Citation:

Jong, C. de & Sande Bakhuyzen, E.F. van de, On the influence exercised by the systematic connection between the parallax of the stars and their apparent distance from the galactic plane upon the determination of the precessional constant and of the systematic proper motions of the stars, in: KNAW, Proceedings, 18 I, 1915, Amsterdam, 1915, pp. 683-695

Astronomy. — "*On the influence exercised by the systematic connection between the parallax of the stars and their apparent distance from the galactic plane upon the determination of the precessional constant and of the systematic proper motions of the stars.*" By Prof. E. F. VAN DE SANDE BAKHUYZEN and C. DE JONG.

(Communicated in the meeting of Sept. 25, 1915.)

Since the researches made by KAPTEYN, it may be regarded as an established fact, that stars of a given magnitude are at a greater mean distance from us, in proportion as they are nearer to the galactic plane. At the galactic poles the mean parallax is found to be about one and a half times as great as in the galactic plane itself. As in the researches so far undertaken concerning the precessional constant and the systematic proper motions of the stars this connection had not been taken into consideration, it is obvious that the determination of these quantities may be affected by systematic errors.

For some time it had been the intention of one of us to institute a nearer investigation of this matter, all the more because it might throw light upon a difference, found by NEWCOMB, between the values of the precession-constant, as deduced on the one hand from Right Ascension- and on the other one from Declination-observations. Later on it was noticed, that NEWCOMB himself had indicated the possibility of such an explanation of the difference, (*Prec. Const.* p. 67 and 73) and also that EDDINGTON in his well known monograph published last year, "*Stellar movements and the structure of the universe*", in pointing out the desirability of taking the differences of distance into consideration, had already made a beginning in this direction. At the same time, he only deals with the influence of the inequality of the distance upon the determination of the apex of the Parallactic motion (p. 81—83), and only develops it in the case of the investigation being based upon stars which are evenly distributed over the entire celestial sphere.

A new research, therefore, embracing the whole question, was by no means superfluous. We have undertaken it, and in the following paper we communicate our results. The term "*Systematic proper motions*" is here taken in a somewhat limited sense; it includes only those motions which are functions of the spherical place of the star, although the coefficients may still be dependent upon their distance from us, and perhaps also upon the spectral type, (we leave that here out of account). Systematic movements which are the consequence of star-streams, or may be ascribed to an equivalent non-spherical distribution of the individual motions, which we might call *systematic proper motions of the second kind* [1]), are excluded from our investigation.

In the first place, then, the dependence of the parallax upon the galactic latitude must be expressed in a simple formula; for the derivation of this we have used the table given by KAPTEYN and WEERSMA in their paper *Publ. Groningen* **24**, 15 In that table values for the mean parallax are given for the magnitudes 3.0 to 11.0, and for galactic latitudes: between — 20° and + 20°, between ± 20° and ± 40° and between ± 40° and ± 90°. For all magnitudes the same ratio is assumed: between π_β and π_0 and with sufficient accuracy for our purpose — the table is given as "quite provisional" — we could put: $\pi_\beta = \pi_0\,(1 + c\, sin^2\,\beta)$.

The three columns of KAPTEYN and WEERSMA's table were assumed to apply to gal. latitudes of ± 10°, ± 30° and ± 60°, and it appeared that the coefficient c must be given a value between 0.60 and 0.70 We assumed therefore

$$\pi_\beta = \pi_0\,(1 + 0.65\, sin^2\,\beta)$$

or,

$$R_\beta = \frac{R_0}{1 + 0.65\, sin^2\,\beta}$$

The relation assumed by EDDINGTON is equivalent to a formula of the same form with $c = 0.60$.

Our value for R must now be substituted in the equations for the systematic proper motion, whereby, for the present, we confined ourselves to the terms dependent upon a precession-correction and upon the parallactic motion.

