}{\partial x_p}\right.$$
$$\left.\left. -\frac{\partial}{dx_m}(\sqrt{-g}\,F^{mn}f_{pn}) + \tfrac{1}{2}\sqrt{-g}\,F^{mn}\frac{\partial f_{mn}}{\partial x_p}\right\}\right].$$

Using the equation of continuity of the electric current

$$\Sigma(m)\frac{\partial}{\partial x_m}(\sqrt{-g}\,W^m) = 0$$

and transforming with

$$\frac{\partial g^{kl}}{\partial x_p} = -\Sigma(am)\,g^{ka}g^{lm}\frac{\partial g_{am}}{\partial x_p},$$

$$\Sigma(mn)\tfrac{1}{2}F^{mn}\frac{\partial f_{mn}}{\partial x_p} = -\frac{\partial M}{\partial x_p} + \tfrac{1}{2}\Sigma(almn)\,F^{mn}f_{ln}g^{al}\frac{\partial g_{am}}{\partial x_p},$$

we find, introducing the symbol E^m_a, for the variation

$$\int dx_1\,dx_2\,dx_3\,dx_4\,\Sigma(almp)\left[\frac{\partial}{\partial x_m}\{\sqrt{-g}(-\lambda W^m\varphi_a - \lambda\delta^m_a M - E^m_a\,\delta r^a\} + \right.$$
$$+\,\delta r^p\left\{\sqrt{-g}\,k_p + \lambda\sqrt{-g}\,W^m\left(\frac{\partial\varphi_p}{dx_m} - \frac{\partial\varphi_m}{\partial x_p}\right) + \right.$$
$$\left.\left.\frac{\partial}{dx_m}(\sqrt{-g}E^m_p) - \tfrac{1}{2}\sqrt{-g}g^{al}\frac{\partial g_{am}}{\partial x_p}E^m_l\right\}\right] \quad . \quad . \quad . \quad . \quad (14)$$

For a virtual displacement which is zero at the boundaries of the

[1]) DAVID HILBERT. *Die Grundlagen der Physik, I.* K. Ges. Wiss. Göttingen, Math. Phys. 1915.

extension, HAMILTON's principle requires that

$$0 = \sqrt{-g}\, k_p + \lambda \Sigma (m) \sqrt{-g}\, W^m \left(\frac{\partial \varphi_p}{\partial x_m} - \frac{\partial \varphi_m}{\partial x_p} \right) +$$

$$+ \Sigma (lma) \left\{ \frac{\partial}{\partial x_m} (\sqrt{-g} E_p^m) - \tfrac{1}{2} \sqrt{-g}\, g^{al} \frac{\partial g_{am}}{\partial x_p} E_l^m \right\} \quad . \quad . \quad (15)$$

These may be called the *equations of motion for the field.* We see that the acting external force and the force which the carrier of the charges exerts on the field[1]) must be opposite to the covariant divergency of a tensor multiplied by $\sqrt{-g}$. The equations correspond exactly to those which we found for the matter. For that reason we are justified in considering the tensor E_p^m as the *dynamical tensor* of the *stresses, momenta and energy* in the electro-magnetic field.

15. For the *virtual displacement of the gravitational field* it is easy to find the variation of the part of the integral containing $\sqrt{-g}\, H$. The integral being a scalar, we have

$$\int \sqrt{-g}\, H\, dx_1 dx_2 dx_3 dx_4 = \int \sqrt{-g'}\, H' dx_1' \, dx_2' \, dx_3' \, dx_4',$$

for the transformation of § 13. H being a scalar, we also have

$$H' = H (x'_p - \delta r^p).$$

$$\sqrt{-g'} = \sqrt{-g} \frac{\partial (x_1 \ldots x_4)}{\partial (x_1' \ldots x_4')} = \sqrt{-g(x'_p - \delta r^p)} \left\{ 1 - \Sigma (p) \frac{\partial \delta r^p}{\partial x_p} \right\}.$$

So that after the displacement we find (by omitting the accents)

$$\delta \int \sqrt{-g}\, H\, dx_1 \, dx_2 \, dx_3 \, dx_4 = \int dx_1 \, dx_2 \, dx_3 \, dx_4 \, \Sigma (p) \frac{\partial}{\partial x_p} (-\sqrt{-g} H \delta r^p). \; (16)$$

