Citation:

P. Zeeman, Fresnel's coefficient for light of different colours. (Second part), in:
KNAW, Proceedings, 18 I, 1915, Amsterdam, 1915, pp. 398-408

Magnetic rotatory power.

Name of element	Observed by	
	PERKIN	HUMBURG
H	0.254	—
C (in Ketones)	0.850	—
O (in OH)	0.191	—
Br	3.562	3.563
Cl	1.734	1.675
I	7.757	—
N	0.717	—
Na	0.558	—
K	1.535	—
Li	1.124	—
Ca	2.143	—
Mg	2.029	—

Distinct periodic curves are obtained. The halogen elements occupy the topmost points.

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Physics. — "FRESNEL's *coefficient for light of different colours.*" (Second part). By Prof. P. ZEEMAN.

(Communicated in the meeting of May 29, 1915)

A first series of experiments was made with yellow, green, and violet (4358) mercury light. As FRESNEL's coefficient changes only slowly with the wavelength, such a high homogeneity of the incident light is unnecessary. With regard to the intensity of the light it is even recommendable to work with a limited part of a continuous spectrum. In a second series of experiments I therefore analysed the light of an electric arc (12 Amp.) with a spectroscope of constant deviation, which I had arranged as a monochromator by taking away the eye-piece and replacing it by a slit. The monochromator had been calibrated with mercury and helium lines. The prism stood on a table, which could be turned by means of a screw. Each reading on the scale attached to this screw gave the *mean* wavelength of the light used with an accuracy of a few ÅNGSTROM-units. By repeating the calibration during the experiments it was proved, that this mean wavelength could always be reproduced with the above mentioned accuracy. This now is more than sufficient, as for instance in the green part of the spectrum a change of $\lambda = 5400$ into $\lambda = 5500$ and at the greatest possible velocity of the water the shift of the interference fringes becomes 0.660 instead of 0 675 of the distance between two fringes. Even a change of 10 Å.U. in the wavelength of the light used corresponds to 0,0015 only of the distance between the interference fringes, while the probable error of the final result is of the order of magnitude of 0.005.

In order to determine the place of the interference fringes I used two or rather three different methods and in a few experiments only eye observations were made. In one series of experiments a wire-net, which could be turned and shifted was adjusted in the focal plane *s* (see Fig. 1)[1]. In the focal plane of the telescope *f* we took photos of the interference fringes, while care was taken that one wire was parallel to the fringes and that the other passed through the middle of the field.

An advantage of this method is, that the interchanging of the photographic plates in the focal plane of *f* does not disturb the relative position of the interference fringes and the wires. With this method however it is rather difficult to adjust the wire-net accurately as it is so far away from the observer. Moreover the net must be very fine because of the strong magnifying power of the telescope. On the proposal of Prof. WOOD I used in a second method ROWLAND'S artifice[2]) for the comparison of spectra. ROWLAND puts in front of the photographic plate a brass plate with longitudinal aperture of the same width as the thickness of the plate, which could turn round a horizontal axis in front of the photographic plate. The rotation could easily be limited to an angle of 90°. By means of two fine quartz wires adjusted perpendicular to the plane of the brass plate the position of the plate could be measured accurately and corrected if necessary.

Two photos taken by this method are reproduced in the Plate (Fig. 4 and 5). The outer system of interference fringes has been obtained while the water was streaming in one direction; the inner system corresponds to a current in the opposite direction.

Fig. 5 shows also the shadow of the fine quartz wires.

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1) See the first part.

2) AMES, Phil Mag. (5) **27**, 369. 1889.

Though this method gives a clear survey of the shift of the interference fringes and e. g. shows immediately, that the shift for red light (Fig. 4) differs from that for violet (Fig. 5), it is not very fit to obtain quantitative results. By a detailed investigation I found, that the uncertainty of the measurements was greater than I had expected from eye observations. A disadvantage of this method is first, that for the measurement of the negative we must once point on an interference fringe and then on the two pieces of a broken fringe. For spectral lines this does not matter much, but the difficulty becomes greater for the more hazy interference fringes. It is however an essential disadvantage of this method that pointings cannot be made on corresponding points of the interference fringes.

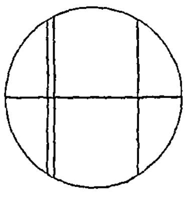


Fig. 4.

