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Meteorology. — *"On the relation between meteorological conditions in the Netherlands and some circumjacent places. Atmospheric Pressure."* By Dr. J. P. VAN DER STOK.

(Communicated in the meeting of May 29, 1915).

1. For the knowledge of the climate of a country as also for the forecasting of the weather, it is of importance to investigate in how far a relation exists between the meteorological conditions within a limited region and in circumjacent places, chosen for this purpose, and to what degree local influences are felt.

Statistical methods, leading to empirical, numerical relations, involve the objection that many peculiarities, especially secondary phenomena, disappear by the collective treatment, but by their means existing relations may become more prominent, which necessarily remain unobserved by those who, for many years, have made a special study of the individual phenomena and, if no new relations are brought to light, quantitative rules are substituted for qualitative knowledge. As the most simple and principal problem, the question will be examined, what relation exists between the oscillations of the atmospheric pressure at de Bilt and the oscillations at a few surrounding places.

The isobars for different months and the corresponding average values of the wind show that this relation can hardly be the same in different seasons. We come to the same conclusion by investigating the relation existing between barometric oscillations within the region of high pressure near the Azores and of low pressure near Iceland, by which the climate of Western Europe is considerably affected.

Each factor indicates that the observations made during the months of January, February, and December are the fittest material for this inquiry which, therefore, is restricted to the wintermonths.

2. The method followed is simple, but necessarily laborious.

If the deviations from the average barometric height at a central point and the circumjacent stations be denoted by $x_1, x_2 \ldots x_n$, then, the quantities under consideration being small, a linear relation may be assumed to exist

$$x_1 = b_{12}x_2 + b_{13}x_3 \;\ldots\; b_{1n}x_n = F_1 , \quad . \quad . \quad . \quad . \quad (1)$$

and the coefficients b can be calculated by means of the method of least squares from the $n-1$ equations formed by multiplying the equations (1) successively by $x_2, x_3 \ldots x_n$ and addition of the total number of equations.

Putting

$$\frac{\Sigma x_i^2}{n} = \sigma_i^2 \qquad r_{pq} = \frac{\Sigma x_p x_q}{n\sigma_p\sigma_q} \quad . \; . \; . \; . \; . \quad (2)$$

where n denotes the number of equations, σ the standard deviation and r_{pq} the correlation-coefficient (c.c.) between x_p and x_q, the n—1 equations deduced from (1) can be substituted by the equivalent set of equations:

$$\left.\begin{array}{l} r_{12} = a_2 + a_3 r_{23} + a_4 r_{24} + \ldots a_n r_{2n} \\ r_{13} = a_2 r_{23} + a_3 + a_4 r_{34} + \ldots a_n r_{3n} \\ \ldots\ldots\ldots\ldots\ldots \\ r_{1n} = a_2 r_{2n} + a_3 r_{3n} + a_4 r_{4n} + \ldots a_n \end{array}\right\} \quad . \; . \; . \; . \quad (3)$$

By the quantities a thus calculated, the quantities b become

$$b_{12} = \frac{\sigma_1}{\sigma_2} a_2 \quad , \quad b_{13} = \frac{\sigma_1}{\sigma_3} a_3 \ldots b_{1n} = \frac{\sigma_1}{\sigma_n} a_n \, .$$

Obviously the equation (1) holds good only to a limited degree because the data are necessarily incomplete; a measure of the completeness is obtained by putting

$$R_1 x_1 = F_1$$

from which

$$R_1^2 = \frac{\Sigma F_1^2}{\Sigma x_1^2}$$

or, by substitution of the values (2):

$$\begin{aligned} R_1^2 = {} & a_2^2 + a_3^2 + a_4^2 \ldots a_n^2 \\ & + 2a_2a_3r_{23} + 2a_2a_4r_{24} \ldots 2a_2a_nr_{2n} \\ & + 2a_3a_4r_{34} + 2a_3a_5r_{35} \ldots 2a_3a_nr_{3n} \\ & \ldots\ldots\ldots\ldots\ldots \\ & + 2a_{n-1}a_nr_{n-1\;n} \end{aligned}$$

The quantity R represents the general c.c. of equations (1) and the probable error of one determination of x_1 becomes

$$w = \alpha\sigma_1 \sqrt{1 - R^2} \qquad \alpha = 0.67449$$

The partial c.c., defined as the c.c. between x_p and x_q when all other values x are zero, is calculated by solving also the equations

$$x_2 = F_2 \quad , \quad x_3 = F_3 \ldots x_n = F_n$$

and is given by the expression

$$\varrho_{pq} = \sqrt{b_{pq}\, b_{qp}} \, .$$

the sign of ϱ being that of the quantities b.