In what follows we shall use the results of § 8. With

$$\delta g^{ab} = - \Sigma (mn)\, g^{am} g^{bn} \delta g_{mn} \, , \quad . \quad . \quad . \quad . \quad . \quad (17)$$

we apply the formulae (4*a*, 4*b*, 11) and find after a short transformation for the total variation

$$\int dx_1 \, dx_2 \, dx_3 \, dx_4 \, \Sigma (almp) \left[\frac{\partial}{\partial x_m} \{ \sqrt{-g} \, (-\delta_a^m H + T_a^m + E_a^m) \, \delta r^a \} + \right.$$

$$\left. + \delta r^p \left\{ \sqrt{-g} k_p - \frac{\partial}{\partial x_m} (\sqrt{-g} T_p^m + \sqrt{-g} E_p^m) + \tfrac{1}{2} \sqrt{-g} g^{al} \frac{\partial g_{am}}{\partial x_p} (T_l^m + E_l^m) \right\} \right] \; (18)$$

As in the preceding cases HAMILTON's principle now teaches us that, whenever the displacement vanishes at the boundary of the extension, we must have

[1]) Per unit of volume.

$$0=\sqrt{-g}\,k_p+\Sigma(alm)\left\{\frac{\partial}{\partial x_m}(\sqrt{-g}\,Z_p^m)-\tfrac{1}{2}\sqrt{-g}\,g^{al}\frac{\partial g_{am}}{\partial x_p}\,Z_l^m\right\}\quad . \quad (19)$$

where

$$Z_p^m=-(T_p^m+E_p^m)=\frac{1}{\varkappa}\,\Sigma\,(b)\,g^{mb}(G_{pb}-\tfrac{1}{2}g_{pb}\,G)\quad . \quad . \quad (20)$$

These might be called the *"equations of motion" for the gravitational field.* Comparing this with our former result, we are induced to consider the tensor Z_p^m as the *dynamical tensor of the stresses, momenta and energy in the gravitational field.* We see that it is just equal and opposite to those of the matter and of the electro-magnetic field taken together.

16. By formula (16) we can prove, that the *covariant divergency of* Z_p^m *must be identically zero.* The variation of $\int\sqrt{-g}\,H\,dx_1\,dx_2\,dx_3\,dx_4$ may also be calculated by means of the formulae of § 8. If we choose the δr^p and their first and second derivatives equal to zero at the boundary, then according to (16) the variation must vanish. From 4c and d together with (17) and (11) we find

$$\delta\int\sqrt{-g}\,H\,dx_1 dx_2 dx_3 dx_4=\Sigma(ab)\int\frac{1}{2\varkappa}\sqrt{-g}\,(G_{ab}-\tfrac{1}{2}g_{ab}G)\delta g^{ab}dx_1 dx_2 dx_3 dx_4=$$

$$=\int dx_1 dx_2 dx_3 dx_4\,\frac{1}{\varkappa}\,\Sigma(abmkl)\left[\frac{\partial}{\partial x_m}\left\{\sqrt{-g}\,g^{bm}\,(G_{ab}-\tfrac{1}{2}g_{ab}\,G)\,\delta r^a\right\}-\right.$$

$$\left.-\,\delta r^a\left\{\frac{\partial}{\partial x^m}(\sqrt{-g}\,g^{bm}(G_{ab}-\tfrac{1}{2}g_{ab}\,G))-\tfrac{1}{2}\sqrt{-g}\,g^{kl}\frac{\partial g_{km}}{\partial x_a}\,g^{mb}(G_{lb}-\tfrac{1}{2}g_{lb}\,G)\right\}\right]$$

This can only be equal to zero if the coefficient of δr^a, i.e. $\sqrt{-g}$ times the covariant divergency of Z_a^m is zero, so that

$$\Sigma(bklm)\frac{1}{\varkappa}\frac{\partial}{\partial x_m}\{\sqrt{-g}\,g^{bm}(G_{ab}-\tfrac{1}{2}g_{ab}\,G)\}-$$

$$-\frac{1}{2\varkappa}\sqrt{-g}\,g^{kl}\frac{\partial g_{km}}{\partial x_a}\,g^{mb}\,(G_{lb}-\tfrac{1}{2}g_{lb}\,G)\equiv 0\quad . \quad . \quad (21)$$