Quite satisfying results I got with the third method, concerning which I shall give some details. In the focal plane of the telescope a system of wires as is shown in fig. 4 was adjusted. There are three vertical wires (and one horizontal wire), so that we can always choose the best one as a fixed mark and read along the horizontal wire. It is very improbable that the three wires are all badly situated with respect to the interference fringes. Just behind the cross wires the photographic plate is adjusted on *a plate-holder which is mounted independently of the telescope with the cross wires.* The photographic plate can be brought in the right position and slidden to take successive photos without touching the telescope. Examples of the obtained photos are reproduced on the Plate (Fig $1a$—$3b$), 4 or 5 times enlarged. The photos $1a$ and $1b$, $2a$ and $2b$ etc. belong together. Comparing two such photos the shift of the interference fringes is evident. The displacement is also given on the Plate in parts of the distance between two fringes. As mentioned above the measurement was made along the horizontal wire.

The width of the interference fringes can be chosen according to the circumstances. p gives the pressure of the water in kilograms per cm^2, measured during the streaming of the water with a manometer coupled to the main tube, just before it divides into two less wide ones. The times of exposition for the making of the negatives amounted between 3 and 5 minutes. It therefore sufficed to read the pressure of the water each 30 seconds. The mean of these readings was taken as the pressure during the measurement. The variations in the pressure most times amounted only to some hundredth parts of a kilogram. If by accident (what happened very seldom) the variation in pressure was greater, the corresponding measurement was not used.

If $2l$ is the length of the whole water-column that is in motion, the double shift to be expected is

$$\frac{8l\left(1-\frac{1}{\mu^2}-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}\right)\mu^2}{c\,.\,\lambda}\,w_{max}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

expressed in parts of the distance between two fringes, w_{max} is the axial velocity, while μ, λ, and c are respectively the index of refraction of the water, the wavelength of the light used, and c the velocity of light in vacuo.

For l has been taken 302,0 cm ; that is the distance between corresponding points of contact of the dotted lines with the axis in the head-pieces at the ends of the tube (see fig. 3). If the current in the tubes was governed by the laws of POISEUILLE for viscous fluids, the maximum velocity would be equal to twice the mean velocity and the distribution of the velocities over the transverse section would be represented by a parabola. In our experiments however the velocity of the water was more uniform; we are in the region of the turbulent motion. From the axis of the tube towards the side the velocity decreases much more slowly than in the case of a parabolic distribution and finally only decreases very rapidly. In the neighbourhood of the axis of the tube there is thus a considerable region, where the velocity may be regarded as being constant, at least more constant than in the case of a distribution of the velocities according to POISEUILLE. From numerous and very careful researches of American engineers [1]) the ratio of the mean velocity to that along the axis of the tube has been deduced. The result was always found in the neighbourhood of 0,84, so that the mean velocity w_0 becomes $w_0 = 0{,}84\, w_{max}$.

The mean velocity for a definite pressure was determined by measuring the quantity of the fluid that streamed out in a certain time or rather the time (about half an hour) necessary to let stream out 10 m^3. By the latter method the determination was independent of the excentricity of the scale division, which gives the volume of the water that has passed through the watermeter. For the pressures used between 1.95 and 2.40 kg/cm^2 it was proved, that

[1]) WILLIAMS, HUBBEL and FRENKELL, Trans. Am. Soc. of Civ. Eng. Vol. 47. 1902. LAWRENCE and BRAUNWORTH ibid. Vol. 57. 1906.

Cf. also R. BIEL. Heft 44 der Mitteilungen über Forschungsarbeiten herausgegeben v. Ver. deutsch. Ing. 1907.

the connection between the mean velocity (the volume) and the pressure could be represented by a parabolic curve. So it was possible to reduce observations at a pressure p to a standard pressure (for which 2,14 k.g. cm² was chosen) by multiplying the shift of the interference fringe, measured at the pressure p, by $\sqrt{\frac{2.14}{p}}$ or graphically by means of the curve.

Before relating the obtained results I shall give in extension an arbitrary example of one of the 32 determinations of the change of phase. The four cocks in Fig. 2B (first communication) will be called A, B, C, D respectively.

Photo n°. 154 wavelength 4580 Å.U.

