

Citation:

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As LINDEMANN'S formula can be deduced¹⁾ from the principle of similarity, applied to solid substances, the data mentioned for k_L can serve at the same time as a comparison of these two metals with respect to that principle. In this comparison the change of θ with the temperature has, however, not been taken into account.

Physics. — "*Further experiments with liquid helium. O. On the measurement of very low temperatures. XXV. The determination of the temperatures which are obtained with liquid helium, especially in connection with measurements of the vapour-pressure of helium.*" By H. KAMERLINGH ONNES and SOPHUS WEBER. (Comm. 147*b* from the Physical Laboratory at Leiden).

(Communicated in the meeting of June 26, 1915).

1. *Introduction.* In this paper will be given some new determinations of vapour-pressures of helium based on more accurate temperature-measurements, as also a contribution to the knowledge of the correction for the thermal molecular pressure, which has to be applied with constant volume-thermometers for low temperatures with gas under diminished pressure, if the manometer is kept at the ordinary temperature.²⁾

This correction was discussed in Comm. 124*b* (Dec. 1911); an estimate of its magnitude, which was necessary to form a judgment of the value of the temperature-determinations, showed that its influence would only exceed the limits of accuracy then given, viz. 0.1 of a degree, in the measurement of the lowest temperatures.

The present determinations of the boiling point of helium made with the aid of a helium thermometer with mercury-manometer arranged for more accurate measurements have enabled us to test the accuracy of the temperature-determination in the previous series of experiments with liquid helium, in which this point had also been determined. It appears that the difference of the previous results from the present can be explained by the correction for the thermal molecular pressure. This correction remains below the value, then given for the limit of accuracy. As to the correction at the lowest temperatures which were measured, this also appears to have about the estimated magnitude. The previous measurements are thus as a whole confirmed.

¹⁾ H. KAMERLINGH ONNES, Comm. N^o. 123 (June '11).

²⁾ Compare for a different arrangement the conclusion of § 7, Comm. Suppl. 84 (Sept. 1913), where the investigation contained in the present communication was also announced.

We have now for the first time made measurements with a helium thermometer in which a heated-wire manometer according to KNUDSEN serves as manometer (comp. § 7 Comm. Suppl. 34). With this thermometer it will be possible to go to still lower temperatures than heretofore. In these measurements the correction for the thermal molecular pressure became even now of paramount importance. We succeeded in calculating a formula for this correction, albeit with the aid of a hypothesis regarding the effective molecular free path which leads to a semi-empirical relation.

The new constant in this formula which is a characteristic constant for helium could be chosen such, that for all our measurements with the thermometer with hot-wire-manometer a satisfactory agreement was obtained with the thermometer with mercury-manometer.

2. *Survey of the difficulties inherent in the determinations of the lowest temperatures.* All measurements of temperature in the helium-region will ultimately have to be reduced to readings on a helium-thermometer supposed to be filled with helium in the AVOGADRO-condition. Hence the importance of knowing, how to arrive at accurate determinations with a helium-thermometer, even at very low pressures. For measuring those temperatures, at which the vapour-pressure of helium approaches a very small value, no other helium-thermometers but those with gas at very low pressure can be used, as the pressure in the thermometer must in any case remain below the vapour-pressure corresponding to the temperature to be measured.

Various circumstances thus render it difficult to raise the accuracy to the level which would be permitted by the high degree of constancy of the temperature of the helium-bath which can be attained when the experiment is not unduly prolonged.

It will be necessary to take care that the following conditions are fulfilled.

1. The dead space, or rather that part of the dead space the temperature of which is uncertain, must be made as small as possible.

2. The adjustment of the equilibrium must take place in a sufficiently small time in order to prevent the temperature-changes of the helium-bath affecting the measurements.

3. The deviations of the equation of state for the thermometric gas from the AVOGADRO-condition must not come too much into account.

4. The correction for the thermal molecular pressure must not become too large and this pressure must not reach a region, for which the correction is less accurately known.