The usual equations are

$$\mu_\alpha\, cos\,\delta = \Delta\, m\, cos\,\delta + \Delta\, n\, sin\,\delta\, sin\,\alpha + \frac{X}{R}\, sin\,\alpha - \frac{Y}{R}\, cos\,\alpha$$

$$\mu_\delta = -\frac{Z}{R}\, cos\,\delta + \Delta\, n\, cos\,\alpha + \frac{X}{R}\, sin\,\delta\, cos\,\alpha + \frac{Y}{R}\, sin\,\delta\, sin\,\alpha$$

Substituting in these the value of R, expressed in R_0, and afterwards, according to the formula

$$sin\,\beta = sin\,\delta\, cos\, i\, - cos\,\delta\, sin\,(\alpha - \theta)\, sin\, i$$

[1]) The frequency-surface may be more general than the ellipsoid, but must, according to our definition, have a centre, as the part of the movement that depends upon the spherical place (Systematic Prop. mot. 1st kind) is subtracted from the total movement.

in which θ and i represent the node and inclination of the galactic plane in respect to the equator, expressing everything in equatorial coordinates, we get, after the expansion of the powers and products of the goniometrical functions of α, leaving the value of θ, i and c for the present undetermined :

$$\mu_\alpha \cos\delta = \Delta m \cos\delta - \tfrac{1}{4} c \sin 2i \cos\theta \frac{X}{R_0} \sin 2\delta - \tfrac{1}{4} c \sin 2i \sin\theta \frac{Y}{R_0} \sin 2\delta$$

$$+ \Big[\Delta n \sin\delta + \frac{X}{R_0} + \tfrac{1}{4} c \sin^2 i\,(2 + \cos 2\theta) \frac{X}{R_0} \cos^2\delta +$$

$$+ c \cos^2 i \frac{X}{R_0} \sin^2\delta + \tfrac{1}{4} c \sin^2 i \sin 2\theta \frac{Y}{R_0} \cos^2\delta \Big] \sin\alpha$$

$$- \Big[\frac{Y}{R_0} + \tfrac{1}{4} c \sin^2 i \sin 2\theta \frac{X}{R_0} \cos^2\delta +$$

$$+ \tfrac{1}{4} c \sin^2 i\,(2 - \cos 2\theta) \frac{Y}{R_0} \cos^2\delta + c \cos^2 i \frac{Y}{R_0} \sin^2\delta \Big] \cos\alpha$$

$$+ \Big[\tfrac{1}{4} c \sin 2i \sin\theta \frac{X}{R_0} \sin 2\delta + \tfrac{1}{4} c \sin 2i \cos\theta \frac{Y}{R_0} \sin 2\delta \Big] \sin 2\alpha$$

$$+ \Big[\tfrac{1}{4} c \sin 2i \cos\theta \frac{X}{R_0} \sin 2\delta - \tfrac{1}{4} c \sin 2i \sin\theta \frac{Y}{R_0} \sin 2\delta \Big] \cos 2\alpha$$

$$- \Big[\tfrac{1}{4} c \sin^2 i \cos 2\theta \frac{X}{R_0} \cos^2\delta - \tfrac{1}{4} c \sin^2 i \sin 2\theta \frac{Y}{R_0} \cos^2\delta \Big] \sin 3\alpha$$

$$+ \Big[\tfrac{1}{4} c \sin^2 i \sin 2\theta \frac{X}{R_0} \cos^2\delta + \tfrac{1}{4} c \sin^2 i \cos 2\theta \frac{Y}{R_0} \cos^2\delta \Big] \cos 3\alpha$$

$$\mu_\delta = - \frac{Z}{R_0} \cos\delta + \tfrac{1}{2} c \sin 2i \sin\theta \frac{X}{R_0} \cos\delta \sin^2\delta -$$

$$- \tfrac{1}{2} c \sin 2i \cos\theta \frac{Y}{R_0} \cos\delta \sin^2\delta - \tfrac{1}{2} c \sin^2 i \frac{Z}{R_0} \cos^3\delta - c \cos^2 i \frac{Z}{R_0} \cos\delta \sin^2\delta$$

$$+ \Big[\frac{Y}{R_0} \sin\delta - \tfrac{1}{4} c \sin^2 i \sin 2\theta \frac{X}{R_0} \cos^2\delta \sin\delta + \tfrac{1}{4} c \sin^2 i\,(2 + \cos 2\theta) \frac{Y}{R_0} \cos^2\delta \sin\delta$$

$$+ c \cos^2 i \frac{Y}{R_0} \sin^3\delta + c \sin 2i \cos\theta \frac{Z}{R_0} \cos^2\delta \sin\delta \Big] \sin\alpha$$