17. This identity, which implies four connexions between the components of $(G_{ab}-\tfrac{1}{2}g_{ab}G)$, is important because it shows that the ten differential equations

$$G_{ab}-\tfrac{1}{2}g_{ab}\,G=0$$

which determine the gravitational field at those places of our extension where there is neither matter nor an electro-magnetic field, are not independent of each other. In such extensions void of matter the gravitation potentials may therefore be subjected arbitrarily to four additional connexions. EINSTEIN has shown that this indefiniteness in the extensions void of matter can never give rise to an indefiniteness in the observations that can be made with material instruments.

The identity further confirms that in the absence of an external force the laws of conservation of energy and momentum hold for the matter. Indeed, from the field equation (6), which is given in (20) in another form, together with (21) it is evident that

$$0 = \Sigma(klm) \frac{\partial}{\partial x_m} \{\sqrt{-g}\,(T_a^m + E_a^m)\} - \tfrac{1}{2} \sqrt{-g}\, g^{kl} \frac{\partial g_{km}}{\partial x_a} (T_l^m + E_l^m). \quad (22)$$

We may even conclude that no other force can be exerted on the matter and the electro-magnetic field by any agency if this does not change the gravitation field at the same time.

18. The second term on the left-hand-side of (21) can be transformed. We may write for it

$$\Sigma(lb) \frac{1}{2\varkappa} \sqrt{-g}\, \frac{\partial g^{lb}}{\partial x_a} (G_{lb} - \tfrac{1}{2} g_{lb} G).$$

According to (4*d*) this comes to the same as

$$\Sigma(lbcd)\, g_a^{lb} \left[\frac{\partial \sqrt{-g}\, H}{\partial g^{lb}} - \frac{\partial}{\partial x_c} \left(\frac{\partial \sqrt{-g}\, H}{\partial g_c^{lb}} \right) + \frac{\partial^2}{\partial x_c\, \partial x_d} \left(\frac{\partial \sqrt{-g}\, H}{\partial g_{cd}^{lb}} \right) \right].$$

The same may also be expressed as follows

$$\Sigma(lbcd) \frac{\partial}{\partial x_a} (\sqrt{-g}\, H) - \frac{\partial}{\partial x_c} \left\{ g_a^{lb} \frac{\partial \sqrt{-g}\, H}{\partial g_c^{lb}} - g_a^{lb} \frac{\partial}{\partial x_d} \left(\frac{\partial \sqrt{-g}\, H}{\partial g_{cd}^{lb}} \right) + g_{da}^{lb} \frac{\partial \sqrt{-g}\, H}{\partial g_{cd}^{lb}} \right\}.$$

If now we put

$$\sqrt{-g}\, z_a^c = \Sigma(lbd)\, g_a^{lb} \frac{\partial \sqrt{-g} H}{\partial g_c^{lb}} + g_{ad}^{lb} \frac{\partial \sqrt{-g} H}{\partial g_{cd}^{lb}} - g_a^{lb} \frac{\partial}{\partial x_d} \frac{\partial \sqrt{-g} H}{\partial g_{cd}^{lb}} - \delta_a^c \sqrt{-g} H,$$

then we have according to the preceding equation and (21), (20):

$$\Sigma(m) \frac{\partial}{\partial x_m} \{\sqrt{-g}(T_a^m + E_a^m)\} + \Sigma(c) \frac{\partial}{\partial x_c} (\sqrt{-g}\, z_a^c) = 0 \quad . \quad . \quad (23)$$

So we find in $\sqrt{-g}\, z_a^c$ a complex, the "quasi-divergency" (no invariant) of which is the opposite of the quasi-divergency of the dynamical tensor of matter and electro-magnetic field. LORENTZ [1]) and DE DONDER [2]) have deduced another similar complex

[1]) l. c. XXV, p. 473.