It will be seen that these requirements cannot all be fulfilled at the same time. A small value of (1) goes together with a large value of (2) and similarly a minimum of (3) corresponds to a maximum of (4).

All we can do therefore is choosing the construction of the thermometer such that in the intended measurements an optimum is attained as regards satisfying the mutually conflicting requirements.

A calculation of the order of magnitude of each of the aforementioned disturbances, uncertainties or corrections will in general sufficiently enable us to reach our object.

It is clear, that we have to devote our attention particularly to the capillary which connects the reservoir (at low temperature) with the manometer (at the ordinary temperature).

As regards (1), the uncertainty regarding the distribution of temperature along the capillary makes itself principally felt in the lowest, coldest part of the capillary, where the density of the gas is highest; the narrower this part in proportion to the rest, the smaller the uncertainty will be.

It would of course be advisable, if possible, to avoid the calculation of the correction for that part of the dead-space which is dependent on the capillary by placing an auxiliary capillary according to CHAPPUIS beside the capillary of the thermometer.¹⁾ In our case we were unable to utilize this device owing to want of space in the cryostat. It was all the more important, therefore, to take the lower part of the capillary as narrow as possible, from which it follows in view of (2), that the capillary must be taken wider higher up.

At the lowest temperatures the question becomes of importance, whether helium still follows the gaseous laws. On the one hand the term $\frac{B}{v}$ in the equation of state $pv = A + \frac{B}{v}$ has to be considered, where B for a reduced temperature say of 0.2 acquires a fairly high value, so that the correction to be made on account of B may obtain an important influence. As long as the equation of state for helium is no better known than is at present the case and the calculation has to be made with the "mean" equation of state according to the law of corresponding states, great uncertainty exists with regard to this correction. On the other hand it might be a question, whether A may still be taken directly proportional to T or whether

¹⁾ We are dealing here exclusively with the constant-volume thermometer. A subsequent Communication will deal with the use of thermometers at constant pressure.

an absolute zero-point pressure according to the theory of quanta ought not to be introduced.

In both respects the difficulty might be sufficiently avoided by simply taking the melting-point pressure of the thermometer sufficiently small, but in that case, as already pointed out, the thermal molecular pressure begins to give difficulties which ultimately exceed all the others. In fact this pressure depends upon the ratio $\frac{2R}{\lambda}$, where R is the radius of the capillary and λ the mean free path.

Whereas we know the condition of the pressure-equilibrium between the bulb of the thermometer and the manometer, when the temperatures of both are given for the two extreme cases $\frac{2R}{\lambda} = \frac{1}{\infty}$ and $\frac{2R}{\lambda} = \infty$, this is no longer the case for intermediate values of this fraction. In ordinary gas-thermometers with a melting-point pressure of about the normal atmospheric pressure, the condition $\frac{2R}{\lambda} = \infty$ is very nearly satisfied and the pressures p_1 at the top and p_2 at the bottom of the capillary, where the temperatures are T_1 (normal) and T_2 (to be measured) respectively, may be taken as equal. As we shall see, this is by no means allowed when temperatures are to be measured at which the vapour-pressure of helium is no more than a few millimeters. In thermometers which are adapted to this object considerable corrections have to be dealt with, as will appear in the measurements to be discussed in this paper, indeed the question naturally arises, whether in this case it is not preferable in the temperature-measurement to start from the condition of equilibrium for $\frac{2R}{\lambda} = \frac{1}{\infty}$ viz. $\frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{p_1}{p_2}$.

