Physics. — *"On the diffraction of the light in the formation of halos"*. By Dr. S. W. VISSER. (Communicated by Prof. J. P. KUENEN).

(Communicated in the meeting of March 31, 1917.)

I. *Introduction.*

The halos which originate by refraction are often seen distinctly coloured. This fact is best known as regards the circumzenithic arc, the parhelia and the tangential arcs, but the large circle is also often coloured and in the ordinary circle of 22° too, distinct colours occur.

In this paper the last circle will be principally dealt with. According to the common refraction-theory of halos, as developed by PERNTER [1]) amongst others in his well-known work, the red on the inside of the circle should be distinctly visible, whereas the green and blue would already be very pale. The observations on the other hand show, that the colours are often practically invisible, but that sometimes they appear with great brightness, and the same holds for the other halo-phenomena mentioned above. These bright colours are explained by PERNTER as regards the parhelia and the tangential arcs by the presence of a large number of ice-crystals, whereby the intensity of the light would be increased and the colour become visible [2]). But if there is nothing but white light or nearly white light, an increase of intensity cannot produce anything but more white: the distribution of colour cannot undergo any change by a greater intensity of light.

A remark by PERNTER himself shows, how little the colours fit into the usual theory [3]):

"It should be mentioned, that in a description of the phenomenon, as observed on September 4th 1900 at Aix la Chapelle by SIEBERG I find the following statement as to the colours of the smaller circle: "In addition it was distinctly coloured red, yellow, green and blue from inside to outside". As this observation would contradict all others, if it referred to the complete ring, I assume, that the colours were observed, where the parhelia were situated on the ring, in which these colours have also been observed by others".

But the colours of the parhelia are not in accordance with the theory either and moreover SIEBERG's observation is not at all in

[1]) J. M. PERNTER and F. M. EXNER, Meteorologische Optik, Wien 1910 pag. 319.
[2]) PERNTER. l.c. p. 318, 320, 321.
[3]) PERNTER, l c. p. 228.

contradiction with those of others. This will all be shown further on.

In this paper an attempt will be made by a modification of the theory in which diffraction will be taken into account to explain the colour-phenomena as observed.

We shall begin by tabulating a number of the various colours that have been recorded, chiefly taken from the publication of the "Koninklijk Nederlandsch Meteorologisch Instituut": "Thunder storms, Optical Phenomena etc. in Holland according to voluntary observations 1901—1914" [1]). This will be followed by a discussion of the simple refraction-theory for ice-crystals with a refracting angle of 60° in order to arrive at the colours which might occur in the ordinary circle. A diffraction-theory will then be developed and finally the colours will be deduced for a specially well developed halo of 22°, which will appear to agree very well with the observations.

II. *Survey of some of the colours observed in halos.*

I shall confine myself to those records, in which colours are mentioned by name. Lyrical rhapsodies like: "brilliantly, very intensely, strongly, magnificently coloured", especially numerous for the parhelia, cannot be utilized. In the fourteen volumes of "Thunderstorms etc." which I consulted the colours of the parhelia are named in only five cases!

With regard to the circumzenithic arc BESSON expresses himself as follows [2]):

Les couleurs sont souvent remarquablement pures: on distingue en bas le rouge, puis le jaune en passant par l'orangé, puis le vert, puis mais pas toujours le bleu et le violet. Cette dernière couleur est fréquemment absente, mais il n'est pas rare, qu'elle soit visible très nettement. Fait important à noter: le violet, quand il existe, est très pur, il n'est pas surmonté ou mêlé de blanc, comme dans les arcs tangents au halo de 22°. La coloration offre une intensité des plus variables: faible parfois au point d'être à peine perceptible, elle est d'autres fois aussi éclatante que celle du plus bel arc-en-ciel.

These references, incomplete though they are, are sufficient to show the great variety of colouring in the halo-phenomena. Especially important from this point of view are those cases in which different observers mention the same colours.

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[1]) Onweders, optische verschijnselen, enz. in Nederland naar vrijwillige waarnemingen 1901—1914. (Cited in the following table as: "Onw.").

[2]) L. BESSON, Sur la Théorie des Halo's, Paris 1909, p. 53.

CZA = circumzenithic arc; UTA = upper tangential arc; CH = circumscribed halo; C 22° = circle of 22°; C 46° = circle of 46°; P = parhelion (paraselene).

	Date	Colours	Accompanying halos	Place of observation	Reference
	a. Circle of 22°.				
1	1903, Apr. 17	strongly coloured	CZA brown, blue; UTA; P	Vrijenban	Onw.
2	Sep. 23	brown-red, violet		Zoutkamp	"
3	1904, Apr. 2	golden brown, yellow, green, violet	UTA uncoloured; B coloured	Nymegen	"
4	Aug. 14	faintly orange	lateral tangential arcs uncoloured	Valkenburg	"
5	1905, March 28	red predominant	UTA red predominant; C 46° green predominant	Zutfen	"
6a	Oct. 7	yellow, violet	C 46° red, green	Zutfen	"
b	"	red, yellow, green, violet	P; faint UTA	Nymegen	"
7	1906, Feb. 12	red predominant	UTA very brightly coloured; C 46° all colours	Zutfen	"
8	1914, Apr. 11	orange, dark-green, white (moon)	P faint, white	Renesse	"
9a	1886, May 3	very brilliant, rainbow hues, except green	TA same colours, fainter	Boulogne s. Seine	La Nature 14, p. 379, 1886
b	"	strongly coloured	CH; C 46° red, blue, faint	Angers	"
c	"	red, yellow, green, blue, violet, inside violet	CH in the same colours	Argentan	"
10a	1887, Jan. 28	red, orange, yellow, green, blue, indigo, violet	C 46°; CZA; UTA	Pithiviers	La Nature 15, p. 161, 1887
b	"	magnificent rainbow	P; double inferior tangential arc	Souppes	"
c	"	prismatic colours	C 46° prismatic colours; P bright pink; CZA	Fontainebleau	"
d	"	spectrum-colours	UTA; P	Orleans	"
	b. Parhelia on the circle of 22°.				
11	1901, Jan. 29	red, orange, yellowish white	two above one another	Zutfen	Onw.

