The sextuple involution, which $[\varrho^6]$ determines in a plane φ, has *three singular points S of order two* lying in a straight line s and (in the intersections of β^4) *four singular points of order one*, which are completed into sets of six by the pairs of an involution lying on s.

Any trisecant t of a ϱ^6 is trisecant of ∞^1 curves of the congruence and in particular of a figure (ϱ^2, β^4). The *congruence of the singular trisecants* is therefore identical with the congruence of the chords of β^4, is consequently a (2, 6).

The cone projecting a ϱ^6 out of one of its points has in common with σ^3 the 6 intersections of the two curves; the remaining 9 points determine each a singular bisecant b.

The surface Π^7 belonging to a point S of σ^3 consists of Σ^2, the plane σ (of which any straight line is singular bisecant) and a cone $(b)^4$. Consequently the *singular bisecants* b form *a congruence* (9, 12).

A plane φ contains a curve φ^5 being the locus of the points of contact of curves ϱ^6. As φ^5 has 34 points in common with $\varDelta^8$, outside σ^3, the curves ϱ^6 touching φ form a Φ^{34}, which is moreover intersected by φ in a curve φ^{24}. As φ^5 is intersected by an arbitrary Σ^2 in 10 points, σ^3 is decuple curve of Φ^{34}; so φ^{24} has three octuple points S. From this it ensues further that φ^5 and φ^{24}, apart from the points S, have 96 points in common, so that φ is *osculated* by 48 curves ϱ^6.

As φ^5 has outside σ^3 140 points in common with Ψ^{34} there are 140 curves ϱ^6 *touching two planes*.

The bilinear congruences of twisted curves ϱ^5 and ϱ^4, which are determined by nets of cubic surfaces I have considered in communications published in volume XVII, p. 1250, in volume XVIII, p. 43 and in vol. XVI, p. 733 and 1186 of these *Proceedings*. The congruence of twisted cubics determined by a $[\Phi^3]$ was extensively treated by STUYVAERT (Bull. Acad. de Belgique, 1907, p. 470—514).

Mathematics. — "*Associated points with respect to a complex of quadrics.*" By CHS. H. VAN OS. (Communicated by Professor JAN DE VRIES).

(Communicated in the meeting of May 29, 1915).

Let a triply infinite linear system (*complex*) be given of quadrics Φ^2. The surfaces passing through a point P form a net and have moreover *seven* points Q in common. If we associate those points to P we get a correspondence, which will be considered here.

§ 1. We first prove the proposition: Any straight line l joining two associated points P and Q contains an involution of pairs of associated points. Any pencil of the complex has *one* Φ^2 in common with the net determined by P and Q, and intersects l therefore along an involution containing the pair of points P, Q. If two pencils have *one* Φ^2 in common (if they "intersect" as we shall say for the sake of brevity) the associated involutions have moreover *one* pair of points in common and so coincide. If the two pencils do not intersect a third may be introduced intersecting each of them and it may be seen that the involutions coincide in that case too. All pencils therefore intersect l along the same involution, any pair of points of it consequently determines an infinite number of pencils, sets apart a net out of the complex, by which the proposition has been proved.

§ 2. Let us determine the locus of the points P coinciding with one of their associated points. For this purpose we determine the number of those points lying on the section ϱ^4 of two Φ^2 of the complex. The sets of eight associated points on ϱ^4 are cut out on ϱ^4 by the Φ^2 of a pencil (Φ^2) from the complex. Now a pencil (Φ^2) contains *sixteen* (Φ^2), touching a twisted quartic of the first kind; this is easily seen by making the curve to degenerate into a quadrilateral, each of the sides of which touches then at two Φ^2, while through each angle passes *one* Φ^2, which must be counted twice.[1]) The number of points lying on ϱ^4 amounts therefore to 16, their locus is therefore a *surface of order four*, Δ^4.

§ 3. What is the locus of the points Q, if P describes a straight line l?.

Any Φ^2 of the complex intersects l in two points P, and so contains also the 14 points Q associated to them; the locus of these points is therefore a *curve of order seven*, ϱ^7. It has in common with l the four intersections of l and Δ^4.