$$+ \Big[\Delta n + \frac{X}{R_0} \sin\delta + \tfrac{1}{4} c \sin^2 i\,(2 - \cos 2\theta) \frac{X}{R_0} \cos^2\delta \sin\delta +$$

$$+ c \cos^2 i \frac{X}{R_0} \sin^3\delta - \tfrac{1}{4} c \sin^2 i \sin 2\theta \frac{Y}{R_0} \cos^2\delta \sin\delta - c \sin 2i \sin\theta \frac{Z}{R_0} \cos^2\delta \sin\delta \Big] \cos\alpha$$

$$- \Big[\tfrac{1}{2} c \sin 2i \cos\theta \frac{X}{R_0} \cos\delta \sin^2\delta - \tfrac{1}{2} c \sin 2i \sin\theta \frac{Y}{R_0} \cos\delta \sin^2\delta -$$

$$- \tfrac{1}{2}\, c \sin^2 i \sin 2\,\Theta \frac{Z}{R_0} \cos^3 \delta \Big] \sin 2\,\alpha$$

$$+ \Big[\tfrac{1}{2}\, c \sin 2\, i \sin \Theta \frac{X}{R_0} \cos \delta \sin^2 \delta + \tfrac{1}{2}\, c \sin 2\, i \cos \Theta \frac{Y}{R_0} \cos \delta \sin^2 \delta +$$

$$+ \tfrac{1}{2}\, c \sin^2 i \cos 2\,\Theta \frac{Z}{R_0} \cos^3 \delta \Big] \cos 2\,\alpha$$

$$- \Big[\tfrac{1}{4}\, c \sin^2 i \sin 2\,\Theta \frac{\lambda}{R_0} \cos^2 \delta \sin \delta + \tfrac{1}{4}\, c \sin^2 i \cos 2\,\Theta \frac{Y}{R_0} \cos^2 \delta \sin \delta \Big] \sin 3\alpha$$

$$- \Big[\tfrac{1}{4}\, c \sin^2 i \cos 2\,\Theta \frac{X}{R_0} \cos^2 \delta \sin \delta - \tfrac{1}{4}\, c \sin^2 i \sin 2\,\Theta \frac{Y}{R_0} \cos^2 \delta \sin \delta \Big] \cos 3\alpha$$

If in these general formulae we substitute:

$$\Theta = 18^{\mathrm{u}} 45^{\mathrm{m}} = 281^\circ$$

$$i = 63^\circ$$

$$c = 0.65.$$

we get :

$$\mu_\alpha \cos \delta = \Delta\, m \cos \delta - 0.02 \frac{X}{R_0} \sin 2\,\delta + 0.13 \frac{Y}{R_0} \sin 2\,\delta$$

$$+ \Big[\Delta\, n \sin \delta + \frac{X}{R_0} + 0.13 \frac{X}{R_0} \sin^2 \delta + 0.14 \frac{X}{R_0} \cos^2 \delta - 0.05 \frac{Y}{R_0} \cos^2 \delta \Big] \sin \alpha$$

$$- \Big[\frac{Y}{R_0} - 0.05 \frac{X}{R_0} \cos^2 \delta + 0.38 \frac{Y}{R_0} \cos^2 \delta + 0.13 \frac{Y}{R_0} \sin^2 \delta \Big] \cos \alpha$$

$$- \Big[0.13 \frac{X}{R_0} \sin 2\,\delta - 0.02 \frac{Y}{R_0} \sin 2\,\delta \Big] \sin 2\,\alpha$$

$$+ \Big[0.02 \frac{X}{R_0} \sin 2\,\delta + 0.13 \frac{Y}{R_0} \sin 2\,\delta \Big] \cos 2\,\alpha$$

$$+ \Big[0.12 \frac{X}{R_0} \cos^2 \delta - 0.05 \frac{Y}{R_0} \cos^2 \delta \Big] \sin 3\,\alpha$$

$$- \Big[0.05 \frac{X}{R_0} \cos^2 \delta + 0.12 \frac{Y}{R_0} \cos^2 \delta \Big] \cos 3\,\alpha$$

$$\mu_\delta = - \frac{Z}{R_0} \cos \delta - 0.26 \frac{X}{R_0} \cos \delta \sin^2 \delta - 0.04 \frac{Y}{R_0} \cos \delta \sin^2 \delta -$$