	Date	Colours	Accompanying halos	Place of observation	Reference
12	1904, Sep. 28	white	C 22° brown-red; C 46° red, blue	Zoutkamp	Onw.
13	1905, Sep. 19	uncoloured	UTA red, yellow, blue (broad), violet (narrow)	Nymegen	"
14	1910, Sep. 7	red specially bright, remaining colours very lively, blue and violet, also distinguishable	C 22°; CZA; LOWITZ's arc	Zutfen	"
15	1911, Nov. 3	red orange, light-green	C 22°, C 46°; UTA and CZA in the same colours	Renesse	"

Compare also 10c.

c. Upper tangential arc

	Date	Colours	Accompanying halos	Place of observation	Reference
16	1903, May 5	red to violet	C 22°; CH	Zutfen	"
17a	1904, Apr. 26	goldish brown, green, blue	P; C 22°; C 46°	Vrijenban	"
b	"	brown-red, yellow, green, violet		Delft	"
18	Oct. 6	red, green, some blue		Zutfen	"
19	1905, Nov. 2	goldish brown, clear white, blue	C 22°; C 46°; P	Vrijenban, Delft	"
20	1907, Oct. 3	red, violet	C 46° red, green; C 22; P faint; CZA	Several	"
21	1910, Jan. 30	red-orange, light green	C 22°; C 46°	Renesse	"
22	Feb. 8	red to violet	C 46° pale red	Groningen	"
23	May 18	brown-red, violet (moon)	C 22°, corona	Zutfen	"

Comp. also Nrs. 3, 5, 7, 9, 13, 15, 25.

d. Inferior tangential arc.

	Date	Colours	Accompanying halos	Place of observation	Reference
24	1904, May 21	red-orange, yellow, green, blue, violet	C 22°; parhel. circle	Zoutkamp	"
25	1909, Apr. 22	yellow predominant	UTA very brightly coloured; C 22°	Zutfen	"

e. Circle of 46°.

	Date	Colours	Accompanying halos	Place of observation	Reference
26	1901, March 22	red, yellow, green		Munnikeburen	"

	Date	Colours	Accompanying halos	Place of observation	Reference
27	1906, Oct. 8	conspicuously lively colours	C 22°	Zutfen	Onw.
28	1911, March 8	faint red	C 22°	Renesse	"
29	Oct. 1	red		de Bilt	"
30	1914, Dec. 29	red	CZA red, blue; P; pillar	de Bilt	"
31	1890, March 3	all the colours of the rainbow	C 22°; P; CZA rainbow colours	Parc Saint Maur	La Nature, 18, p. 238, 1890

Compare also: Nrs. 5, 6a, 7, 9b, 12, 20, 22.

f. Circumzenithic arc.

	Date	Colours	Accompanying halos	Place of observation	Reference
32	1911, Oct. 1	all colours to violet	UTA	Zutfen	Onw.
33	Nov. 15	red, yellow, green	C 22°	Munnikeburen	"

Compare also: 1, 15, 30, 31.

III. *The refraction of light in ice-crystals of a refracting angle of* 60°.

We may confine ourselves to the phenomenon as it presents itself in one plane brought through the eye and the sun. By a rotation of this plane about the line eye-sun the circular phenomenon will be generated. It will also be allowable to consider exclusively those crystals whose refracting edge is at right angles to the plane chosen, seeing that the colours can only take rise on the inner edge of the halo, where the light which moves perpendicularly to the axis of the crystals chiefly contributes to the formation of the circle.

In order to deduce the colour which will be seen in a given direction it is necessary to determine the intensities of the various spectral colours in that direction: from these the resulting colour can be calculated.

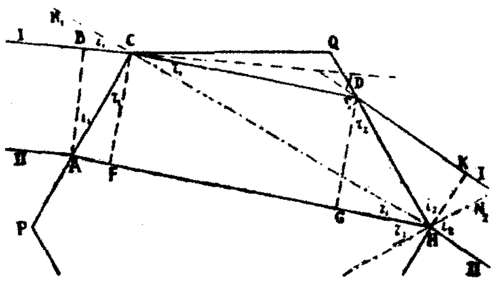

Fig. 1.

For an angle of incidence i_1 in the hexagonal prism $PCQH$ (fig. 1; only the faces required for the construction are shown) the passing beam is confined within the rays I and II. In the direction of a given angle of deviation D there are two different beams of widths AB

and HK respectively. The sum of the intensities of these emerging beams determines the intensity in the given direction. These two intensities may be replaced by those of the incident beams HK and AB. It is true that some light is lost by reflection, but no great error will be introduced by assuming that — on the inner side at any rate — all colours are weakened approximately in the same ratio by the reflections. The colour to be observed is not influenced by these losses: only the total intensity will be lessened. The intensities of the incident beams may be taken proportional to their widths and, as in the end we are only concerned with the ratios, the intensity may be put equal to the sum of the widths $AB + HK$ itself.