A plane V passing through l intersects ϱ^7 outside l moreover in 3 points Q, each associated to a point P of l. The 3 joining lines PQ, which we shall indicate by g_1, g_2 and g_3 contain each an involution of associated points.

The locus of the points P of V, for which one of the associated points Q lies in V consists of these straight lines and of the section c^4 of V with Δ^4. Now this locus is the section of V with the surface

[1]) Vide ZEUTHEN, *Lehrbuch der abzählenden Methoden der Geometrie*, Teubner 1914.

REVIEW OF THE THEORETICALLY PREDICTED SYMMETRY OF RÖNTGEN-PATTERNS OF UNIAXIAL CRYSTALS, FOR PLATES PARALLEL TO THE BASAL FACE, AND TO THOSE OF THE FIRST AND SECOND PRISM.

I. Tetragonal System.

Seriesnumber of the Class of Symmetry:	*Indication of the Crystal-Symmetry:*	*Elements of Symmetry in the considered Crystals:*	*Symmetry of the Röntgenpattern for a plate parallel to* {001}:	*Symmetry of the Röntgenpattern for a plate parallel to* {100}:	*Symmetry of the Röntgenpattern for a plate parallel to* {110}:	*Representative Crystalspecies:*
9	Tetragonal-bisphenoidal	$\bar{A}_4$ (also = A_2)	A single quaternary axis	A single horizontal plane of symmetry	A single horizontal plane of symmetry	No mineral known
10	*Tetragonal-pyramidal	A_4	A single quaternary axis	A single horizontal plane of symmetry	A single horizontal plane of symmetry	*Wulfenite*
11	Tetragonal-scalenohedrical	$\bar{A}_4$ (also = A_2) $2A_2'$; $2S_v''$	A quaternary axis; 2×2 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	*Urea; Potassium-hydrophosphate*
12	*Tetragonal-trapezohedrical	A_4; $2A_2'$, $2A_2''$	A quaternary axis; 2×2 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	*Nickelsulphate* ($6H_2O$)
13	Tetragonal-bipyramidal	A_4; HS; C	A single quaternary axis	A single horizontal plate of symmetry	A single horizontal plane of symmetry	*Scheelite; Erythrite*
14	Ditetragonal-pyramidal	A_4; $2S_v'$; $2S_v''$	A quaternary axis; 2×2 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	*Penta-Erythrite*
15	Ditetragonal-bipyramidal	A_4; $2A_2'$; $2A_2''$; HS; $2S_v'$; $2S_v''$; C	A quaternary axis; 2×2 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	*Rutile; Cassiterite; Potassiumferrocyanide* (mimetic)

II. Trigonal System.

Seriesnumber of the Class of Symmetry:	*Indication of the Crystal-Symmetry:*	*Elements of Symmetry in the considered Crystals*	*Symmetry of the Röntgenpattern for a plate parallel to* {0001}:	*Symmetry of the Röntgenpattern for a plate parallel to* {10$\bar{1}$0}:	*Symmetry of the Röntgenpattern for a plate parallel to* {$\bar{1}$2$\bar{1}$0}:	*Representative Crystalspecies:*
16	*Trigonal-pyramidal	A_3.	A single ternary axis	No symmetry at all	No symmetry at all	*Sodiumperjodate* ($3H_2O$)
17	Trigonal-rhombohedrical	A_3 (also = $\bar{A}_6$); C	A single ternary axis	No symmetry at all	No symmetry at all	*Phenakite; Dolomite*
18	*Trigonal-trapezohedrical	A_3; $3A_2$	A ternary axis; three planes of symmetry	A single vertical plane of symmetry	The perpendic. to the plate is a single binary axis	*Quarz; Cinnabar*
19	Trigonal-bipyramidal	A_3; HS	A single senary axis	A single horizontal plane of symmetry	A single horizontal plane of symmetry	No mineral known
20	Ditrigonal-pyramidal	A_3; $3S_v$	A single ternary axis	A single vertical plane of symmetry	The perpendic. to the plate is a single binary axis	*Turmaline*
21	Ditrigonal-scalenohedrical	A_3 (also = $\bar{A}_6$); $3A_2$; $3S_v'$; C	A single ternary axis	A single vertical plane of symmetry	The perpendic. to the plate is a single binary axis	*Calcite*
22	Ditrigonal-bipyramidal	A_3; $3A_2$; HS; $3S_v$	A senary axis; and 2×3 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	No mineral known