[2]) TH. DE DONDER, *Les équations différentielles du champ gravifique* d'EINSTEIN *créé par un champ electromagnétique de* MAXWELL-LORENTZ. Verslagen, Kon. Ac. Wet. Amsterdam, XXV, p. 150.

$$\sqrt{-g}\,\overset{c}{s}_a = \Sigma(lbd)\, g_{lb,a}\frac{\partial\sqrt{-g}H}{\partial g_{lb,c}} + g_{lb,ad}\frac{\partial\sqrt{-g}H}{\partial g_{lb,cd}} - g_{lb,a}\frac{\partial}{\partial x_d}\frac{\partial\sqrt{-g}H}{\partial g_{lb,cd}} - \overset{c}{\delta}_a\sqrt{-g}H,$$

which is found as easily as $\sqrt{-g}\overset{c}{z}_a$ by transformation of the second term of the identity (21).

If we wish we may take the components of one of these complexes for the stresses, momenta etc. in the gravitation field. According to (21) we have however identically

$$\Sigma\,(m)\frac{\partial}{\partial x_m}(\sqrt{-g}\;\overset{m}{Z}_a) \equiv \Sigma\,(c)\frac{\partial}{\partial x_c}(\sqrt{-g}\;\overset{c}{z}_a) \equiv \Sigma\,(c)\frac{\partial}{\partial x_c}(\sqrt{-g}\;\overset{c}{s}_a)\,,$$

so that we have also

$$\Sigma\,(m)\frac{\partial}{\partial x_m}\left\{\sqrt{-g}\,(\overset{m}{T}_a + \overset{m}{E}_a)\right\} + \frac{\partial}{\partial x_m}(\sqrt{-g}\;\overset{m}{Z}_a) = 0\,. \quad . \quad (24)$$

Now it is quite a matter of taste and, as to the calculations one of opportunity, which of the three equations (22), (23) or (24) will be regarded as the expression of the laws of conservation of energy and of momentum and whether $\overset{c}{z}_a$, $\overset{c}{s}_a$ will be regarded as a dynamical quasi-tensor, or $\overset{m}{Z}_a$ as the dynamical pure tensor of the gravitation field; or finally whether it is better not to introduce a dynamical tensor in the gravitational field at all.

Connexion with LORENTZ's *theory of electrons.*

19. Finally we shall shortly show how the deduced formulae are connected with the classic formulae of the theory of electrons. For this purpose we must treat the case of constant gravitation potentials having the values

$$g_{ab}\,(=)\;\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0\,. \\ 0 & 0 & 0 & c^2 \end{matrix}$$

To these corresponds the value $g = -c^2$ and the values of the algebraic complements

$$g^{ab}\,(=)\;\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0\,. \\ 0 & 0 & 0 & \frac{1}{c^2} \end{matrix}$$

Our formulae are based on HAMILTON's principle for the motion of a point which falls freely. In the case now under consideration it takes the form

$$0 = \delta \int_1^2 - m\, ds = \delta \int_1^2 - m \sqrt{c^2 - v^2}\, dt\,.$$

Comparing this with what we were used to write in the old mechanics of relativity

$$0 = \delta \int_1^2 - mc^2 \sqrt{1 - v^2/c^2}\, dt\,,$$

we see that in our formulae the function of LAGRANGE has been taken c times smaller. Correspondingly definite forces, energy, stresses etc. will have to be represented by numbers which are c times smaller than they were formerly.

If for instance the unit of electric charge is left as it is defined in the theory of electrons (in these units an electron has e.g. a charge $-\sqrt{4\pi} \times 4{,}65 \times 10^{-10}$), and if at the same time the unit of the intensity of the electric field d and that of the magnetic field h are left unchanged, we shall have to write for the force per unit of volume

$$f = \varrho \left\{ \frac{1}{c}\, d + \frac{1}{c^2}\, [v \,.\, h] \right\}.$$

If we wish our equations (3) for the electric field

$$\Sigma\,(b)\, \frac{\partial}{\partial x_b} (\sqrt{-g}\, F^{ab}) = \sqrt{-g}\, W^a\,,$$

in which the components of the *current* $\sqrt{-g}\, W_a$ are ϱv_x, ϱv_y, ϱv_z, ϱ, to agree with the well known relations

$$c\, \frac{\partial h_z}{\partial y} - c\, \frac{\partial h_y}{\partial z} - \frac{\partial}{\partial t}\, d_x = \varrho v_x\,, \quad etc.,$$

the components of the *contravariant field tensor* must be

$$F^{ab}\, (=) \begin{matrix} 0 & h_z & -h_y & -\frac{1}{c}\, d_x \\ -h_z & 0 & h_x & -\frac{1}{c}\, d_y \\ h_y & -h_x & 0 & -\frac{1}{c}\, d_z \\ \frac{1}{c}\, d_x & \frac{1}{c}\, d_y & \frac{1}{c}\, d_z & 0. \end{matrix}$$