For the probable error of the c.c., PEARSON gives the formula

$$f = {}^2/_3 \, \frac{1-r^2}{\sqrt{n}} \, .$$

It holds good for the case of normal distribution of deviations and the c.c. is considered to be reliable when f is considerably smaller than the c.c. itself; in the following tables

$$q = {}^1/_f.$$

3. The monthly mean values of barometric height in Iceland and the region of the Azores are compiled from Danish and Portuguese annals for the 36 years 1875—1910.

The Iceland values are obtained by taking the average of three stations namely: Berufjord, Grimsey and Stykkisholm.

From the Portuguese observations average values were calculated for two stations: Punta Delgada (Azores) and Funchal (Madeira); for the years 1906—1910 Horta was substituted for Funchal.

The monthly means thus obtained and considered as normal values, are shown in Table I; they are uncorrected for height above sealevel, this correction being unnecessary for the calculation of deviations, and given only to show the correspondence existing between the annual variation of the differences of pressure and the c.c. of table II.

TABLE I.

Monthly means of atmospheric pressure 1875—1910, 700 mm. +

	Azores	Iceland	Δ		Azores	Iceland	Δ
January	65.0	48.3	+16.7	July	65.7	56.4	+ 9.3
February	64.2	50.6	13.6	August	64.4	56.0	8.4
March	63.3	53.0	10.3	September	63.9	53.6	10.3
April	63.6	56.6	7.0	October	62.4	53.8	8 6
May	63.7	59.3	4.4	November	63.0	52.5	10.5
June	65.3	57.7	7.6	December	64.3	48.5	15.8

It appears from these data that the differences of atmospheric pressure are greatest in the winter months and smallest in May. Table II shows the results of the calculation relating to the deviations from the normal values of table I.

TABLE II.
Standard deviations and correlation coefficients Iceland = 1, Azores = 2.

	σ_1	σ_2	r	q	b_{12}	b_{21}
January	6.31 mm.	2.86 mm.	— 0.527	6.5	— 1.164	— 0.239
February	7.00	3.97	— 0.595	8.2	— 1.048	— 0.337
March	5.30	3.07	— 0.620	9.0	— 1.071	— 0.359
April	3.83	2.24	— 0.484	5.6	— 0.827	— 0.283
May	2.96	1.51	— 0.365	3.7	— 0.717	— 0.186
June	3.32	1.39	— 0.396	4.2	— 0.946	— 0.166
July	2.64	1.25	— 0.345	3.5	— 0.727	— 0.164
August	3.01	1.21	— 0.376	3.9	— 0.933	— 0.152
September	3.56	1.18	— 0.485	5.7	— 1.459	— 0.162
October	4.36	2.31	— 0.469	5.3	— 0.885	— 0.249
November	5.52	2.87	— 0.421	4.5	— 0.810	— 0.219
December	5.04	2.97	— 0.541	6.8	— 0.919	— 0.318

These results show, with a certainty much greater than can be obtained by graphic representations that the antagonism between the barometric oscillations in the region of the Azores and the northern parts of the Atlantic Ocean is evident in every month. From the regular course of the values of r, in the summer months as well as in winter, the conclusion may be drawn that a value of $q = 3.5$ indicates a reliable result, for, if the four months: May—August were taken together, the same value $r = 0.37$ would be obtained, but now with a factor of accuracy twice as great, or $q = 7.5$.

In his extensive investigation of correlations between monthly oscillations of atmospheric pressure and temperature at 49 stations in the northern hemisphere during the three winter months of the years 1897—1906, Exner [1]) gives the value $r = -0.479$ ($q = 5.0$) for the c.c. between Stykkisholm and Punta Delgada which correspond well with the data of table II, and the fact that, by using a number of observations four times as great, a *greater* value is found may be considered as proof of the reliability of the results obtained.