$$- 0.26 \frac{Z}{R_0} \cos^3 \delta - 0.13 \frac{Z}{R_0} \cos \delta \sin^2 \delta$$

$$+ \Big[\frac{Y}{R_0} \sin \delta + 0.05 \frac{X}{R_0} \cos^2 \delta \sin \delta + 0.14 \frac{Y}{R_0} \cos^2 \delta \sin \delta + 0.13 \frac{Y}{R_0} \sin^3 \delta +$$

$$+ 0.10 \frac{Z}{R_0} \cos^2 \delta \sin \delta \Big] \sin \alpha$$

$$+\left[\Delta n + \frac{X}{R_0} \sin \delta + 0.38 \frac{X}{R_0} \cos^2 \delta \sin \delta + 0.13 \frac{X}{R_0} \sin^3 \delta + \right.$$

$$\left. + 0.05 \frac{Y}{R_0} \cos^2 \delta \sin \delta + 0.52 \frac{Z}{R_0} \cos^2 \delta \sin \delta \right] \cos \alpha$$

$$-\left[0\ 04 \frac{X}{R_0} \cos \delta \sin^2 \delta + 0.26 \frac{Y}{R_0} \cos \delta \sin^2 \delta + 0.10 \frac{Z}{R_0} \cos^3 \delta \right] \sin 2\,\alpha$$

$$-\left[0.26 \frac{X}{R_0} \cos \delta \sin^2 \delta - 0.04 \frac{Y}{R_0} \cos \delta \sin^2 \delta + 0.24 \frac{Z}{R_0} \cos^3 \delta \right] \cos 2\alpha$$

$$+\left[0.05 \frac{X}{R_0} \cos^2 \delta \sin \delta + 0.12 \frac{Y}{R_0} \cos^2 \delta \sin \delta \right] \sin 3\,\alpha$$

$$+\left[0\ 12 \frac{X}{R_0} \cos^2 \delta \sin \delta - 0.05 \frac{Y}{R_0} \cos^2 \delta \sin \delta \right] \cos 3\,\alpha$$

In many cases it is convenient to modify the formulae so that in place of R_0 they contain the mean distance R_m corresponding to the magnitude or the mean magnitude under consideration. We will define this mean distance as the reciprocal value of the mean parallax, and therefore put:

$$R_m = \frac{R_0}{1 + 0.65 \times \text{mean value} \sin^2\beta}$$

We must then integrate $\sin^2\beta$ over the whole surface of the sphere, and in this way we find: mean value of $\sin^2\beta = \frac{1}{3}$, so that $R_0 = 1.22\ R_m$, and this relation must be substituted in all the terms which are dependent upon the parallactic motion.

To save space, we give below only the values of the numerical coefficients in the new formulae containing R_m.

Coefficients in the formulae containing R_m.

$\mu_\alpha \cos \delta =$

	+1.00	−0.02	+0.11			
+[	+1.00	+0.82	+0.11	+0.11	−0.04]	$\sin \alpha$
−[	+0.82	−0.04	+0.31	+0.11	]	$\cos \alpha$
−[	+0.11	−0.02			]	$\sin 2\alpha$
+[	+0.02	+0.11			]	$\cos 2\alpha$
+[	+0.10	−0.04			]	$\sin 3\alpha$
−[	+0.04	+0.10			]	$\cos 3\alpha$

$$\mu_\delta =$$

$$\begin{array}{llllllll}
 & -0.82 & -0.21 & -0.03 & -0.21 & -0.11 & & \\
+ & [+0.82 & +0.04 & +0.11 & +0.11 & +0.08 &] & \sin \alpha \\
+ & [+1.00 & +0.82 & +0.31 & +0.11 & +0.04 & +0.43] & \cos \alpha \\
- & [+0.03 & +0.21 & +0.08 & & &] & \sin 2\alpha \\
- & [+0.21 & -0.03 & +0.20 & & &] & \cos 2\alpha \\
+ & [+0.04 & +0.10 & & & &] & \sin 3\alpha \\
+ & [+0.10 & -0.04 & & & &] & \cos 3\alpha
\end{array}$$

Using these formulae we can now trace the influence which the systematic difference in the distance of the stars of the same magnitude will have upon the derivation of the precessional constant and of the elements of the parallactic movement, and thus deduce the corrections, which must be applied to results in the derivation of which the differences of distance were not taken into account. When we consider this question more closely, however, we soon see that a sharp determination of the corrections, which would hold for all the determinations of these constants hitherto made, is hardly possible.