Hence it follows for the components of our covariant field tensor

$$\Sigma(ab)\, g_{ap}\, g_{bq}\, F^{ab} = f_{pq}\, (=) \begin{matrix} 0 & h_z & -h_y & c\, d_x \\ -h_z & 0 & h_x & c\, d_y \\ h_y & -h_x & 0 & c\, d_z \\ -cd_x & -cd_y & -cd_z & 0\,. \end{matrix}$$

We know that $f_{pq}=\frac{\partial\varphi_q}{\partial x_p}-\frac{\partial\varphi_p}{\partial x_p}$. Hence it is evident how the scalar potential φ and the vector potential a of the theory of electrons are connected with our *potentials*:

$$\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4 \quad (=) \quad a_x \quad a_y \quad a_z \quad -c\varphi .$$

For the components of the force acting on the charge per unit of volume we found in our formula (10):

$$-K_p=-\sqrt{-g}\,k_p=\lambda\,\Sigma\,(m)\sqrt{-g}\;W^m f_{pm} .$$

To make this agree with the above, we must, with a view to the choice of units, give the value $\lambda=1/c^2$ to the coefficient λ. The formula thus becomes

$$-\sqrt{-g}\,k_p=\frac{1}{c^2}\,\Sigma(m)\sqrt{-g}\;W^m f_{pm} .$$

It keeps this form when we pass to a system of coordinates in which the unit of time is c times smaller and in which the velocity of light becomes equal to 1 (c remains 3.10^{10}). It may be remarked in passing that in the papers of LORENTZ [1]) and TRESLING the factor $1/c^2$ is failing. It is thus seen that they have silently used a unit of charge c times larger than the usual one.

The scalar for the field becomes

$$M=-\tfrac{1}{4}\,\Sigma\,(ab)\,F^{ab}f_{ab}=\tfrac{1}{2}\,(d^2-h^2) ,$$

and the principal function $\lambda\sqrt{-g}M=\frac{1}{2c}\,(d^2-h^2)$. In agreement with what has been said at the beginning of this paragraph this expression is c times smaller than the one we were accustomed to.

The *stresses*, the *negative momenta*, the *energy* and the *energy-currents* become

$$\sqrt{-g}\,E_a^m=-\lambda\Sigma(b)\sqrt{-g}\,F^{mb}f_{ab}-\lambda\sqrt{-g}\,\delta_a^m\,M ,$$

$$\begin{array}{llll}
\frac{1}{2c}(2h_x^2-h^2+2d_x^2-d^2), & \frac{1}{c}(h_xh_y+d_xd_y), & \frac{1}{c}(h_xh_z+d_xd_z), & -\frac{1}{c^2}(d_yh_z-d_zh_y), \\
\frac{1}{c}(h_xh_y+d_xd_y), & \frac{1}{c}(2h_y^2-h^2+2d_y^2-d^2), & \frac{1}{c}(h_yh_z+d_yd_z), & -\frac{1}{c^2}(d_zh_x-d_xh_z), \\
\frac{1}{c}(h_xh_z+d_xd_z), & \frac{1}{c}(h_yh_z+d_yd_z), & \frac{1}{2c}(2h_z^2-h^2+2d^2-d^2), & -\frac{1}{c^2}(d_xh_y-d_yh_x), \\
(d_yh_z-d_zh_y), & (d_zh_x-d_xh_z), & (d_xh_y-d_yh_x), & \frac{1}{2c}(h^2+d^2) .
\end{array}$$

We see that all these components become c times smaller than formerly, as has been remarked already in the beginning of this paragraph.

1) For the comparison with the papers of LORENTZ it may be remarked that $\sqrt{-g}\,F_{ab}=\psi_{ab}$ and $f_{ab}=\overline{\psi_{ab}}$. Further that $\sqrt{-g}\,W_m=w_m$.