4. For an investigation of the relation between oscillations of atmospheric pressure at different places, the "Dekadenbericht" edited

[1]) F. M. Exner, Ueber monatliche Witterungsanomalien auf der nördlichen Erdhälfte im Winter. Sitz. Ber. Akad. d. W. Wien **122**, 1913 (1105—1240).

by the "Deutsche Seewarte" contains valuable data: commencing in 1900, this publication gives ten-day means of barometric heights, in such a way that three average values are always formed for each month. At the same time normal values are given so that deviations from the normals can be formed at once for the purpose of further treatment. In accordance with the results of Table II, the inquiry is restricted to the winter months from December 1900 to February 1914 as being the most disturbed; the number of observations therefore amounts to 126.

From the stations in this publication the following places were chosen, in the equations represented by their rank-number; the values σ are the standard deviations.

1. Helder $\sigma_1 = 6.96$ mm.
2. Valencia (W. coast Ireland) $\sigma_2 = 8.70$ „
3. Clermont (S. France) $\sigma_3 = 5.98$ „
4. Milan (N. Italy) $\sigma_4 = 5.82$ „
5. Neufahrwasser (Baltic Sea coast, Prussia) $\sigma_5 = 6.30$ „
6. Christiansund (W. coast Norway) $\sigma_6 = 8.45$ „

TABLE III.

Correlation-coefficiënts r, factors of precision q and distances D

Helder—Valencia	$r_{12} = 0.770$	$q = 30.8$	$D =$ 9°.2
Helder—Clermont	$r_{13} = 0.727$	25.7	7°.25
Helder—Milan	$r_{14} = 0.511$	11.5	8°.0
Helder—Neufahrwasser . .	$r_{15} = 0.633$	17.6	8°.35
Helder—Christiansund . .	$r_{16} = 0.609$	16.1	10°.3
Valencia—Clermont . . .	$r_{23} = 0.704$	23.2	10°.7
Valencia—Milan	$r_{24} = 0.380$	7.4	14°.3
Valencia—Neufahrwasser .	$r_{25} = 0.247$	4.4	17°.4
Valencia—Christiansund. .	$r_{26} = 0.310$	5.7	14°.7
Clermont—Milan	$r_{34} = 0.645$	18.4	4°.2
Clermont—Neufahrwasser .	$r_{35} = 0.246$	4.4	13°.15
Clermont—Christiansund .	$r_{36} = 0.058$	1.0	17°.5
Milan—Neufahrwasser . .	$r_{45} = 0.370$	7.1	10°.8
Milan—Christiansund . .	$r_{46} = 0.095$	1.6	17°.7
Neufahrw.—Christiansund .	$r_{56} = 0.746$	28.0	10°.4

In Table III (p. 314) the different correlation coefficients are given and the distances between the stations expressed in degrees of the great circle corresponding to about 111 k.m.

For ascertaining meteorological conditions, the regression-equations (preferably called meteorological condition equations) are of greater importance than these general, interdependent correlation coefficients.

$$\left.\begin{array}{ll} & R \\ x_1 = 0.238\,x_2 + 0.520\,x_3 + 0.011\,x_4 + 0\;201\,x_5 + 0.292\,x_6 & 0.943 \\ x_2 = 0.928\,x_1 + 0.416\,x_3 - 0.096\,x_4 - 0.485\,x_5 - 0.112\,x_6 & 0.830 \\ x_3 = 0\;680\,x_1 + 0.109\,x_2 + 0.242\,x_4 - 0.026\,x_5 - 0.336\,x_6 & 0.908 \\ x_4 = 0.038\,x_1 - 0.076\,x_2 + 0\;594\,x_3 + 0.353\,x_5 - 0.150\,x_6 & 0.672 \\ x_5 = 0.457\,x_1 - 0.259\,x_2 - 0.054\,x_3 + 0\;250\,x_4 + 0.396\,x_6 & 0.843 \\ x_6 = 0.929\,x_1 + 0.063\,x_2 - 0.822\,x_3 - 0.150\,x_4 + 0.573\,x_5 & 0\;873 \end{array}\right\} \quad (4)$$

The partial c.c. calculated from the coefficients of these equations are given in Table IV, arranged according to their magnitude.