3. *Description of the two thermometers.*

The thermometer with mercury-manometer (fig. 1) was the improved form of that in Comm. 119 as described in Comm. 124*b*. The bulb Th_1 had about three times greater capacity, 23.95 cc., and the capillary consisted of three parts, the first starting from below $C_a—C_b$ 15.3 cms. of 0.0362 cm. radius, the next $C_b—C_c$ 9.80 cms. of 0.0783 cm., the third $C_c—C_d$ 22.59 cms. of 0.0947 cm. (the upper 5.25 cm. having 0.090 cm. radius). To the top of the glass capillary (being another part of 5.55 cm. of 0.090 cm. radius) was soldered (entering over this same length) a copper capillary of 1.2 mm. diameter, which was connected to the mercury manometer,

first described in Comm. 119 Pl. I and more recently in Comm. 124*b* Pl. I fig. 3 and specially designed for thermometric work. After the improvement of Comm. 124*b* this manometer had only undergone a slight modification: in addition to the glass-tap in the glass capillary leading to the copper one, a side-tap has been added whose object is to connect the thermometer with the mercury-airpump if required.

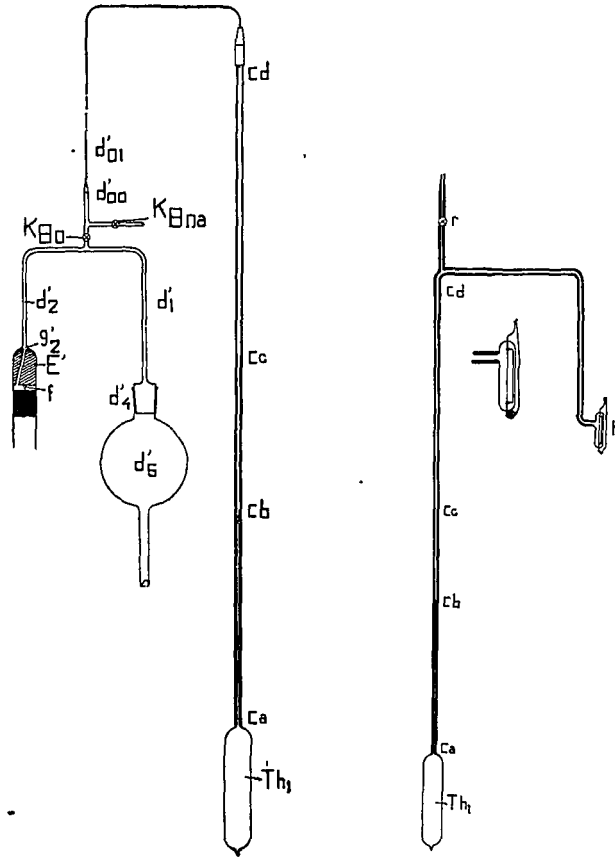


Fig. 1.

Fig. 2.

The part of the dead space of the thermometer which during the measurements remains at room-temperature had a volume of 4.87 cc.

The second thermometer (fig. 3) was provided with a very small heated-wire manometer¹⁾ *k*, designed to measure small pressures with sufficient accuracy. The bulb of the thermometer had the same volume as that of the first thermometer 23.956 cc. and the capillary was constructed in exactly the same way as with the latter. The part of the dead space which in this thermometer did not assume the low temperature was 2.67 cc; the heated-wire manometer stood

¹⁾ H. KAMERLINGH ONNES and SOPHUS WEBER, Comm N^o. 137*b*.

in ice. Previously this instrument had been carefully calibrated with the aid of a set of pipettes with pure helium. We are glad to offer Mr. P. G. CATH our thanks for his assistance in this work.

4. *Results.* The thermometers were mounted side by side in the helium-cryostat, which was vigorously stirred by means of a pump-stirrer. The bulbs were surrounded by brass tubes in order to protect them from radiation through the liquid helium. The vapour pressures of helium were corrected for the aerostatic difference of pressure between the helium liquid surface and the vapour-pressure manometer.