	Photo a.	*Photo b.*
	B, D open; A, C shut.	A, C open; B, D shut.
	Pressure on manometer.	Pressure on manometer.
	2.12	2.15
	2.14	2.14
	2.14	2.15
	2.17	2.18
	2.18	2.17
	2.18	2.16
	2.16	2.18
Mean:	2.16	2.16

Mean pressure during the experiment 2.16 k.g./cm².

We have mentioned already that the given pressures refer to the times 0, 30″, 60″ etc.

Measurement of photo N°. 154 a. Readings with the Zeiss-comparator in m.m.

	on the interference fringes.			on the fixed wire.
	54.217	53.591		52.689
	224	591		686
	220	599		688
	218	593		692
	225	599		689
	219	594		688
	225	598		52.689
	223	600		
Mean:	54.221	53.596	thus middle:	53.908
				52.689

Distance between the fringes } $\delta_1 = 0.625$ Distance from the fixed wire } 1.219

Measurement of photo N°. 154 *b.* Readings:

on the interference fringes.		on the fixed wire.
53.675	53.037	52.264
675	041	263
680	046	266
681	045	262
683	051	52.264
680	046	
686	046	
683	044	
Mean: 53.680	53.044	Thus middle: 53.359
		52.264

Distance between the fringes $\Big\}$ $d_2 = 0.636$ Distance from the fixed wire $\Big\}$ 1.095

Mean distance of the fringes $\left.\begin{matrix}0.625\\0.636\end{matrix}\right\}$ 0.630

Shift of the fringes by the motion $1.219 - 1.095 = 0.124$ or reckoned in the right direction $0.630 - 0.124 = 0.506$.

Thus shift in parts of the fringe distance

$$\Delta = \frac{506}{630} = 0.803 \text{ for } p = 2.16 \text{ k.g./cm}^2.$$

thus $\Delta = 0.799$ for $p = 2.14$ k.g./cm².

The obtained results may be summarized in a table.

Shift of the interference fringes by reversing the direction of the current.

$p = 2.14$ k.g./cm². $w_0 = 465$ cm/sec. $w_{max} = 553.6$ cm/sec.

λ in A°.	Δ_F	Δ_L	$\Delta_{exp.}$	Number of experiments
4500	0.786	0.825	0.826 ± 0.007	6
4580	0.771	0.808	0.808 ± 0.005	6
5461	0.637	0 660	0.656 ± 0.005	9
6440	0.534	0.551	0.542	1
6870	0.500	0.513	0.511 ± 0.007	10

Under Δ_F and Δ_L are given the shifts calculated with the formula with FRESNEL's coefficient without the term of dispersion for the value $w_{max} = 553.6$ cm/sec. belonging to $p = 2.14$. Under Δ_{exp} are found the observed shifts with the probable error in the final reading. The number of experiments is given in the last column.