TABLE IV. Partial correlation-coefficients.

Helder—Clermont	0.594	Valencia—Christiansund. .	0.084
Helder—Christiansund . .	0.521	Helder—Milan	0.020
Neufahrw. - Christiansund .	0.476	Clermont - Neufahrwasser .	− 0.037
Helder—Valencia	0.470	Valencia—Milan	—0.085
Milan—Clermont	0.379	Milan—Christansund . .	—0.150
Helder—Neufahrwasser . .	0.303	Valencia—Neufahrwasser .	− 0.355
Milan - Neufahrwasser . .	0.297	Clermont—Christiansund .	—0.526
Valencia—Clermont . . .	0.213		

From these results it appears that the choice of the stations was good, except Milan which, although at about the same distance from Helder as Clermont, still exercises a much smaller influence.

Clermont and Milan being at a mutual distance of only 4°.2, it is possible that this result is due to purely arithmetical reasons; the method followed involves that two stations near to each other must be considered as one, because it depends on incalculable factors how the common effect is distributed over either point, this being of no importance for the result.

If this were the case, however, the partial c.c. between Clermont and Milan ought to be nearly equal to unity, which is contradicted by the c.c.: 0.379.

It appears, therefore, that Milan is situated out of the circle of influence, which from a meteorological point of view is perfectly clear because here the influence of the Alpine montain chains and the Mediterranean prevails, the equations (4) are, therefore, actually based upon only four points, situated round Helder and the first equation proves that these are sufficient to account for the barometric oscillations in the central point to an extent of 94 %.

As it may be assumed that this percentage would increase by augmenting the number of stations, it appears from this equation that local disturbances have only a subordinate influence. Whether this statement is also applicable to the summer months can only be proved by experiment.

Another result is that the meteorological field cannot be considered as uniform in different directions, the influence of Clermont being twice as great as that of Valencia at a slightly greater distance from Helder.

It may be, further, remarked that the central point, without exception, plays a more important part in the equations for the surrounding stations than, inversely, the latter for Helder; which is easily understood because the central point represents the meteorological conditions common to the whole field of disturbance. In the partial c.c. this asymmetry disappears and for these quantities the question arises whether and to what degree the relations are dependent on the distance.

Assuming that this relation can be taken as linear so that

$$\varrho = 1 - kD$$

where D denotes the distance, expressed in degrees and k a constant, we find for Valencia, Clermont and Christiansund for k respectively:

0.0576 0.0560 0.0465

for Neufahrwasser the somewhat different value: 0.0834.

According to this relation the partial c.c. at equal distances of 5° would be

$$\varrho_{12} = 0.711 \qquad \varrho_{13} = 0.720 \qquad \varrho_{15} = 0.583 \qquad \varrho_{16} = 0.767.$$

Finally the remarkable fact may be noticed that the same negative correlation, observed between the region of the Azores and Iceland at a distance of about 35°, appears to exist, and with the same magnitude, between the stations Clermont and Christiansund at about half the distance.

5. In order to come to a conclusion concerning the results obtained,

it seemed desirable to institute a similar inquiry based upon other data and partly other stations.

For this purpose daily observations made at 7 a. m. as published in different weather bulletins and inscribed in registers at de Bilt, were chosen.

A first group of stations is: 1. de Bilt, 2. Ile d'Aix (W. coast France), 3. Dresden, 4. Lerwick (Shetland Isles). The distances between de Bilt and the surrounding stations are:

$$7^\circ.38 \quad 5^\circ.42 \ , \ 8^\circ.80 \ ,$$

the azimuths:

$$N 217^\circ 11' E \ , \ N\,97^\circ 44' E \ , \ N 338^\circ 59' E \ ,$$

the mutual angular distance, therefore, about 120°.

The data are observations made during the winter months of January, February, December 1912, January, February, December 1913 and January, February 1914, in total 240 observations.

The standard deviations are:

$$\sigma_1 = 8.25 \ , \ \sigma_2 = 7.79 \ , \ \sigma_3 = 7.96 \ , \ \sigma_4 = 10.72 \ mm.$$

The correlation coefficients:

$$r_{12} = 0.709 \quad , \quad r_{13} = 0.868 \quad , \quad r_{14} = 0.579$$
$$r_{23} = 0\ 532 \quad , \quad r_{24} = 0.1475 \ , \quad r_{34} = 0.402$$

The criterion $q = {}^r/_f$ for the reliability of the c.c. calculated, mentioned above, cannot be applied in this case (as it was for ten day and monthly means) because daily observations are by no means to be considered as independent data.