Two series of observations were made. In the first Mr. CHAPUIS did us the honour to take part in the measurements. In this series

Vapour-pressures and thermometer-readings with helium 1st series.						
Vapour-pressure helium-bath in mms	Thermometer with mercury-manometer $p_{0^{\circ}C.} = 25.738$ cms			Thermometer with heated-wire manometer $p_{0^{\circ}C.} = 5.240$ cms		
	Uncorrected T	Number of obs.	Mutual deviations of obs.	Uncorrected T	Number of obs.	Mutual deviations of obs.
756.6	4.205 K.	3	0.5%	4.468 K.	2	0.2%
564.5				4.155	2	0.2
363.3				3.800	2	0.2
359.5	3.535	3	0.25			
4.4	1.478	2	2.5	1.774	2	0.2

Vapour-pressures and thermometer-readings with helium. 2nd series.						
Vapour-pressure helium bath in mms	Thermometer with mercury manometer $p_{0^{\circ}C.} = 25.358$ cms			Thermometer with heated-wire manometer $p_{0^{\circ}C.} = 1.2509$ cms		
	Uncorrected T	Number of obs.	Mutual deviations of obs.	Uncorrected T	Number of obs.	Mutual deviations of obs.
757.5	4.215 K.	2	0.1%			
757.4				5.472 K.	2	0.1%
4.15	1.509	2	1.0	2.558	2	0.1
756.5	4.219	2	0.1			
756.4				5.470	2	0.1

the melting-point pressure of the helium-thermometer with mercury-manometer was 25.738 cms. mercury, that of the helium-thermometer with heated wire manometer 5.240 cms. In the second series these pressures were 25.358 cms. and 1.2059 cm. mercury respectively. The vapour-pressure measurements were conducted in the same manner as before (comm. 119 and 124*b*). The results were as follows (uncorrected T stands for T not corrected for B and for thermal molecular pressure¹):

The first column gives the vapour-pressure of helium at the corresponding temperature, the second the temperature as calculated with the aid of the ordinary gas-laws (with $B = 0$). The great difference between the temperatures found in this way with the two thermometers is very striking, especially in the last series of measurements in which the melting-point pressure of the second thermometer was very low. The influence of the thermal molecular pressure causes a temperature of $5^{\circ}.5$ to be found instead of $4^{\circ}.2$.

5. *Correction for the thermal molecular pressure.* Expressions for the thermal molecular pressure which are valid for the ranges $0 \leq \frac{2R}{\lambda} \leq 1$ or $10 \leq \frac{2R}{\lambda} \leq \infty$ have been developed by KNUDSEN. It is clear, that the choice of the two limits 1 and 10 has been somewhat arbitrary, but we may assume, that, when these limits are attended to, the uncertainty of the results of calculation by means of these formulae, supposing the constants which occur in them to be known, is on the average smaller than 1%. KNUDSEN's formulae do not hold for the intermediate range of $1 \leq \frac{2R}{\lambda} \leq 10$.

The condition of pressure-equilibrium in a tube with a gradient of temperature is in KNUDSEN's notation

$$2\pi R(M + B) + \pi R^2 \frac{dp}{dl} = 0,$$

$\frac{dp}{dl}$ is here the pressure-gradient, R the radius, M the tangential force per cm^2 . exerted by the gas on the wall in consequence of

¹) In controlling the calculations it was found that small errors and uncertainties remain about the data for calculating the gas contained in the capillary, which can change the numbers for the uncorrected T by some thousandths of a degree. The necessary corrections are inside the limits of the experimental errors. So we have left them mixed up with the latter. As soon as we shall have an opportunity to compare the present determinations with more accurate ones, we can perhaps return to this point. (Added in the English translation).

the temperature-slope and B the tangential force which the gas owing to its flow back along the axis of the tube exerts on the wall. M and B according to KNUDSEN¹⁾ are thus given by:

$$M = -k_1 \frac{3\pi}{128} Nm \Omega \lambda \frac{d\Omega}{dl} = -k_1 \frac{3\pi\eta}{128.0,30967} \frac{d\Omega}{dl}$$

and

$$B = \frac{3}{32} k_2 \Omega \frac{ap}{R^2} \frac{dp}{dl} \quad \text{where} \quad a = \frac{\pi}{8} \varrho_1 \frac{R^4}{\eta},$$

N the number of molecules per cc., m the mass of a molecule, η the viscosity and λ the mean free path.