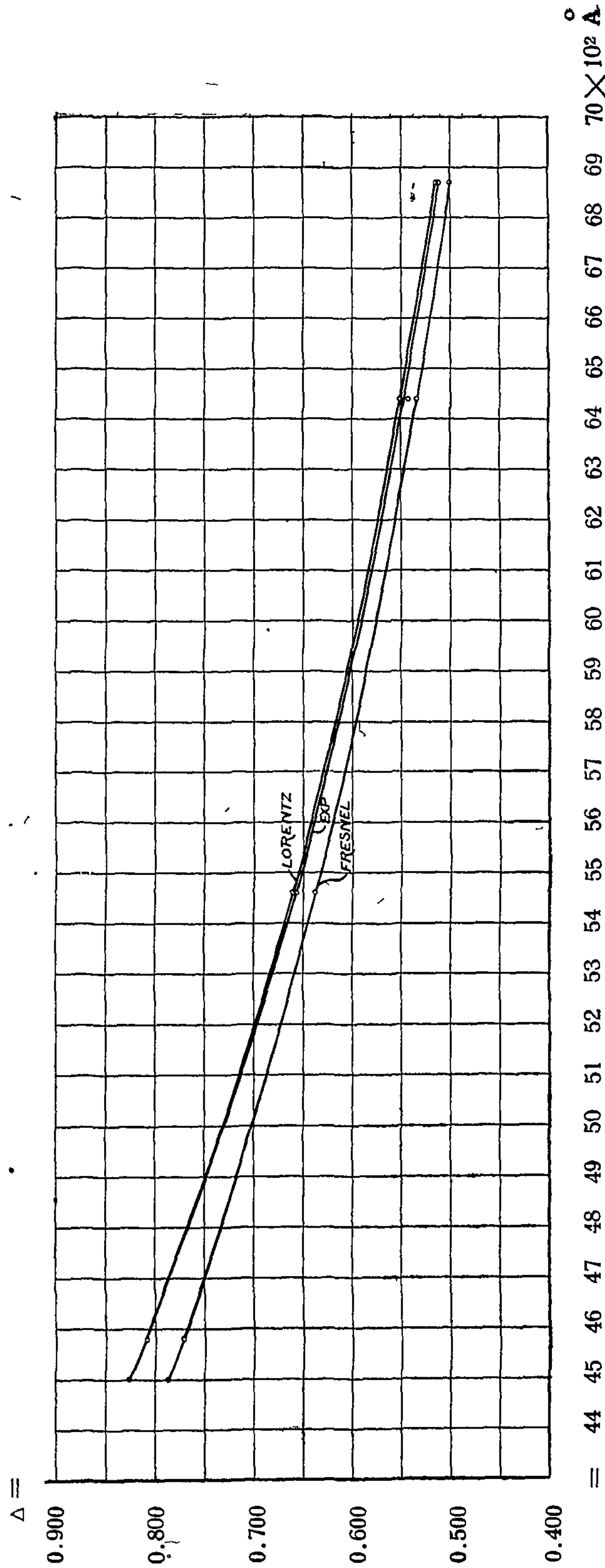
For the reading at λ 6443 no probable error is given as only one reading was made for that colour. The agreement of the experiments with the formula of LORENTZ is evident.

In Fig. 5 I have represented graphically the results obtained. For λ 4500 and λ 4580 the theoretically and experimentally determined points coincide. Perhaps it is interesting to give also the values of FRESNEL's coefficient ε:

λ in A°.	ε_{Fr}	ε_L	ε_{exp}
4500	0.443	0.464	0.465
4580	0.442	0.463	0.463
5461	0.439	0.454	0.451
6870	0.435	0.447	0.445

Here $\varepsilon_{Fr} = 1 - \frac{1}{\mu^2}$, $\varepsilon_L = 1 - \frac{1}{\mu^2} - \frac{\lambda}{\mu}\frac{d\mu}{d\lambda}$ and $\varepsilon_{exp.}$ is found from the numbers in the fourth column of the table concerning the shift of the interference fringes (under $\Delta_{exp.}$) by multiplication by $\frac{\lambda c}{8 l \mu^2 \cdot w_{max.}}$.

A few words may be said concerning the determination of the mean velocity $w_0 = 465$ cm/sec., $p = 214$ k.g./cm², which was important for the interpretation of our observations. We have mentioned already that there was a watermeter in the main tube. This meter (of the WOLTMANN-type) ran very regularly, so that no vibrations were transferred to the system of tubes. It was destined however for large quantities. Its errors were known in rough approximation only. If the meter was supposed to indicate accurately, we found taking into consideration the above mentioned precaution (see p. 401) concerning the reading at a complete rotation of the hands of the counting-piece, $w_0 = 475$ cm/sec, $p = 2.14$ k.g./sec. With this value I found a difference of about 2.1 °/₀ between the results of my experiments and the formula of LORENTZ. In order to investigate, whether this difference might be ascribed to an error in the watermeter, I decided to put a more accurate measuring apparatus at the end of the system of tubes to control the first watermeter. With extreme kindness Mr. Ing. PENNINK, Director of the Amsterdam waterworks put at my disposal a calibrated so-called "Ster" meter, which begins to indicate at a quantity of 10 L. per hour and which indicates accurately for 30 L. and more per hour. If this "Ster" meter was connected to the end of the system of tubes, while the principal cock was quite open, the mechanical vibrations of the systems would



LORENTZ
EXP
FRESNEL
Δ =
0.900
0.800
0.700
0.600
0.500
0.400
44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 × 10² Å

Fig. 5.

have been propagated from the "Stermeter" and have badly influenced the optical observations.