Even if we assume that the same law of mean variation of distance with the gal. latitude holds for all individual magnitudes, which is perhaps still doubtful for the brightest classes [1]), it does not follow that it will also hold for the mean magnitude of a material which extends over several classes, as the distribution of the separate magnitudes may be different for the different regions of the heavens. The working of the simple law may also be disturbed, when, as is often done, and frequently quite rightly, proper motions above certain limits are excluded from the discussion.

Further, it is evident that the correct value of the necessary corrections will be influenced by the manner, followed in each particular case, of establishing and solving the equations. Where the separate determination of the various unknown quantities is just possible, we may try to do so, or by preference take those which would be determined with the least weight from other investigations. There is, moreover, ample room for differences of opinion as to the attribution of the weights, and often in different instances different distributions of weights will recommend themselves. If there is reason to believe that a group of stars belong together physically, this may determine us to attribute to it the weight of only one star, and in general, the discussion may be based upon the individual stars, or

[1]) NEWCOMB in his *Precessional constant* Section XIV p. 43—46, points out the difficulties which the answering of this question presents.

upon larger or smaller trapezia in which the celestial sphere is divided.

Some investigators have made use of different methods and have discussed and combined the respective results; NEWCOMB, in particular, has done this in an admirable manner. It is therefore often difficult, even for the results of one investigator, to fix the exact value of the corrections to be applied to them, and whereas an *accurate* knowledge of the foundation of our investigation, namely the exact mean variation of the distances, is not yet attained, it would certainly not be worth while to make elaborate calculations concerning the influence of this variation. We shall therefore only trace this influence in a few simple suppositions concerning the method of calculation followed. For this we use the formulae expressed in R_m, as it can be seen at once that the values previously obtained for the components of the parallactic motion will agree most nearly with the corrected results for that distance.

In the first place we will consider the influence of the assumed law of distances, upon the results for the precessional constant.

a. Determination of the Precession from Right Ascensions. In this deduction we may either determine the correction of the total luni-solar precession Δp by expressing Δm and Δn in it, or, eliminating Δn by attributing equal weights to the results from groups formed according to the A. R., confine ourselves to the determination of Δm; the influence of Δn disappears of course, when the material used is symmetrically distributed over north and south declinations. If we allow for the influence of Δn, the correction terms which contain $\sin \alpha$ must be taken into consideration, and we must investigate how the influence of these terms will be divided between the term in Δn which contains $\sin \delta$ and that in $\frac{X}{R_m}$, which is constant for all declinations. Now owing to the approximate equality of two coefficients the whole coefficient of $\sin \alpha$ is reduced to $\Delta n \sin \delta + 0.93 \frac{X}{R_m} - 0.04 \frac{Y}{R_m} \cos^2 \delta$ and, even without the rigorous formation of the normal equations, it is clear that, for not too high declinations, the term with $\cos^2 \delta$ will principally influence the parallactic motion.

So it follows that, even if we take the influence of Δn into account, provided our stars are distributed over all R.A. and we do not attribute too great differences of weight to the different groups, we may practically only pay attention to the correction terms which do not depend upon α. Calling the value of Δm (variation in 100 years) which is found, if the correction terms are left out of consideration, $[\Delta m]$, then

$$[\Delta m] = \Delta m - 0.04 \frac{X}{R_m} \sin \delta + 0.21 \frac{Y}{R_m} \sin \delta.$$

If we accept for the mean distance of the BRADLEY-stars (mean magn. 5.5) according to NEWCOMB's results: $X = +0''.20$, $Y = -2''.60$ and according to his table on p. 39, as a mean value $\sin \delta = +0.20$, we get $\Delta m = [\Delta m] + 0''.11$ or

$$\text{corr. } \delta p \text{ NEWC.} = +0''.12.$$

A separate correction of NEWCOMB's 7 zones (p. 39) gives the result corr. $\delta p = +0''.11$.