If λ is not small as compared to R , we may not assume, as is done in the derivation of the formulae, that a molecule in a collision with a second molecule possesses the velocity corresponding to the temperature at a point at a distance λ ; in that case the collisions with the wall have also to be taken into account. The paths described by the molecules since the last collision are then found as follows:

In a disk of unit length cut out from the tube there are $\pi R^2 N$ molecules and therefore $\pi R N \frac{\Omega}{\lambda}$ mutual collisions occur per second and $2\pi R \frac{1}{2} N \Omega$ collisions with the wall; the joint number of collisions is thus

$$2\pi R \frac{1}{2} N \Omega + \pi R^2 N \frac{\Omega}{\lambda} \quad \text{per second,}$$

and each molecule collides $\left(\frac{1}{2R} + \frac{1}{\lambda}\right) \Omega$ times, while describing a path Ω . The path described without collision is therefore on the average

$$\lambda_1 = \frac{1}{\frac{1}{\lambda} + \frac{1}{2R}} = \frac{\lambda}{1 + \frac{\lambda}{2R}} \quad ^2)$$

This leads to the following condition of equilibrium

$$2\pi R \left(-\frac{3\pi}{128} k_1 Nm \Omega \frac{\lambda}{1 + \frac{\lambda}{2R}} \frac{d\Omega}{dl} + \frac{3\pi k_2}{256.0,30967} \cdot \frac{R^2}{\lambda} \frac{dp}{dl} \right) + \pi R^2 \frac{dp}{dl} = 0$$

as $\eta = 0,30967 Nm \Omega \lambda$ or

¹⁾ M. KNUDSEN, Ann. d. Phys. 33, p. 1435, 1910. 31, p. 633, 1910 and 31, p. 205, 1910 and SOPHUS WEBER, Leiden, Comm. 137c.

²⁾ The temperature change of the coefficient of accommodation for collisions with the wall is disregarded on account of its smallness.

$$\frac{dp}{p} = \frac{3}{4} k_1 \frac{d\Omega}{\Omega} \frac{1}{\left(1 + \frac{2R}{\lambda}\right) \left(1 + \frac{3\pi k_2}{0,30967 \cdot 256} \frac{2R}{\lambda}\right)} \dots (1)$$

As for $\frac{2R}{\lambda} = 0$ we have $\frac{dp}{p} = \frac{d\Omega}{\Omega}$ or $\frac{p_1}{p_2} = \sqrt{\frac{T_1}{T_2}}$, it follows, that $k_1 = \frac{4}{3}$ for $\frac{2R}{\lambda} = 0$.

In the case, that $\frac{2R}{\lambda}$ becomes large, we obtain

$$\frac{dp}{p} = \frac{k_1}{k_2} \cdot \frac{8}{\pi} \cdot 0,30967 \frac{\lambda^2}{R^2} \frac{dT}{T}$$

or introducing

$$\lambda^2 = \frac{\pi}{8} \frac{1}{0,30967^2} \frac{\eta^2}{p^2 \varrho_0} \frac{T}{273}$$

where ϱ_0 is the density of the gas at 0° and 1 dyne per c.m.², we get the formula

$$p dp = \frac{1}{0,30967 \cdot 273^2} \frac{\eta_0^2}{\varrho_0 R^2} \cdot \frac{k_1}{k_2} \left(\frac{1 + \frac{c}{273}}{1 + \frac{c}{T}} \right) T dT$$

calculating, like KNUDSEN, with SUTHERLAND'S formula (which however is no longer applicable at temperatures below those of liquid air) and calling the viscosity at 0° C. η_0 .

KNUDSEN has determined the value of k_1 and k_2 for hydrogen and oxygen and found $\frac{k_1}{k_2} = 2.3$ and $k_2 = 1$.

It is easily shown, that our formula (1) differs from KNUDSEN'S formula only by the factor $\frac{1}{1 + \frac{\lambda}{2R}}$, which has no influence for

high values of $\frac{2R}{\lambda}$.