III. Hexagonal System.

Seriesnumber of the Class of Symmetry:	*Indication of the Crystal-Symmetry:*	*Elements of Symmetry in the considered Crystals*	*Symmetry of the Röntgenpattern for a plate parallel to* {0001}:	*Symmetry of the Röntgenpattern for a plate parallel to* {10$\bar{1}$0}:	*Symmetry of the Röntgenpattern for a plate parallel to* {$\bar{1}$2$\bar{1}$0}:	*Representative Crystalspecies:*
23	*Hexagonal-pyramidal	A_6	A single senary axis	A single horizontal plane of symmetry	A single horizontal plane of symmetry	*Nephelite*
24	*Hexagonal-trapezohedrical	A_6; $3A_2$; $3A_2'$	A senary axis and 2×3 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	*Antimonylbarium-tartrate + Potassiumnitrate*
25	Hexagonal-bipyramidal	A_6; HS; C	A single senary axis	A single horizontal plane of symmetry	A single horizontal plane of symmetry	*Apatite*
26	Dihexagonal-pyramidal	A_6; $3S_v$; $3S_v'$	A senary axis and 2×3 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary azis	*Zincite; Wurtzite*
27	Dihexagonal-bipyramidal	A_6; $3A_2$; $3A_2'$; HS; $3S_v$; $3S_v'$; C	A senary axis and 2×3 planes of symmetry	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	Two perpendic. planes of symmetry; the perpendic. to the photograph. plate is a binary axis	*Beryl*

It may be generally remarked here, that planes of symmetry perpendicular to the photographic plate, will be manifested in the Röntgenpattern by their resp. *intersections* with the plane of the photographic plate; and that in the case, where the perpendicular to the plate corresponds to the direction of a binary axis, this will appear in the pattern, as if a symmetry-centre in the photo were present. Binary axes in a plane parallel to that of the photographic plate are of course not revealed in the diffraction-pattern.

N.B. The symmetry-elements of the Crystals are indicated as follows: A_n = symmetry-axis of the first order, with a period of $\frac{2\pi}{n}$; $\bar{A}_n$ = symmetry-axis of the second order (axis of composed symmetry) of the period $\frac{2\pi}{n}$; *HS* = a horizontal plane of symmetry; S_v = vertical plane of symmetry; unequivalent axes and planes are discerned by accents; *C* = centre of symmetry. The optical axis is always supposed to be *vertical*; the cristallographical principal axis of the same direction is discerned as the *c*-axis. In the case of the trigonal crystals, the symbols of Brávais are used; in the case of hexagonal and trigonal crystals both, the direction of the face (10$\bar{1}$0) is supposed to be parallel to that of (100) in the tetragonal crystals, and just so that of ($\bar{1}$2$\bar{1}$0) parallel to that of (010) in the case of tetragonal forms. In some trigonal crystals, the plates were cut parallel to (01$\bar{1}$0) and (2$\bar{1}\bar{1}$0), what does not involve any appreciable difference for the considered problem, but makes it necessary to compare more directly the corresponding patterns with those obtained from tetragonal crystals cut parallel to (110) and (1$\bar{1}$0). The symmetry-classes indicated by * are those, whose crystals can appear in enantiomorphous forms. (Enantiomorphism).

of the points Q, which are associated to the points P of V, this is consequently a *surface of order seven*, Φ^7.

This order is also easily found from the number of intersections with a ϱ^4 of the complex; the latter intersects V in 4 points P, contains therefore 28 points Q, associated to it.

The joining lines of associated points apparently form a congruence (7,3).