In the second place we compute the correction which must be applied to the value of Δm, deduced by DYSON and THACKERAY from the comparison of GROOMBRIDGES catalogue with the second 10 year catalogue. Taking 7.0 as the mean magnitude of the GROOMBRIDGE-stars, and accordingly (see NEWC. p. 34) adopting for X a small value, putting $Y = -\frac{10}{13} \cdot 2''.60 = -2''.00$, and accepting (*Monthly Not.* **65**, 440) as mean declination of the stars $+52°$, we find for the correction to be applied to $[\Delta m]$: $+0''.42 \sin 52° = +0''.33$.

In general, if the difference of distances is disregarded, the precessional constant deduced from the right ascensions will be too small if we had used stars of *north declination* and *too large* if the stars had *south declination.*

b. Determination of the precession from the Declinations. To trace the errors made in this case, by the assumption of equal distances, we must consider the terms containing $\cos \alpha$. We have two principal terms of this form: $\Delta n \cos \alpha$ and $\frac{X}{R_m} \sin \delta \cos \alpha$. Almost always, and unless the mean decl. of the stars in question is large, it will be preferable to determine the sum of Δn and the influence of X and then to substitute the value of X derived from the R. A. This is also NEWCOMB's method, and we shall accordingly assume that this has been done and put:

$$\text{coeff. of } \cos \alpha - \frac{X}{R_m} \sin \delta = [\Delta n]$$

then, after an easy transformation:

$$[\Delta n] = \Delta n - (0.07 - 0.20 \cos^2 \delta) \sin \delta \frac{X}{R_m} + 0.04 \cos^2 \delta \sin \delta \frac{Y}{R_m} +$$
$$+ 0.43 \cos^2 \delta \sin \delta \frac{Z}{R_m}.$$

For NEWCOMB's result from the BRADLEY-stars we find, taking according to NEWCOMB $\frac{Z}{R_m} = +1''50$:

$$\Delta n = [\Delta n] - 0''.00 + 0''.02 - 0''.13 = [\Delta n] - 0''.11$$

so that

$$\text{corr. } \delta p \text{ NEWC.} = -0''.29.$$

As the first correction term is always small and the three others have as factor $cos^2 \delta \, sin \, \delta$, while the sum $-0.20 \frac{X}{R_m} - 0.04 \frac{Y}{R_m} - -0.43 \frac{Z}{R_m}$ has a considerable negative value, *the precessional constant from declinations* will be found *too large* for stars with a *north declination*, or when in the compared catalogues stars with a *north declination* are preponderant, while stars with a *south declination* will yield *too small a value.*

We have therefore arrived at the remarkable result that, in deriving the precessional constant in the ordinary way, in which no attention is paid to the dependence of the distances upon the galactic latitude, from catalogues with preponderating north declinations the lunisolar precession p is found *larger* from the *declinations* than from the *R.A.*, while the true value must lie between these two, and nearer to the result from the R.A., and thus, to some extent at least, the discrepancy found by NEWCOMB is accounted for. The values finally assumed by NEWCOMB for δp and those corrected according to our investigation are as follows:

	NEWCOMB	Corrected
δp from R.A.	$+0''.36$	$+0''.48$
,, Decl.	$+1\,.12$	$+0\,.83$

The difference found by NEWCOMB is thus reduced to half, and no longer presents a serious difficulty.

It should be mentioned once more, that, after the completion of our calculations, the explanation found here appeared to have been suggested by NEWCOMB himself as a possible cause of the discrepancy; so far his remarks upon this subject do not appear to have received sufficient attention.

Distinguishing by the names of "vernal region" and "autumnal region" the regions between R.A. $19^h.5$ and $5^h.5$ and between $7^h.5$ and $17^h.5$, he says on p. 67: "A very little consideration "will show that if the stars of a given apparent magnitude are "farther away within the vernal region than within the autumnal "region, then the smaller parallactic motions in the former region "will tend to diminish the precession found from the right ascensions "and increase that found from the declinations", while later on p. 71 in drawing up his final conclusions he says: "I have already "remarked that a possible cause for the discrepancy . . . . .". As a matter of fact the galaxy, for the northern heaven is in the vernal region, and for the southern in the autumnal one.