The only purpose however was to compare the indications of the two meters. By two independent, quite corresponding measurements on different days it was proved, that the large meter gave 10000 L., when the accurate "Ster" meter registrated 9810 L. only. This is a difference of 1.9°/₀. Now the error of the "Ster" meter itself is about 0.2°/₀ as had been determined by direct measurement of the volume transmitted to a large tank on the grounds of the waterworks. Altogether, taking the error of 0.2°/₀ with the right sign, the error in the indication of the large watermeter amounts to 2.1°/₀. We have seen already, that theory and experiment agree extremely well, if we introduce this correction, which reduces the values of w_0 from 475 c.m./sec. to 465 c.m./sec. for $p = 2.14$ k.g./c.m².

The value of w_0 at $p = 2.14$ K.g./c.m.² may thus be regarded as well established and the same may be said of the value of l, at least within the limits of the accuracy of the final result. About the factor 0.84 however some doubt may exist. Therefore it seems to us interesting to show, that even if the *absolute* value of the LORENTZ dispersion-term might have been determined less accurately than has been the case, there might have been drawn a conclusion about the necessary existence of this term [1]). *This conclusion is independent of the values given to l and $w_{max.}$*.

For, writing down equation (4) for two different colours with the wavelengths λ_1 and λ_2 we see, that $l, w_{max.}$ and c fall out by the division. The ratio of the shifts $\Delta\lambda_1$ and $\Delta\lambda_2$ becomes then according to LORENTZ

$$\frac{\Delta\lambda_1}{\Delta\lambda_2} = \frac{\left(1 - \frac{1}{\mu_1^2} - \frac{\lambda_1}{\mu_1}\frac{d\mu_1}{d\lambda_1}\right)\frac{\mu_1^2}{\lambda_1}}{\left(1 - \frac{1}{\mu_2^2} - \frac{\lambda_2}{\mu_2}\frac{d\mu_2}{d\lambda_2}\right)\frac{\mu_2^2}{\lambda_2}} \quad \ldots \ldots \quad (5)$$

and according to FRESNEL

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[1]) I will still make one remark. If we wished to explain the difference of 5 °/₀ between our observations and the formula of FRESNEL by an error in the factor 0,84, we should have to change this factor into 0,88 in order to obtain coincidence of the experimental curve and that of FRESNEL. But such a great inaccuracy does by no means exist in that factor.

$$\frac{\Delta_{\lambda_1}}{\Delta_{\lambda_2}} = \frac{\left(1-\frac{1}{\mu_1^2}\right)\frac{\mu_1^2}{\lambda_1}}{\left(1-\frac{1}{\mu_2^2}\right)\frac{\mu_2^2}{\lambda_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Taking $\lambda_1 = 4500$, $\lambda_2 = 6870$ we find from (6) $\frac{\Delta_{\lambda_1}}{\Delta_{\lambda_2}} = 1{,}572$, from (5) $\frac{\Delta_{\lambda_1}}{\Delta_{\lambda_2}} = 1{,}608$, whereas the experiment (Table p. 403) gives $\frac{\Delta_{\lambda_1}}{\Delta_{\lambda_2}} = 1{,}616$.

For $\lambda_1 = 4580$, $\lambda_2 = 6820$ the ratios become respectively 1,542, 1,575, 1,581.

So there is only a difference of 0,5 resp. 0,4 % between the formula of LORENTZ and the experiments, but a difference of 2,2 resp. 2,0 % between these and *the formula of* FRESNEL. [1])

Even if we had not succeeded in giving to l, w_{max} and the coefficient 0.84 very probable values, even then the result of our experiments had been very favourable to equation (5).

Further we must mention, that the light beam was limited by rings of tin-foil to a width of 11 m.m., whereas the glass plates allowed a beam of 18 m.m. diameter to pass along the axis, the horizontal tubes through which the water flows being of an inner diameter of 40 m.m. By this precaution the optically effective change of the velocity over the section of the tube is diminished and this is also the case with the broadening of the interference fringes caused by the curving of the wavefronts by inequality of the velocities in them.

Sometimes (not always) there is a small change in the distance between the interference fringes after reversing the direction of the water current. It is easily proved, that, neglecting quantities of the second order, we get a right result by dividing the mean value of the distance between the interference fringes before and after change of the current in the shift of the fringes.