§ 4. If the straight line l is one of the straight lines PQ, considered in § 1, a Φ^2 of the complex will intersect the straight line l in two associated points, consequently contain six points only, which are associated to points of l. The locus of those points is therefore a *twisted cubic* ϱ^3. The curve ϱ^7 has been replaced here by the figure composed of l and the ϱ^3 counted twice. The latter intersects l in two of the four points which l has in common with Δ^4; the two others are the double points of the involution lying on PQ.

Let us bring through PQ a plane V, in which PQ stands therefore for the straight line g_1. This plane intersects ϱ^3 moreover in a point R outside g_1; the joining lines of R with the two points on g_1 associated to it, must be the straight lines g_2 and g_3. We see therefore that the three intersections of g_1, g_2 and g_3 are mutually associated and that each plane V *contains one set of three associated points*.

A ϱ^4 of the complex passing through two associated points lying on g_1, intersects Φ^7 further in the 6 points associated to them and in the 14 points associated to its two other intersections with V. As the total number of intersections must be 28, the 6 points mentioned first are nodes of Φ^7. The three ϱ^3 belonging to g_1, g_2 and g_3 are therefore *nodal curves of* Φ^7.

A ϱ^4 passing through the three intersections of g_1, g_2 and g_3 intersects Φ^7 further in the 5 points associated to them and in the 7 points associated to the fourth intersection of ϱ^4 and V. From this it easily ensues that the five points mentioned are *triple points of* Φ^7.

§ 5. If P lies on Δ^4 one of the associated points coincides with P. If R is one of the others the locus of R may be inquired into.

A ϱ^4 of the complex intersects Δ^4 in 16 points, contains therefore the $16 \times 6 = 96$ points R associated to them; that locus is consequently a *surface of order* 24, Δ^{24}.

Δ^4 and Δ^{24} intersect in a curve of order 96; it will, however, degenerate:

1. in het locus of the points P, coinciding with *two* of the points associated to them. Δ^4 and Δ^{24} *touch* each other along this curve.

2. in the locus of the points P, coinciding with *one* of their associated ones while two more of the other points associated to them coincide as well.

§ 6. In order to find the first of these curves we investigate the locus of the points R, associated to the points of the section c^4 of V with Δ^4.

A Φ^2 of the complex intersects c^4 in 8 points, contains therefore $8 \times 6 = 48$ points R, so that the locus of R is a curve of order 24, ϱ^{24}.

The curve ϱ^{24} intersects V in 24 points, of which 2 lie on each of the three straight lines g, and these are associated to the intersections of g with the associated ϱ^3; there remain 18, which must lie on c^4, and in each of which the point P coinciding already with Q coincides now moreover with R.

The locus wanted is therefore a *curve of order* eighteen, ϱ^{18}.

§ 7. The ϱ^{24} found just now intersects Δ^4 in 96 points; 36 of them are lying in the just found intersections with c^4, the 60 remaining ones lie on Δ^4, coincide consequently with *one* of the associated ones while two others coincide on c^4. We see therefore that the second of the curves mentioned in § 5 is really of order 60.

§ 8. The Φ^2 of the complex passing through a point P of Δ^4, have a common tangent t in P. As they form a net two more points are necessary to determine one of them.

We now take these points infinitely near P, and in such a way, that they do not lie with t in *one* plane. The surface Φ^2 thus determined has two different tangent planes in P, must therefore be a *cone* which has P as vertex. Δ^4 *is therefore nothing but the locus of the vertices of the cones of the complex.*

§ 9. The involution I^8 considered here is a particular case of an I^8 investigated by Prof. JAN DE VRIES[1]). Three arbitrary pencils (Φ^2) had been given there. Through a point P passes out of each of them *one* Φ^2; these 3 Φ^2 will intersect moreover in 7 points outside P. If we associate these to P we get the I^8 meant.

The I^8 considered above is acquired by taking the 3 pencils as belonging to one and the same complex; in that case the three Φ^2

[1]) These Proceedings volume XXI, p. 431.

passing through P determine a net and have the base-points of this net in common.

For the more general I^8 the proposition of § 1 does not hold good; consequently the joining lines of associated points form a complex of rays instead of a congruence of rays.