Calling the width of the side of the prism a, the angles of incidence and refraction i_1, r_1 and $i_2\, r_2$ (see fig.), we find:

$$AB + HK = \left(\frac{\cos i_1}{\cos r_1} + \frac{\cos i_2}{\cos r_2}\right) \sin r_1 \, . \, a \sqrt{3}.$$

$$\Big[HK = EH \cos i_2 = EG \frac{\cos i_2}{\cos r_2} = CF \frac{\cos i_2}{\cos r_2} = CH \sin r_1 \frac{\cos i_2}{\cos r_2} =$$

$$= \sin r_1 \frac{\cos i_2}{\cos r_2} . a\sqrt{3}; \quad AB = AC \cos i_1 = CH \, tg\, r_1 \cos i_1 = \sin r_1 \frac{\cos i_1}{\cos r_1} . a \sqrt{3} \Big].$$

For a given value of a the intensity L, leaving out a constant factor, may be put equal to:

$$L = \left(\frac{\cos i_1}{\cos r_1} + \frac{\cos i_2}{\cos r_2}\right) \sin r_1 \quad \ldots \ldots \quad (1)^{1)}.$$

This function can be calculated for all values of i and n. Computing the deviation D corresponding to a given value of i_1 the intensity for the direction determined by D is found by substituting in (1) that value of i_1 and the corresponding values of r_1, i_2 and r_2.

The various refractive indices n of ice which are required were derived from measurements by PULFRICH[2]) by graphical interpolation (with a small extrapolation, utilizing a remark of PULFRICH's, that the dispersion of ice is equal to that of water) taking for n the mean of the values for the ordinary and extraordinary rays (for the whole spectrum the difference in the minimum deviation between the two amounts to only 6′).

The values required for the purpose are as follows (P refers to

[1]) For angles of incidence greater than that of the symmetrical case the indices 1 and 2 interchange.

[2]) C. PULFRICH, Wied. Ann. 34, p. 336, 1888.

a measurement by PULFRICH; D_0 = angle of minimum deviation):

λ	n	D_0	λ	n	D_0
B 0.687	1.3071(P)	21°37′	x 0.494	1.3136	22°7′
C 0.656	1.3079(P)	21°41′	F 0.486	1.3140(P)	22°8′
D 0.589	1.3098(P)	21°50′	y 0.449	1.3157	22°16′
E 0.527	1.3121(P)	22°0′	G 0.431	1.3168	22°21′

x and y are two special wave-lengths which are of importance for the deduction of the colour-effect following further on.

For these eight colours and for angles of incidence between about 41° (corresponding to the angle of minimum deviation, which differs for the various colours) and 62° the expression (1) was computed. In this set angles of deviation up to 25° are included. Larger angles of incidence were found to be unnecessary; and, moreover, the loss of light by reflection probably begins to exercise its influence in this region.

The results are contained in the accompanying table. A special calculation for the line F which differs very little from x was not carried out. Fig. 2 gives the dependence of the intensity on the deviation.

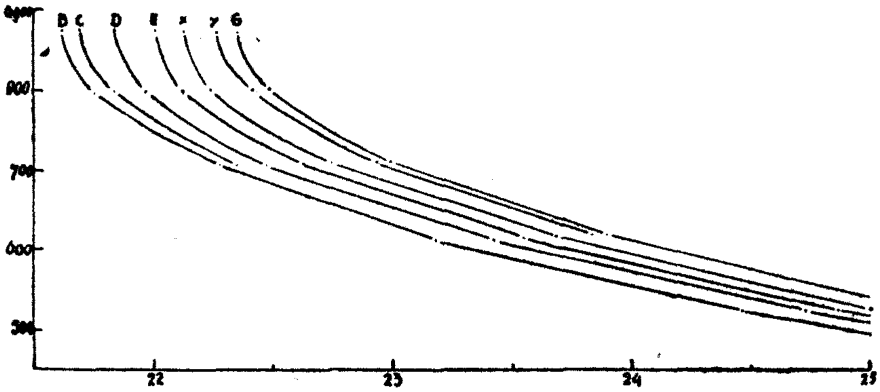

Fig. 2.

By means of these curves the intensity was determined in directions from 21°30′ upwards ascending by 15′. It is necessary in this computation to bring into account the finite extension of the sun. The same approximation was applied as used by PERNTER in the case of the rainbow; in fact PERNTER's method of computing the colours was followed throughout [1]): the curves are shifted in three steps of 5′ both to the left and to the right and read each time. The figures thus obtained, corresponding to seven points of the sun at intervals of 5′, are added up. The practical execution of the method comes to reading the curves from 5′ to 5′ and each time

[1]) PERNTER, l.c. pag. 529 sqq.

i	B		C		D		E		$x(F)$		ν		G	
	L	D	L	D	L	D	L	D	L	D	L	D	L	D
about 41°	0.875	21°37′	0.874	21°41′	0.873	21°50′	0.872	22° 0′	0.871	22° 7′	0.870	22°16′	0.870	22°21′
45° 0′	798	45	798	50	799	58	800	7	800	15	801	25	801	29
50 0	704	22 17	705	22 21	705	22 29	706	38	707	45	708	56	708	23 0
55 0	608	23 11	609	23 17	610	23 26	611	23 33	612	23 41	614	23 50	615	54
60 0	521	24 28	522	24 33	522	24 44	522	24 52	523	25 0	523	25 8	524	25 13
62 0	483	25 9	483	25 14	485	25 22								

combining seven readings: the sum represents the intensity of the light at the middle point.

The sums thus arrived at are then reduced to the intensities with which the eight colours concerned occur in the light of the sun by multiplying each by a special coefficient.

The final calculation of the resulting colour by PERNTER's method, which is based on MAXWELL's colour-equations, consists in dividing the intensity found for each of the eight colours over the three primary colours red, green and violet (.630 μ, .528 μ, and .475 μ) and thence to deduce the colour-equations which yield the final colours, each with the percentage of white with which it is mixed.

	S	*R*	*G*	*V*
B	23	1.000	0.000	0.000
C	94	0.904	0.011	0.085
D	262	0.557	0.446	—0.003
E	153	—0.006	0.993	0.013
x	118	−0.068	0.602	0.466
F	130	—0.061	0.346	0.715
y	152	0.020	0.007	0.973
G	68	0.000	—0.059	1.059
W		240	383	377

Column S gives the proportional numbers for sunlight, R, G and V those for the primary colours. The bottom row gives the proportion of the primary colours in white (W).