As NEWCOMB further. according to observations of the sun and of Mercury, considered as probable a correction of the assumed centennial motion of the equinox in the system N_1 by $+0''.30$, he finally assumed $\delta p = +0''.82$. With this correction, our results become

$$\begin{array}{lr} \delta p \text{ from A.R.} & +0''.78 \\ \text{from Decl.} & +0\ .83 \\ \hline \delta p \text{ mean} & +0''.80 \end{array}$$

so that the discrepancy would then vanish entirely. If we do not accept the latter correction, our final result is

$$\delta p \text{ mean } +0''.66.$$

There is a striking agreement between the mean of the results from α and δ, as they are found by us, with that which NEWCOMB found by eliminating the parallactic motion from the motions of the individual stars, by a method corresponding in principle to one given before by KAPTEYN (use of the proper-motion-component τ). NEWCOMB found in this way:

$$\delta p = +0''.64$$

or, if he accepted the corrected motion of the equinox, by estimation, $+0''.84$.

From this we get a strong impression that the principal uncertainty which still remains in the *precessional constant according to the* BRADLEY-*stars*, is not due to the method of treatment, but to possible errors in the catalogues compared and particularly on the one hand to an error in the equinox and on the other hand to periodic errors in the declinations, the $\Delta\delta_\alpha$.

The precession in R.A. (the value for m) deduced from the GROOMBRIDGE-stars by DYSON and THACKERAY, was already much larger than the m according to NEWCOMB, and the discrepancy becomes still greater by applying our corrections. Beside this result they deduced a value for Δn from the R.A. and Decl.-observations together, which is grounded upon the principle that from large and from small proper motions the same R.A. of the apex must be found. It cannot be seen at once, how the difference in distance of the stars will affect the results by this method. This investigation gave $\Delta p_{\text{Newc.}} = +0''.43$, while the R.A. after applying our correction gave $\Delta p_{\text{Newc.}} = +0''.76 + 0''.33 = +1''.09$. In these results too, catalogue-errors probably play a considerable part.

Finally we must draw attention to the terms which we found, depending upon 2α and 3α, amongst which there are some which may attain values which can certainly not be neglected.

We have in R.A. the terms:

$$+ 0.11 \sin 2\delta \frac{Y}{R_m} \cos 2\alpha - 0.10 \cos^2 \delta \frac{Y}{R_m} \cos 3\alpha$$

that is for stars of the magnitude $5^m.5$:

$$- 0''.29 \sin 2\delta \cos 2\alpha + 0''.26 \cos^2 \delta \cos 3\alpha$$

and in Decl., to confine ourselves to the terms in 2α,

$$-0.08 \cos^3 \delta \frac{Z}{R_m} \sin 2\alpha - 0.20 \cos^3 \delta \frac{Z}{R_m} \cos 2\alpha$$

that is for stars $5^m.5$

$$- 0''.12 \cos^3 \delta \sin 2\alpha - 0''.30 \cos^3 \delta \cos 2\alpha.$$

These terms will, when we do not take account of them in our calculations, be added to the corresponding ones arising from periodic catalogue-errors, and show all the more clearly, that no conclusions can be easily drawn from limited areas of R.A., and that it is advisable in investigations of this kind as far as possible to give equal weights to the different R.A.-groups.

In the second place we investigate the influence of the assumed law of distances upon the determination of the parallactic motion.

We assume here that the X and Y-components are deduced from the *R.A.* only, that is, from the terms which depend respectively upon $\sin \alpha$ and $\cos \alpha$, and that for the determination of X a value of Δn is introduced, which is deduced from other terms (m in α, n in δ). If we then indicate by $\left[\frac{X}{R_m}\right]$ the value which is found when we regard the distance as only dependent upon the magnitude, and act in the same way with regard to the two other components, and if we further apply a few simple transformations, as was already partially done above, we get

$$\left[\frac{X}{R_m}\right] = 0.93 \frac{X}{R_m} - 0.04 \cos^2 \delta \frac{Y}{R_m}$$

$$\left[\frac{Y}{R_m}\right] = 0.93 \frac{Y}{R_m} - 0.04 \cos^2 \delta \frac{X}{R_m} + 0.20 \cos^2 \delta \frac{Y}{R_m}$$

$$\left[\frac{Z}{R_m}\right] = 0\,93\frac{Z}{R_m} + 0.21\,sin^2\,\delta\frac{X}{R_m} + 0.03\,sin^2\,\delta\frac{Y}{R_m} + 0.10\,cos^2\,\delta\frac{Z}{R_m}$$