It is therefore obvious, that the factor k_1 in (1), if this equation is to hold for all values of $\frac{2R}{\lambda}$, cannot be a constant, seeing that for all gases it approaches the value $\frac{4}{3}$ for $\frac{2R}{\lambda} = 0$ and that for high values of $\frac{2R}{\lambda}$ it becomes 2.3 for oxygen and hydrogen.

It is further to be remembered that in the theoretical deduction

of the relation between heat-conduction and friction numerically correct results can only be arrived at by taking for the mean free path in the case of conduction a somewhat higher value than that which follows from internal friction. In other words the velocity of the molecules at a collision is not that which corresponds to the temperature at a distance λ , but at a distance $a\lambda$, where a is 2.5 for monatomic gases and 1.7 for di-atomic gases. If we introduce this into the expression for M we obtain, as found by KNUDSEN, taking $k_2 = 1$, for di-atomic gases $k_1 = 1.7 \times \frac{4}{3}$ for high values of $\frac{2R}{\lambda}$ or $k_1 = 2.3$. For helium we shall have to take $k_1 = 2.5 \cdot \frac{4}{3} = 3.33$ for this gas k_1 thus changes between the limits $\frac{4}{3}$ and 3.3. The question, as to how k_1 depends upon the mean free path will have to be decided by experiment. This problem is analogous to that concerning the relation between heat-conduction and friction, when there is also slipping along the wall. Keeping that in view we have ventured to make a simple assumption which does not clash with the available experimental data and explains the nature of the deviations between our thermometers with different melting-point pressures as well as possible. In how far this assumption may be correct, can only be settled by future experiments. In the mean time it may perhaps be considered as a rough representation of what will be found, when this problem, which is of great importance for the insight into the mechanism of heat-conduction and internal friction, will be specially taken up. The assumption in question is, that

$$k_1 = \frac{4}{3} \frac{1 + c_1 c_2 \frac{2R}{\lambda}}{1 + c_1 \frac{2R}{\lambda}} \dots \dots \dots (2)$$

In this formula c_1 and c_2 are two coefficients, c_1 having a special value for each gas and being 0.550 for helium and c_2 differing for monatomic and diatomic gases. For the former $c_2 = 2.5$ and for the latter $c_2 = 1.7$.

If we abandon the assumption, that $k_1 = \frac{4}{3} \times 2.5$ for large values of $\frac{2R}{\lambda}$, there is an additional constant c_1 available to adapt the formula to our observations. A very good agreement is in that case obtained with $c_1 = 2.865$ and $c_2 = 0.3101$ ¹⁾. The corrections obtained by this method are indicated in the tables of § 6 below by ().

¹⁾ It may be observed, that the ratio of this c_1 to the more theoretical value is the value of the power of T in the viscosity-law for helium.

Returning to our equation (1) we have for a monatomic gas

$$\frac{dp}{p} = \frac{1}{2} \left(\frac{1 + c_1 c_2 \frac{2R}{\lambda}}{1 + c_1 \frac{2R}{\lambda}} \right) \frac{1}{\left(1 + \frac{2R}{\lambda}\right) \left(1 + c_3 \frac{2R}{\lambda}\right)} \frac{dT}{T}$$

where for helium $c_3 = 0.1190$ ¹⁾.

We have now to express T as a function of p and λ .

As the relation $\eta = \eta_0 \left(\frac{T}{273.1} \right)^{0.647}$ holds down to the boiling point of hydrogen (comp. Comm. N^o. 134*b* March 1913) and as the thermometer-corrections are almost entirely due to that part of the capillary which is at a higher temperature than 20° K, we may apply this formula to the whole temperature-range in the form

$$\eta = \eta_0 \left(\frac{T}{273.1} \right)^{1/2 + n}$$

According to the expression for λ given above, we have:

$$d \lg p = (1 + n) d \lg T - d \lg \lambda.$$

or also

$$d \lg p = (1 + n) d \lg T + d \lg y$$

if

$$y = \frac{2R}{\lambda}.$$

With y as independent variable we may therefore write:

$$\int \frac{dp}{p} = - \frac{3}{8} \int \frac{k_1 dy}{y [(1+y)(1+c_3 y)(1+n) - \frac{3}{8} k_1]}$$