The following table contains the intensity as obtained for a few directions:

	B	*C*	*D*	*E*	*x*	*F*	*y*	*G*
21°30′	37.4	78	0	0	0	0	0	0
22°0′	121	505	1244	496	192	213	0	0
22°30′	110	454	1296	782	618	684	707	264
23°0′	101	418	1195	715	564	623	748	339
24°0′	89	367	1046	620	489	540	642	290
25°0′	79	327	928	551	433	479	567	257

The numbers for F were obtained from those for x by a shift of 1' (the difference in the minimum deviation for the two wave-lengths).

The final results are contained in the following table:

D	L	W %	C %	G	Colour	
21°30′	12	1.7	98.3	29.5	red (weak)	
21°45′	116	5.5	94.5	3.7	orange	
22°0′	279	29	71	4.7	yellow	
15′	424	69	31	5.9	green-yellow	
30′	491	96	4	8.4	green	
45′	491	96	4	17.0	blue	
23°0′	471	96	4	16.9	„	white
30′	437	96	4	16.9	„	
24°0′	408	97	3	16.8	„	
25°0′	362	98	2	16.8	„	

In this table the letters have the following meaning:

D angle of deviation; L intensity of the light; W % and C % percentages of white and of colour; G the colour-number in PERNTER's (MAXWELL's) colour-triangle, the last column gives the corresponding colour.

Fig. 3 shows the dependence of L on D; from which it appears that the refraction-theory gives a maximum at a distance of more than 22°30′. The observations on the other hand show, that the

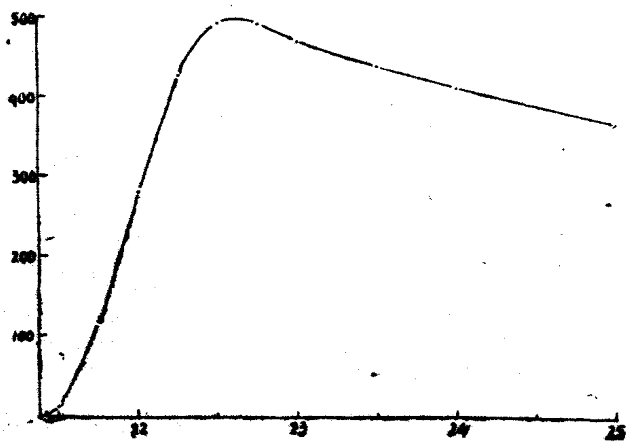

Fig. 3.

maximum is nearer to the sun. PERNTER[1]), taking the mean of a large number of trustworthy measurements, finds 21°50′. This difference of more than $^1/_2$° is much too large. Moreover, the refraction theory leads to the conclusion, that the inside of the circle would be red-orange, yellow, green-yellow and that blue can hardly appear and violet not at all. No increase of the intensity of the light can improve this disagreement with observation; the white always remains 24 times as intense as the blue; specially directed crystals are not capable of producing colours which are not there before. The colouring of the parhelia and tangential curves are not explained. It may also be noticed that on the underlying refraction-theory the size of the crystals which is determined by a cannot have an influence on the colour-phenomenon either, seeing that the width of the beam changes in the same ratio as a for all colours and that the light is parallel.

The conclusion to be drawn is simply this: the refraction theory is not able to give a complete explanation of the halo-phenomena.

I have not carried out a similar calculation for crystals with a refracting angle of 90°, but the circumstance, that the minima of deviation lie further apart in this case[2]), cannot be sufficient to explain the presence of differently coloured rings of 46°.

IV. The diffraction theory.

We may again confine ourselves to the phenomenon as occurring in one plane sun-crystal-eye, with the refracting edge at right angles to this plane. We shall also assume the special case of all crystals having the same size, which must be looked upon as a limiting case which is specially favourable to a development of colour.

As we have seen a beam of definite width emerges from the crystal after refraction, but this beam will be subject to diffraction. A large number of crystals of the same size, irregularly distributed, with parallel edges, all give similar beams, which will give rise to interference phenomena: the light source is seen as it were through

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1) PERNTER, l.c. 230. PERNTER's proof that a ring must be formed at 21°50 is by no means conclusive. He only shows that the yellow light has a minimum deviation of that magnitude (p. 313). The maximum intensity lies further out: about at 22°21′ (min. dev. of violet) + 16′ (sun's radius) = 22°37′, the place where all the colours of the complete spectrum are fully developed, in accordance with what the above more elaborate calculation gives. The light circle seen against the dark background will appear narrower to the eyes. Thereby the difference between observation and calculation becomes smaller, but it is more than doubtful, whether this effect could cause the difference to disappear altogether.

2) PERNTER, l.c. p. 354, 357.

a very large number of rectangular apertures, slits, spread at random. Diffraction fringes will be formed parallel to the refracting edges of the crystals and a rotation of the plane about the line eye-sun produces diffraction circles round the sun.

The places of the diffraction minima are given by the equation

$$\sin\theta = m\frac{\lambda}{a},$$

where θ is the angle of diffraction for the minimum, m the order, λ the wave-length and a the width of the slit.

The first maximum for each colour is formed in its minimum deviation. Towards the outside of the ring each direction represents a maximum for each colour accompanied on both sides by diffraction fringes. The resulting colour cannot be anything there but white and the only colours which can be seen will have to be looked for on either side of the first maximum.