The condition-equations calculated from these values are as follows:

$$\left.\begin{array}{ll} x_1 = 0.395\,x_2 + 0.568\,x_3 + 0.234\,x_4 & R_1 = 0\ 957 \\ x_2 = 1.370\,x_1 - 0.525\,x_3 - 0.346\,x_4 & R_2 = 0.821 \\ x_3 = 1.207\,x_1 - 0.321\,x_2 - 0\ 205\,x_4 & R_3 = 0.905 \\ x_4 = 2.042\,x_1 - 0.873\,x_2 - 0.842\,x_3 & R_4 = 0.751 \end{array}\right\} \quad . \ . \ (5)$$

The partial c.c., the mutual distances, the variation k of the partial c.c. per degree of distance and the partial c.c. for equal distances of 5° from the centre are:

		$k = 5$
$\varrho_{12} = 0.735$	$k_{12} = 0.0358$	$\varrho_{12} = 0.821$
$\varrho_{13} = 0.828$	$k_{13} = 0.0318$	$\varrho_{13} = 0\ 841$
$\varrho_{14} = 0\ 691$	$k_{14} = 0.0351$	$\varrho_{14} = 0.824$
	Mean 0.0342	Mean 0.829

$$\varrho_{23} = -\ 0.411 \qquad D_{23} = 11^\circ.10$$
$$\varrho_{24} = \ \ \ 0.550 \qquad D_{24} = 14^\circ.12$$
$$\varrho_{34} = -\ 0\ 415 \qquad D_{34} = 12^\circ.50$$

6. A second set of four stations is.

1. De Bilt, 2. Valencia, 3. Mulhausen i. E. and 4. Sylt (W. coast Schleswig Holstein).

The distances from de Bilt to the surrounding stations are respectively:

$$9^\circ.48, \qquad 4^\circ.57, \qquad 3^\circ.39$$

the azimuths:

$$N\ 32^\circ\ 40'\ E. \qquad N\ 161^\circ\ 32'\ E, \qquad N\ 275^\circ\ 13'\ E.$$

For these places the angular distance is likewise about 120°, and they differ 60° with the stations mentioned sub 5.

The standard deviations are

$$\sigma_1 = 8.25,\ \sigma_2 = 10.82,\ \sigma_3 = 7.30,\ \sigma_4 = 8.96 \text{ mm.}$$

The correlation coefficients:

$$r_{12} = 0.633\ ,\ r_{13} = 0.818\ ,\ r_{14} = 0.864$$
$$r_{23} = 0.480\ ,\ r_{24} = 0.433\ ,\ r_{34} = 0.528$$

from which the following condition-equations derive:

$$\left.\begin{array}{ll} x_1 = 0.140\, x_2 + 0.494\, x_3 + 0.510\, x_4 & R_1 = 0\ 976 \\ x_2 = 2.417\, x_1 - 0.852\, x_3 - 1.034\, x_4 & R_2 = 0.722 \\ x_3 = 1.457\, x_1 - 0.146\, x_2 - 0.653\, x_4 & R_3 = 0.905 \\ x_4 = 1.595\, x_1 - 0.188\, x_2 - 0.693\, x_3 & R_4 = 0.934 \end{array}\right\} \quad . (6)$$

For the partial c.c., the distances not yet mentioned, the variation k for one degree distance and the c.c. for equal distances of 5°, we find:

		$k = 5$
$\varrho_{12} = 0.583$	$k_{12} = 0\ 0441$	$\varrho_{12} = 0.780$
$\varrho_{13} = 0.848$	$k_{13} = 0.0332$	$\varrho_{13} = 0.834$
$\varrho_{14} = 0.902$	$k_{14} = 0.0290$	$\varrho_{14} = 0.855$
	Mean 0.0354	Mean 0.823

$\varrho_{23} = -\ 0.352$	$D_{23} = 12^\circ\ 03$
$\varrho_{24} = -\ 0.440$	$D_{24} = 11^\circ.45$
$\varrho_{34} = -\ 0.672$	$D_{34} = 7^\circ.17$