The locus of the coincidences is now a surface of order 8; the curve associated to a straight line l is of order 23, the surface associated to a plane V is also of order 23. The question arises how the results obtained above are connected with the properties of those more general I^8.

§ 10. If the 3 pencils (Φ^2) lie in the same complex ∞^1 pencils (Λ^2) may be introduced intersecting the three given pencils. If the Φ^2 of the complex are represented by the points of a tridimensional space, the (Λ^2) are represented by the generatrices of the ruled surface having the images of the given Φ^2 as directrices.

For a point P on the base-curve λ^4 of a (Λ^2) the three Φ^2 from the given pencils passing through P belong to (Λ^2), consequently they have λ^4 in common. *For such a point P the associated points Q become therefore indefinite,* if we start for the definition of the I^8 from the three pencils (Φ^2) instead of directly from the complex.

In order to find the locus of P, we observe that the Φ^2 of the three pencils (Φ^2) belonging to one and the same pencil (Λ^2) are projectively associated to each other, as immediately follows from the representation mentioned. The base-curves λ^4 are consequently sections of corresponding surfaces Φ^2 out of two projectively associated pencils; their locus is therefore a *surface of order four*, Ω^4.

§ 11. If starting from the more general I^8, the given pencils Φ^2 are allowed to change in such a way that they come to lie in the same complex, the occurrence of Ω^4 will apparently cause various degenerations.

As the points associated to a point P of Ω^4 are indefinite they may also be considered as coinciding with P, and consequently the surface Δ^8 of the coincidences of the general I^8 will degenerate into Δ^4 and Ω^4.

A straight line l intersects Ω^4 in 4 points, intersects therefore four λ^4, the ϱ^{23} associated in the general case to l degenerates consequently into the ϱ^7 found above and those four λ^4.

A plane V passing through l intersects ϱ^{23} in general in 15 points outside l, of these 12 lie now on Ω^4, which are associated by 3's to 4 points of l.

From the section of V with the associated surface Φ^{23} the section

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with Ω^4 is therefore separated thrice, and as this section must be counted once more as part of the section with Δ^8, Φ^{23} has degenerated into the surface Φ^7 found above and in the four times counted surface Ω^4.

§ 12. On each of the straight lines PQ considered in § 1 lies an involution of associated points, of which the double points are situated on Δ^4. If these are associated to each other an *involution on* Δ^4 is obtained. It has been deduced in a different way by STURM (Die Lehre von den geometrischen Verwandtschaften, Vol. III, p. 409). He proves among others that in this way to each plane section c^4 of Δ^4 a twisted curve ϱ^6 of order six and rank sixteen is associated.

Chemistry. — "*On the allotropy of the ammonium halides I.*" By Dr. F. E C. SCHEFFER. (Communicated by Prof. A. F. HOLLEMAN).

(Communicated in the meeting of June 26, 1915).

1. *Introduction.* In the literature, in particular in the crystallographical literature, there are a number of papers to be found which lead us to the conclusion that ammonium chloride and ammonium bromide can occur in two different crystalline forms. Thus STAS [1]) found that the transparent crystalline mass which deposits from the vapour of subliming ammonium chloride, comes off from the wall when cooled, and becomes opaque; he also states that the specific weight of the transparent and the opaque ammonium chloride are different. Though STAS does not enter into further details about these phenomena, these experiments would already be sufficient to suggest dimorphy here. It is remarkable that STAS has evidently succeeded in cooling the transparent ammonium chloride, which according to the above is metastable at the ordinary temperature, to room temperature without the conversion taking place, the more so because in the papers that have appeared later no indications are to be found for this possibility. GOSSNER.[2]), who repeated STAS' sublimation experiment, says that generally conversion sets in already during the sublimation, and the clear crystals can only be preserved for a short time.

LEHMANN [3]) was the first to conclude to dimorphy; he tried

[1]) STAS Untersuchungen über die Gesetze der chemischen Proportionen u. s. w. übersetzt von ARONSTEIN. S. 55 (1867).

[2]) GOSSNER, Zeitschr. f. Kryst. 38 110 (1903).

[3]) LEHMANN, Zeitschr. f. Kryst. 10 321 (1885).