In each direction two beams AB and HK (fig. 1) of different width cooperate. Considering that these beams approach each other in width the nearer they are to the minimum-deviation and that in its neighbourhood the deviation of the rays changes very slowly with the angle of incidence, it follows that the diffraction lines very nearly cover each other in the neighbourhood of the minimum-deviation and hence that the colours must be more prominent there than elsewhere. In order to elucidate this effect I have computed the relative widths of the slits AB and HK (equation 1) on both sides of the minimum for angles of incidence between 37° and 50°

i_1	i_2	a_1	a_2	max.—min. a_1	max.—min. a_2	D
40°55′	40°55′	0.4363	0.4363	100′	100′	21°50′
42 0	39 51	4228	4306	103	101	51
43 0	38 53	4104	4250	106	103	53
44 0	37 55	3980	4190	110	104	55
45 0	36 59	3856	4130	113	106	59
46 0	36 3	3734		117		22 3
47 0	35 8	3612		121		8
48 0	34 14	3490		125		14
49 0	33 22	3370		130		22
50 0	32 30	3250		134		30

for $n = 1{,}310$ (yellow); further for the same angles the deviation which gives the place of the central maximum and finally the place of the first diffraction-minimum on both sides of the central maximum, assuming at the minimum-deviation a distance between central maximum and first minimum of $100'$ (a value about equal to the one found in the special case to be dealt with further on corresponding to an absolute width of the slit a of $20.24\ \mu$.)

The positions of the maxima and minima for the *D*-line derived from this are as follows

i	1st min.	1st max.	1st min.	i	1st min.	1st max.	1st min.
36°59′	20°13′	21°59′	23°43′	44°0′	20°5′	21°55′	23°44′
37 55	11	55	39	45 0	5	59	
38 53	9 50″	53	35	46 0	6	22 3	
39 51	9 38	51	32	47 0	8	8	
40 55	10 22	50	30	48 0	9	14	
42 0	8	51	34	49 0	12	22	
43 0	6	53	39	50 0	16	30	

The peculiar movement of the inside minimum is due to the cooperation of the change of the minimum deviation and of the angle of diffraction of the first minimum.

The results show that for angles of incidence between 39° and 48° the inside minima do not deviate by more than 5′ from the smallest value, that the same is true for the outside minima between 39° and 42°, and finally for the central maximum from 38° to 44°. What was found for yellow, also holds mutatis mutandis for the other colours.

By the superposition of these maxima and minima the development of colour will be much promoted in a manner, which is impossible on the ordinary refraction-theory, and by the presence of the diffraction minima the resulting colour is completely modified. This is particularly true for the first minimum on both sides of the central maximum. The theory taken generally shows the possibility of the formation of diffraction rings on both sides of the central maximum; but it goes without saying that these circles have a better chance of becoming visible on the light-free inside of the halo than on the outside which is covered with non-minimal light.

The resulting colours and the intensity of the light in each direction will again have to be found by a calculation similar to the one applied above in the refraction theory.

The fundamental formula shows that the phenomenon depends on the width of the slit a, that is on the size of the crystals. A possible procedure would thus be to calculate the colour for a number of different values of a chosen at random and in this manner try to reproduce the various observations. We shall, however, confine ourselves to a special case in which the observations themselves give an indication as to the size of the crystals which were operative.

V. *The halos of May* 19 1899 *and of September* 19 1905.

On two occasions HISSINK at Zutfen observed very interesting halos which are described in "Onweders etc." as follows.

May 19 1899. "At 10.10 a.m. the small arc and the complete circumscribed halo became visible. For some time clouds prevented the observation, but when it cleared the circle became visible once more. At 11.52 a.m. an additional ring b, also circular, appeared, principally inside the upper half of the main ring and at 12.15 p.m. another circle c inside the former, whereas at 12.2 p.m. a further one d showed itself again nearer the sun. The two rings b and c were red on the side of the sun and showed round the red a greenish-yellowish tint, surrounded by violet. The small circle d had its outer edge coloured like the former, and its red on the side of sun was also similar, but the space on the inside of the circle was dark blue with a dull-brown hue."

By estimation HISSINK determined the radii at: $d = 7°.5$; $c = 17°.5$; $b = 19°.5$ (putting the ordinary circle at 22°).

Sept. 19. 1905. "The halo observed on this day at Zutfen was a very rare one. It included a the large circle, b the upper tangential arc, c the small circle, d a circle with a radius of about 19°30′, e a circle with a radius of about 18° and f the left parhelion.

As regards the colour of the various parts it should be principally mentioned, that the large circle was comparatively brightly coloured and that the violet of the tangential arc near the point of contact was particularly striking.

The circles d and e are the most interesting, the radii being determined by HISSINK by measuring the radius of the small circle with an octant, which gave 22°, and subsequently the distances between it and the arcs d and e. The latter were found to be 2°32′ and 4°2′, which would give 19°28′ and 17°58′ for the radii of these circles. Direct measurement of the radii gave 19°32′ and 18°2′

76

respectively. The means 19°30′ and 18°0′ must therefore have a comparatively high degree of certainty".

Similar circles have been observed on other occasions. BURNEY on June 9 1831 saw a ring of a radius of 20°[1]). HISSINK himself saw one on Sept. 5 1899 of a radius estimated at 19°. On the ordinary theory all such circles are explained by means of specially shaped crystals with refracting angles which produce a circle at the distance required. The following crystal-faces come into consideration[2]) for the above cases:

Refracting angle		D_0 (yellow)
50°28′	two pyramidal faces at the same end of the crystal	17°26′
53°50′	two pyramidal faces at opposite ends of the crystal	18°56′
54°44′	a base face with a pyramidal face at the other end or a prism face with a pyramidal face exactly opposite.	19°20′

The distances of the rings for yellow are then 2°30′, 2°54′ and 4°24′ respectively.

The first one agrees exactly with HISSINK's measurements, whereas the last is too large. The colours give difficulties which are not solved in this manner.

Starting from the supposition, that the rings of 18° and 19°30′ are nothing but secondary diffraction rings, I have made a calculation of the colours in the following manner.

PERNTER[3]) gives the positions of the maxima and minima for the diffraction through slit-shaped apertures in connection with the theory of coronae, as follows:

		position	intensity
1st maximum		0.0000	1.000000
	1st minimum	1	0
2nd maximum		1.4303	0.047191
	2nd minimum	2	0
3rd maximum		2.4590	0.016480.