Either group proves that barometric oscillations in a central point may be determined with great accuracy from only three well chosen stations; the condition-equations for de Bilt (x_1) show even a greater value of R than the corresponding equations (4) and the equations for the three easterly stations: Dresden, Mulhausen and Sylt all show a value greater than 0.9. As one would perhaps be inclined to overrate the value of such a c.c. for an actual calculation, it seems not superfluous to remark that if — as in this case — the standard deviation is relatively great, a large value of c.c. may leave a pretty large margin of uncertainty. According to the formula given in § 2 the probable errors of a determination from (5) and (6) for de Bilt with $R = 0.957$ and 0.976 resp. are 1.62 and 1.21 m.m.; they prove however, as well as equ. (4) that local influences play an unimportant part.

In the same manner as from (4), it appears from (5) and (6) that the influence of the eastern stations Mulhausen, Dresden and Sylt is considerably greater than that of the western stations: Valencia, Ile d'Aix and Lerwick.

For the partial c.c. between Helder and Valencia we have found 0.470 (Table IV) whereas for that between de Bilt and Valencia, as deduced from (6), we find 0.583, an agreement which can be considered fairly satisfactory if we take into account that the data used in computing these values are totally different.

As mentioned in § 3, for the first series general normal values have been used, given in the "Berichte" so that it is possible that in this case the sum of the deviations for each station is not exactly equal to zero which, of course, would influence the value of the c.c.

It is, however, more probable that the cause of this disagreement must be ascribed to an insufficiency of the number of observations used in § 5 and § 6, because the values of k found in the first investigation (§ 4) are all greater than those derived from the groups treated in § 5 and § 6, from which a generally smaller value of the c.c. would follow. Owing to the mutual dependence a number of 240 daily observations cannot be considered as equivalent to 126 tenday means and it is a general law in statistical investigations that the computed relations show a tendency to give smaller limiting values as the data increase in number.

7. Finally the question may be put, what will the condition equation become when the two groups of three surrounding stations are taken together so that the deviation of atmospheric pressure in the central point is determined by 6 circumjacent stations within angular distance of about 60°.

The numeration of the stations then becomes:

1.	de Bilt	5.	Dresden
2.	Valencia	6.	Sylt
3.	Ile d'Aix	7.	Lerwick
4.	Mülhausen		

The c.c. computed in § 5 and § 6 and all products can be used for this purpose so that the labour entailed for this calculation was relatively small.

The values not yet given are:

$r_{23} = 0.670$ $r_{36} = 0.359$ $r_{67} = 0.744$

$r_{25} = 0.360$ $r_{45} = 0.781$

$r_{27} = 0.543$ $r_{47} = 0\,134$

$r_{34} = 0.888$ $r_{56} = 0.848$

And the condition equation becomes:

$$x_1 = 0.140\ x_2 - 0.069\ x_3 + 0.624\ x_4 - 0.101\ x_5 + \\ + 0.538\ x_6 + 0.015\ x_7 \quad \ldots \ldots \quad (7)$$

It appears from (7) that the methods of computation followed in this inquiry fails in this case in so far that, owing to the insufficient distances between successive stations, negative coefficients now appear in the equations. Obviously they are due to a mutual distribution of common influence which must be considered as unreal and as a mere arithmetical result.

Equation (7), therefore, shows a great resemblance to the first of he equations (6); the coefficients are alternatively small or even negative and if we reduce the equation to one with three terms by an equal distribution of the odd over the even coefficients so that for example:

$$coeff.\ x_2 = 0.624 - \frac{0.069 + 0\,101}{2} = 0.539,$$

we find the following equation little different from (6)

$$x_1 = 0.113\ x_2 + 0\,539\ x_3 + 0.495\ x_4$$

In equation (7) the prevailing influence of the stations Mulhausen and Sylt is still more conspicuous than in the results of other groups.

A calculation of the remaining equation and of partial c.c. would in this case have no meaning.

Taken as a whole equation (7) is to be considered as an improvement because the general correlation-coefficient is very large namely

$$R = 0.9953$$

from which follows, for the calculation of one value, the probable error:

$$w = 0.539\ mm.$$