The correction consists in our case in the sum of three corrections for the different parts of the capillary, each with a different R . For each of the three parts the integral might be easily found by mechanical quadrature, taking into account the changing value of k_1 , as soon as the limits of the integration are known. We may also for the sake of simplicity divide each part into smaller parts such, that in the integration a mean value may be assumed for k_1 . The limits are each time determined by the value of the viscosity

¹⁾ It follows from this expression, that there is a maximum value of $\frac{dp}{dT}$ (S. WEBER Comm. N^o. 137*c* Sept. 1913). In arranging the measurements in question care must be taken that at the place where this maximum occurs the distribution of temperature is known as accurately as possible.

The determination of this maximum may possibly be of importance in the investigation of the relation between k_1 and $\frac{2R}{\lambda}$.

of helium corresponding to the temperature and the density, as also by the value of the radius at the ends of the given portions of the tube¹⁾).

As the density depends on p as well as on T and as p varies along the tube, the limits at the ends of the various parts will depend upon the local values of p themselves: of these only that at the top of the capillary is immediately known, whereas at the bottom the density is approximately known, it is true, but neither p nor T . It is therefore necessary to proceed by successive approximation and starting at the top to calculate the diminution of pressure assuming as a first approximation $p = \text{constant}$ equal to the value at the top of that portion of the tube, and then, using the distribution of pressure which is found and the known distribution of temperature to improve the calculation, etc.

The uncertainty regarding the distribution of temperature along the capillary is of course a source of error, but as a rule the errors arising from this uncertainty are not of any importance, especially because usually, according as this uncertainty is greater for a given portion of the tube, its contribution to the total correction for the molecular pressure becomes smaller. Finally for that portion which reaches down to the range of temperatures which have to be determined by the thermometer itself the contribution to the correction can be entirely neglected. The most important contribution to the correction is due to the upper part of the capillary.

5. *Corrected temperatures.* Applying the corrections on the basis of the pressure-distribution along the capillary, as found by the above calculation, the following results are obtained: (see tabel II p. 505).

The values between brackets () refer to the calculation with the more empirical values of c_1 and c_2 , introduced solely with a view to the observations without taking into account the theoretical limiting values.

Calculating the correction of the helium-thermometer with mercury-manometer by means of the formulae tested in the above series of observations, we find (considering only the most reliable observations (see table III p. 505).

¹⁾ If afterwards a changing value of n were found for helium at the lower temperatures, as in other gases, the same formula will be applicable, for each piece into which the tube is divided its own value of n being introduced.

TABLE II. Temperature-measurements in the helium-region with the heated wire helium-thermometer.				
Vapour-pressure of the bath	Series I. $p_0^\circ \text{C.} = 5, 240 \text{ cm.}$		Series II. $p_0^\circ \text{C.} = 1, 2059 \text{ cm.}$	
	T uncorr.	T corr.	T uncorr.	T corr.
756.6 mm.	4.468 K.	4.260 K. (4.230)		
757.4			5.472 K.	4.245 K. (4.207)
564.5	4.155	3.937 (3.912)		
363.3	3.800	3.587 (3.568)		
4.4	1.774	1.495 (1.490)		
4.16			2.558	1.445 (1.461)
757.4			5.472	4.245 (4.207)

TABLE III. Vapour-pressures of helium measured by the helium-thermometer with mercury-manometer and corrected for the thermal molecular pressure.				
Vapour-pressure of helium in mms.	Accurate series 1913.		Series 1911.	
	T uncorr.	T corr.	T uncorr.	T corr.
760			4.29 K.	4.22 K. (4.21)
757.5	4.215 K.	4.204 K. (4.205)		
565			3.97	3.90 (3.89)
359.5	3.535	3.519 (3.516)		
197			3.26	3.18 (3.17)
51			2.34	2.25 (2.24)
4.15	1.509	1.480 (1.475)		
3			1.47	1.36 (1.35)

In the same manner the vapour-pressure above the boiling point is found as follows (Comm. N^o. 124*b*, p. 16): (see tabel IV p. 506).