Applying these results to HISSINK's measurements in 1905, where

$$a = 21°50' \qquad b = 19°30' \qquad c = 18°0'$$
$$a - b = 2°20' \qquad a - c = 3°50'$$

the angle of diffraction θ of the first minimum is found to be

1) PERNTER, l.c. page 266.
2) Onweders etc. 26 p. 83. 1905.
3) PERNTER, l. c. pag. 452.

as calculated from $a-b$ $\quad \frac{1}{1.43} \times 2.33 = 1.630° = 1°38'$

as calculated from $a-c$ $\quad \frac{1}{2.46} \times 3.83 = 1.557° = 1°34'$

the mean 1°36′ having an uncertainty of 2′. This agreement seems to support the underlying supposition as to the nature of the subsidiary circles.

	B	*C*	*D*	*E*	*x*	*F*	*y*	*G*
23° 0	17.84	77.5	224.2	169.7	171 4	191.3	302.1	160.4
22 45	41.10	179.0	561.7	408.2	377.7	422.8	605.9	308.2
30	71.90	315.5	990.8	691 0	606.9	680.6	887.4	422.7
15	105.3	457.0	1416	925.5	761.8	846.4	1003	435.4
0	135.3	573.0	1700	992.3	765.5	841.1	870.9	331.7
21 45	152.8	633.0	1743	926.7	617.3	665 5	575.9	212.6
30	154.2	614.0	1531	690.4	391.1	402.4	272.6	72.8
15	136.9	533.5	1140	407.9	181.3	174.7	74.60	15.31
0	109.6	349.0	700.5	167.9	52.4	46.5	16.62	10.43
20 45	72.95	248.5	329.7	41.18	10.53	13.10	30.13	17.64
30	42.65	124.0	103.9	9.92	20.57	25.78	41.67	16.23
15	19.40	44.50	20.88	26.97	33.30	37.13	29.87	7.19
0	5.56	9.15	30.88	44.16	29.25	29.33	10.92	2.27
19 45	0.91	6.35	67.90	38.71	14.14	12.32	4.94	3.95
30	2.33	17.60	79.71	19.27	4.21	3.70	11.11	6.39
15	5.19	27.10	61.30	5.68	5.05	6.62	14.79	5.16
0	7.09	27.40	28.85	4.56	10.17	12.26	10.34	2.01
18 45	6.64	19.30	8.20	11.37	10.83	11.97	3.82	0.93
30	4.28	9.10	6.60	15.51	7.60	6.37	2.46	2.36
15	1.97	2.45	17.35	12.36	2.91	2.00	5.67	3.44
0	0.56	1.85	26.32	5.71	1.85	2.51	6.40	2.50
17 45	0.42	4.90	25.50	1.72	4.13	5.48	4.83	0.99
30	1.23	8.80	15.98	1.76	6.03	6.42	1.71	0.44
15	2.14	9.75	6.01	3.03	5.28	4.34	1.75	1.06
0	2.25	6.95	1.35	8.40	2.60	1.54	3.65	1.87

Assuming, that the maxima arise under the influence of the strong yellow light, the width of the slit a is found to be

$$a = \frac{\lambda}{\sin\theta} = \frac{0.539}{\sin 1°36'} = 21.10\ \mu.$$

The side of the base-face thus becomes 0.279 mm. which appears to be quite a possible size [1]).

Using this value of a the colour-effect can now be derived in the following manner. The expression $\frac{a \sin\theta}{\lambda}$ is first calculated ascending by 15′ for the same eight colours as used before. SCHWERD [2]) has computed the intensity as a function of $\frac{a \sin\theta}{\lambda}$ from $\frac{a \sin\theta}{\lambda} = 0$ up to $\frac{a \sin\theta}{\lambda} = 6$. By means of a graphical representation of these results the intensities for all the above values of the expression can be arrived at and the results are then reduced to the relative intensities in sunlight. They are then drawn graphically for the eight colours taking as abscissae the distance from the sun, at which the fringes are formed, the central maximum being taken at the minimum deviation. The curves are further used as explained before with a view to the dimension of the sun.

The results are contained in the foregoing table. (See p. 1189).

The figures in the horizontal rows give the relative intensities for the directions contained in the first column.

These data are then reduced to colour-equations and finally the colour-numbers in the colour-triangles as well as the percentage of white are computed. Fig. 4 gives the change of the intensity.

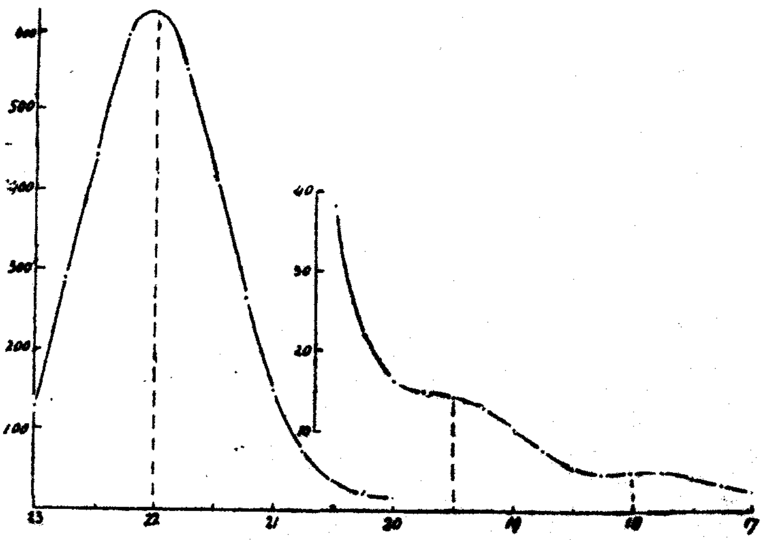

Fig. 4.