These equations contain in the correction-terms only $cos^2\,\delta$ and $sin^2\,\delta$, so that they do not disappear even by integration over the whole sphere. We see thus, that, even when the stars used are spread evenly over the whole sphere, 1st the velocity-components for the mean distance, corresponding to $sin^2\,\beta = \frac{1}{3}$, are not equal to those which are found in the assumption of equal distances, and 2nd that the changes which X, Y, and Z undergo are not proportional to the quantities themselves, so that the place deduced for the apex also undergoes a change. As we have: mean value of $cos^2\,\delta = \frac{2}{3}$, m. v. of $sin^2\,\delta = \frac{1}{3}$, we find for the entire sky:

$$\left[\frac{X}{R_m}\right] = 0.93\,\frac{X}{R_m} - 0.03\,\frac{Y}{R_m}$$

$$\left[\frac{Y}{R_m}\right] = 1.06\,\frac{Y}{R_m} - 0.03\,\frac{X}{R_m}$$

$$\left[\frac{Z}{R_m}\right] = 1.00\,\frac{Z}{R_m} + 0.07\,\frac{X}{R_m} + 0.01\,\frac{Y}{R_m}.$$

Starting from the same values of the three components for the BRADLEY-stars, as were accepted before, the corrected values for the mean distance are as follows:

	Original	Corrected	Correction
X	+ 0″.20	+ 0″.14	— 0″.06
Y	— 2 .60	— 2 .43	+ 0 .17
Z	+ 1 .50	+ 1 .51	+ 0 .01

and the R.A. and Decl. of the apex become:

	Original	Corrected	Correction
A	274° 24′	273° 20′	—1° 4′
D	+ 30 0	+ 31 48	+1 48

As we said at the beginning of this paper, this particular problem appeared to have been already treated by EDDINGTON in his *Stellar movements* p. 81—83. He found, starting from practically the same data, but by an entirely different method, that A in particular will need a correction, viz. of about —2.°4. The two results for A agree tolerably well, and ours is also not accurate to a few minutes. We find also an appreciable value for the correction of D, although the Z-component remains almost unchanged.

The result found for the whole sky is equal to that for $\delta = \pm\,35°15'$. As a second example we will calculate the corrections for $\delta = 0$.

$$\left[\frac{X}{R_m}\right] = 0.93\frac{X}{R_m} - 0.04\frac{Y}{R_m}$$

$$\left[\frac{Y}{R_m}\right] = 1.13\frac{Y}{R_m} - 0.04\frac{X}{R_m}$$

$$\left[\frac{Z}{R_m}\right] = 1.03\frac{Z}{R_m}$$

and herewith we find, starting from the same original values as above,

	Original	Corrected	Correction
X	$+0''.20$	$+0''.11$	$-0''.09$
Y	$-2\ .60$	$-2\ .29$	$+0\ .31$
Z	$+1\ .50$	$+1\ .46$	$-0\ .04$
A	274°24′	272°45′	−1°39′
D	+30 0	+32 32	+2 32

The corrections to be applied differ not much, therefore, from those in the first case.

As the components of the parallactic motion are thus found to require appreciable corrections, those found above for the precession are no longer quite correct, but their errors are of the same order as other unavoidable inaccuracies in the calculation.

The result of our research is thus to show that in researches concerning precession and systematic proper motions it is necessary to take into account the dependence of the mean distance upon the galactic latitude: its influence upon both the precessional constant, and the parallactic prop. motion cannot be neglected.

By taking this influence into account it is possible to bring into fair agreement NEWCOMB's results for the precessional constant found from observations of R.A. and from those of Decl. For the present, therefore, it is not necessary to follow HOUGH and HALM, who proceed from a new definition of the precession, by which this is not to be determined with reference to the whole of the stars, but with reference to the mean of the two star streams regarded as of different strength in different parts of the sky: a method which, moreover, as it would appear, involves great difficulties.

This, of course, does not mean that we can now rely upon the precession, determined relatively to a large complex of stars, giving us the true *mechanical precession*. To throw more light upon this subject many more extensive researches will be necessary, in which attention must also be paid to general rotations possibly occurring in our system of stars, as first proposed by SCHÖNFELD. It seemed premature to include terms of this kind in our present calculations.