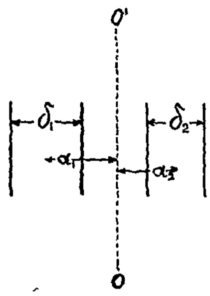


Fig. 6.

Let δ_1 and δ_2 be the distances between the fringes in the two cases and a_1 and a_2 the shifts of them from the original position OO'. From the measurements we find $\frac{a_1 + a_2}{\delta_1 + \delta_2}$ or rather the double of this. We want to know $\frac{a_1}{\delta_1} = \frac{a_2}{\delta_2}$. The difference between the first and the second expression gives the error we make. Let us put

[1]) Our conclusion is confirmed by a recent, more accurate series of observations. [Note to the translation].

$$\delta_1 = \delta - x$$
$$\delta_2 = \delta + x$$

where x represents a small quantity.

We calculate how much $\frac{a_1 + a_2}{2\delta} - \frac{a_1}{\delta - x}$ differs from zero.

It is easily found that $\frac{a_1 + a_2}{2\delta} - \frac{a_1}{\delta - x} = \frac{-a_1 x^2}{2\delta^3} = -\frac{a_1}{\delta} \cdot \frac{x^2}{2\delta^2}$, an error of the second order of magnitude.

In the example on p. 402 $\frac{x}{\delta}$ is equal to $\frac{1}{100}$, so that the change in the distance between the fringes might be still 4 or 5 times greater, without making the error larger than 1 per thousand.

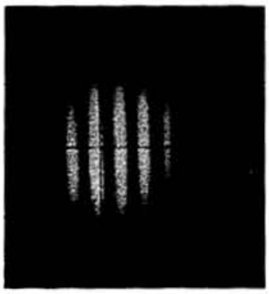
In the above cited paper [1]) JAUMANN derives on p. 462 with his theory for the FRESNEL-coefficient the formula $\frac{1}{2}\,\frac{n_0^2 - n^2}{n_0^2}$, where n_0 is the index of refraction for very long waves and n that for the colour considered.

For water $n_0^2 = 80.0$ and $n^2_{Na} = 1.78$, so that JAUMANN finds for the FRESNEL-coefficient of sodium light 0.488. This value does not agree with the result of our experiments and these are so accurate, that we may say with security, that the theory of JAUMANN is in conflict with reality. There is still another point of disagreement between experiment and this theory. The latter gives for decreasing wavelength a *decrease* of the FRESNEL-coefficient, while the experiments (see p. 404) prove the contrary.

Resuming we may say, that we have repeated FRESNEL's experiment with different colours and have proved the exactness of the FRESNEL coefficient $1 - \frac{1}{\mu^2} - \frac{\lambda}{\mu} \cdot \frac{d\mu}{d\lambda} = \varepsilon_L$ within the limits of the experimental errors It is perhaps interesting to notice that the relative values of ε_L for different colours have also been confirmed by these experiments, because these relative values are independent of the effective length of the moving watercolumn and of the exact value of a numerical coefficient that was put equal to 0,84. So the measurements from which the absolute value of the FRESNEL-coefficient has been derived, might be considered as an experimental determination of the ratio $w_{\max} \cdot w_0$ The FIZEAU-effect would from this point of view form the fixed theoretical base, as it is an effect of the first order, quite based on the ascertained fundamental equations of electrodynamics.

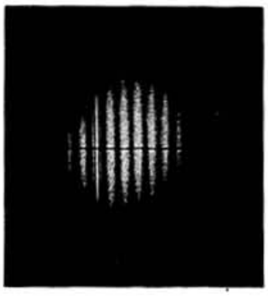
[1]) See the first part of this paper.

Current first direction.



1a $\lambda = 6870$ p = 2.13

2a $\lambda = 4580$ p = 2.16



3a $\lambda = 4580$ p = 2.26

Current opposite direction.

1b $\lambda = 6870$ p = 2.13

From a and b : $\Delta = 0.522$

2b $\lambda = 4580$ p = 2.16

$\Delta = 0.803$

3b $\lambda = 4580$ p = 2.26

$\Delta = 0.812$

4 $\lambda = 6870$ p = 2.21

$\Delta = 0.53$

5 $\lambda = 4500$ p = 2.30

$\Delta = 0.86$

HELIOTYPIE, VAN LEER, AMSTERDAM