1) PERNTER l. c. pag. 287.

2) PERNTER l. c. pag. 453.

The results of the calculation will be found in the following table, where the symbols have the same meaning as in the corresponding table on the refraction-theory.

D	*L*	*W* %	*C* %	*G*	Colour	Observations
23° 0′	131	62	38	17.6	violet	violet N
22 45	290	68	32	17.3	blue—violet	
30	467	75	25	17.2	blue	blue N
15	595	86	14	16.0	blue	
0	621	62	38	4.3	yellow	
21 45	553	77	23	5.1	yellow	yellow N
30	414	56	44	4.7	yellow	
15	266	22	78	4.7	yellow	
0	150	17	83	2.8	red	
20 45	76	30	70	2.1	red	red N
30	38	54	46	29.3	red	
15	22	83	17	26.3	purple	violet Z
0	16.1	72	28	11.0	green-blue	
19 45	14.9	45	55	6.7	green-yellow	green-yellow Z
30	14.5	44	56	4.2	yellow	
15	13.1	59	41	1.6	red	red Z
0	10.3	73	27	29.9	red	violet Z
18 45	7.3	74	26	2.7	red	
30	5.4	68	32	6.3	green-yellow	
15	4.6	67	33	6.3	green-yellow	green-yellow Z
0	4.8	62	38	5.2	yellow	
17 45	4.8	68	32	2.8	red	
30	4.2	64	36	2.8	red	red Z
15	3.3	74	26	29.6	red	
0	2.9	73	27	6.8	green-yellow	

In the column under "Observations" Z refers to the circles observed at Zutfen by HISSINK in 1899, *N* to the colours of an upper tangential arc seen at Nymegen (N°. 13 of the Table) on

September 19 1905, the same afternoon as HISSINK's second observation. The application of the calculation to the upper tangential curve is allowable, at least in the neighbourhood of the point of contact, as the refraction takes place at that point in exactly the same manner as in the ordinary circle; the colours may thus be looked upon as belonging to the latter.

The agreement with the colours as observed at Nymegen and Zutfen is nearly complete: only the second violet is absent in the calculated set. The observer at Nymegen reports: red, yellow, blue (wide), violet (narrow). The calculation for 22°10′ and 22°5′ gives green-blue (colour-numbers 13.1 and 13.2) very near blue. Green is absent and blue has a width of 40′. The violet is nearly exhausted at 23°0′ and does not exceed a width of 15′. This agreement in the colours gives a strong support to the diffraction-theory as above developed.

A circle and a tangential curve without green are also reported from Boulogne sur Seine (N°. 9*a*).

As regards the agreement with HISSINK's circles: the colours given in the table are those observed on May 19 1899 and these need not be identical with those of 1905. Indeed, the characteristic feature of diffraction-rings is that their distance is variable, depending as it does on the dimensions of the refracting crystal. Perhaps the small remaining differences with the results of calculation may herein find their explanation. Intermediate calculations gave:

20°10′	*G* 13,2	green-blue
20°5′	12,7	green-blue
18°40′	4,2	yellow
18°35′	5,7	yellow

The intensities of the maxima are small and the maxima are but little prominent. They can only become visible by the differences in colour and only with a very high intensity of the main circle.

Professor VAN EVERDINGEN when asked for further information replied:

"that in the observations at Zutfen, both in 1899 and 1905, the colours of the small circle were described as *very bright*, as also those of the surrounding (circumscribed) halo or upper tangential curve".

It seems to me, that the above results render it extremely probable, that HISSINK's circles have to be taken as diffraction-rings; but in that case other similar rings must also arise by diffraction (compare the two cases mentioned on page 1188).

It is not impossible, that similar diffraction-rings may also occur outside the main circle. PERNTER mentions two observations of that kind [1]), but the data are too incomplete for a calculation to be based on them. In this connection the observation at Souppes (Nr 10 of the table) is important: in this case two concentric arcs are reported, the wider one of which is the inferior tangential arc. The other one may, as it seems to me, be looked upon as an external diffraction-ring of this arc.

As regards the main maximum, the theory gives it as lying at 22°0′ in complete agreement with the observations which give 21°50′ as the mean.

It is very probable, that by a calculation of the system of colours for other values of the width a the other observations may also be reproduced. In this connection the fact should be noted that in the various reports some combinations of colours occur repeatedly and will probably have to be ascribed to crystals of the same size. Some instances may be given here:

"Spectral colours":	Circle of 22°:	9 and 10
	Circle of 46°:	7 and 31
	Parhelion	14
Red, yellow, green, violet:	Circle of 22°:	3 and 6
	Upper tangential arc:	17*b*
Red, violet:	Circle of 22°:	2
	Upper tangential arc:	20 and 23
Red, blue:	Circle of 46°:	9*b*
	Circumzenithic arc:	1 and 30
Red, green:	Circle of 46°:	6*a* and 23
	Upper tangential arc:	21

The case of red, yellow, blue, violet (circle of 22°, 9*a* and upper tangential arc, 13) is dealt with above.

The very lengthy calculations which would be required for the further testing of the theory, have not been carried out so far and we shall confine ourselves to some general remarks.

1. As in the rainbow we have in the colours a means of determining the size of the refracting particles. In order to obtain say a well developed violet it is necessary that the maximum intensity of violet coincides about with the extinction of red and green. A very rough approximation to the dimensions in this case is arrived at as follows.

Supposing the colours B, C, D and E to have their first minimum

1) PERNTER, l.c. p. 260. GRESHOW's halo, Oct. 20 1747, radius 26°; and an observation by WHISTON, radius 29°.

in the direction of the central maximum of G, the corresponding angles of diffraction are (see table on page 1182)

$$B\ 44' \quad C\ 40' \quad D\ 31' \quad E\ 21'$$

respectively and the widths of the slit

$$B\ 53.7\,\mu, \quad C\ 56.4\,\mu, \quad D\ 65.2\,\mu, \quad 86.1\,\mu.$$

The mean width $65.4\,\mu$ may be looked upon as giving a close approximation to the correct value. This gives .075 mm for the width of the side-face of the prism and .15 mm for the diagonal of the base.