The corrections to be applied to the temperature-values as given in previous communications will be seen to be but small. Both the

TABLE IV. Vapour-pressures measured by the helium-thermometer with mercury-manometer, corrected for thermal molecular pressure (above the boiling point).		
p	T uncorr.	T corr.
767 mm.	4.28 K.	4.22 K. (4.21)
1329	4.97	4.91 (4.90)
1520	5.10	5.05 (5.04)
1569	5.15	5.10 (5.09)
1668	5.22	5.15 (5.16)
1718	5.25	5.20 (5.19) crit.

boiling point and the critical point go down a little, but the change is within the limits of accuracy as previously given. The conclusions formerly drawn from the temperature-measurements thus remain valid, especially the rapid change of the constant f in VAN DER WAALS'S vapour-pressure law which we inferred at the time.

With the chosen pressures in the helium-thermometer with mercury-manometer the correction for B becomes of minor importance.

At the boiling point of helium it is too small to have any influence. According to Comm. N^o. 119b § 5 in $pv = RT + \frac{B}{v}$ we found $B_{4^{\circ}21\text{ K}} = -0.000047$ and we thus have with $p_{0^{\circ}\text{ C}} = 25.5$ cc. at $4^{\circ}21\text{ K}$. $\frac{T_{\text{corrected for } B}}{T} = 1 + 0.000128$.

It is true, that at lower temperatures, as discussed in § 1, B becomes much larger. An extrapolation according to the "mean" equation of state, in itself certainly little justified, would give. $B_{15^{\circ}\text{ K.}} = 56.1 B_{4^{\circ}21\text{ K.}}$ Even on this supposition an error of only 2% or 0.03 degrees would have to be expected from B being neglected. This deviation is smaller than the uncertainty of the correction for the thermal molecular pressure.

When the melting-point pressure in the thermometer with heated-wire manometer is as low as it was taken in the above 2nd series, the uncertainty regarding the last-named correction becomes predominant. From this it appears, that a very accurate knowledge of

1) W. H. KEESOM, Suppl. N^o. 30, p. 12.

the thermal molecular pressure will be needed, if values of B are to be derived from the comparison of thermometers with different initial pressure. The same is true with respect to possible corrections for deviations, as predicted by the theory of quanta.

7. *Approximate formula for the vapour-pressure of helium.* We did not succeed in representing our observations by NERNST's vapour-pressure formula, treated as interpolation-formula.

The BOSE-RANKINE form ¹⁾

$$\lg p_{\text{cm.Hg}} = A + B \frac{1}{T} + C \frac{1}{T^2} + D \frac{1}{T^3}$$

gave with

$A = +3.7290$, $B = -7.9780$, $C = -0.13628$, $D = +4.3634$
the results shown in Table V

T	$p_{\text{obs.}}$	p_{calc}
1.475 K.	0.415 cm.	0.419 cm.
3.516	35.95	35.50
4.205	75.75	76.38
4.9	132.9	136.5
5.16	166.8	162.1

Even with this formula containing four constants the observations appear to agree only very imperfectly.

Physics. — *Methods and apparatus used in the cryogenic laboratory.*

XVI. *The neon-cycle.* By H. KAMERLINGH ONNES. (Comm. 147c from the Physical Laboratory at Leiden).

(Communicated in the Meeting of June 26, 1915).

1. *Introduction.* In several accurate investigations on the law of dependence on the temperature of the properties of substances the difficulty is encountered, when going below 55° K., that not till 20° K. is reached liquid baths of the desired constancy are again available. The gap between 55° K. and 20° K. in a range which other-

¹⁾ C. A. CROMMELIN, Comm. N^o. 138c.