Very small crystals will produce very broad maxima, in comparison to which 44′ — the difference between the maxima of red and violet — may be looked upon as small, in consequence of which the various colours will cover each other and nothing will be seen but white with a red inner edge.

2. It may be useful to point out the analogy with the rain-bow. In that case large drops give narrow diffraction-maxima and distinct colours, small drops broad maxima, diluted colours and the rare white rainbow. Similarly with the halo: the larger the ice-crystals, the more distinct the spectral colours will be. The "white halos" are by far the most common.

Still there are some very fundamental differences between rainbow and halo. Whereas in the former case the wave-front becomes curved, it remains flat in the latter case. Whereas in the rain-bow the maxima are strongly developed, though only on one side by which the extremely common secondary bows on the side of the violet are formed, these maxima are comparatively weak in the halo and possible on both sides. They will have the best chance of being seen in the dark region inside the red, but in the white on the outside they will but seldom succeed in making themselves visible.

3. In connection with the colours of halos the shape of the crystal is of some importance. Let us consider a crystal plate with a broad side-face but of small height. The width of the side-face determines the width of the slit which plays a part in the formation of the ordinary ring, the height determines the width of the slit for the circle of 46°, as this halo is formed by a refracting angle of 90°. A plate of the above shape is specially suited to the production of colours in the ordinary circle, but unsuitable as regards the large circle. With an elongated prism the colour-production in the circle of 22° is again dependent on the width of the side face, but for the circle of 46° the determining dimension is now the short diagonal

of the base which is $\sqrt{3}$ times or 1.7 times longer than the width of the side-face. A crystal of that kind is therefore more suited to the production of colour in the circle of 46° than in that of 22°. And as a matter of fact a number of halos enumerated on pages 1176 to 1178 show striking differences in the degree of colouring in the two circles: 9*b* belongs to the former kind, 7, 12, 27 and 31 to the latter.

4. A further important conclusion seems to me justified, although I have not tested it in detail. As we have found (p. 1185), in the neighbourhood of the minimum deviation we can turn the incident beam or, what comes to the same, the crystal over a comparatively large angle before its having any influence on the diffraction-fringes. But if that is true, the difficulty disappears which lies in the necessity of having to assume a constant, vertical axis in the usual explanation say of the circum-zenithic arc. [1])

The "strikingly pure colours", the "pure violet" of which BESSON speaks, are a consequence of diffraction, but not of a constant direction of the refracting edge.

5. We also found (page 1185) that in the external minimum a much smaller variation was admissible. The same will hold with regard to the next maximum: another ground, therefore, to expect, that diffraction-rings outside the main circle will be very great exceptions.

6. In the large circle of 46° the difference in the minimum for red and violet is 2°6′ [2]): the spectrum is thus spread out over an angle three times as wide as in the circle of 22°. But the slit becomes smaller in the ratio $\frac{\cos i_{46^0}}{\cos i_{22^0}} = \frac{\cos 67°51'}{\cos 40°55'} = \frac{1}{2}$. The colour-effect is thus enhanced in the ratio $^3/_2$. With a favourable shape of the crystal, the effect may be increased another 1.7 times and the conditions so become $2\,^1/_2$ times more favourable as regards production of colour in the circle of 46°. This agrees with the fact, that in this circle striking colours have been seen comparatively frequently.

7. In the formation of halos where the light no longer passes the crystal at right angles to the refracting edge, which corresponds to a broadening of the beam, the diffraction pattern agrees with that of a larger crystal with the light moving in a plane at right angles to the refracting edge. The chance of colour is increased. In agreement with this the tangential curves to the circles of 22° and of 46° (circum-zenithic curve) are pretty frequently distinctly coloured.

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[1]) See a.o. L. BESSON, l c.

[2]) PERNTER, l. c. p. 354.

8. The diffraction tells us something of the size of the crystals and by this means possibly of the temperature at which they have been formed: "with falling temperature the size of the crystals diminishes" [1]). In that way the halo-colours, which have been too much neglected, may possibly contribute to a better knowledge of the higher atmosphere.

VI. Conclusions.

The above investigation seems to me to justify the following conclusions:

1. The simple refraction-theory cannot explain the halo-phenomena completely, in particular as regards the great variety of the colours.
2. The diffraction-theory gives a simple explanation of the colours which appear and allows special conclusions to be drawn regarding the influence of the size and the shape of the crystals. It alone gives the ordinary circle its correct place of 22°.
3. The rings which have been observed in the neighbourhood of 22° are secondary diffraction-rings: their radii are not constant.
4. The diffraction-theory will probably be able to afford a better insight into the formation of the circumzenithic arc.
5. It is necessary that the colours be accurately recorded by each observer in order to permit a further testing of the theory and a complete deduction of the origin of the observed phenomenon.

Chemistry. — *"In-, mono- and divariant equilibria"*. XVI. By Prof. F. A. H. SCHREINEMAKERS.

(Communicated in the meeting of March 31, 1917).

The regions in the P, T-diagram.

In communication VIII we have already briefly discussed those regions; now we shall consider them more in detail. When the equilibrium

$$E = F_1 + F_2 + \ldots + F_n \quad \ldots \ldots \quad (1)$$

consists of n components, then it is generally divariant; consequently it is generally represented in the P, T-diagram by a region. We shall consider this region E in its whole extensity, viz. without taking into consideration that some parts may become metastable by the occurrence of other phases.

With a definite equilibrium E we may distinguish:

[1]) PERNTER, l. c. p. 289.