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$$\lambda = 1{,}1 \,.\, 10^{-7},$$

which agrees very well with the theoretical one 1,13. 10^{-7}.

We must observe, however, that we cannot assign to our measurements a greater precision than of 10°/₀.

It seems to us that within these limits the theoretical conclusions have been fairly confirmed by our observations..

The experiments have been carried out in the "Physikalisch-Technische Reichsanstalt". We want to express our thanks for the apparatus kindly placed at our disposition.

Physics. — *"On a possible influence of the* FRESNEL-*coefficient on solar phenomena"*. By Prof. P. ZEEMAN.

(Communicated in the meeting of September 25, 1915).

We shall prove here, that the presence of the term $-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}$ of LORENTZ in the expression for the FRESNEL coefficient (cf. also my paper Vol. **18**, p. 398 of these Proceedings) may give rise to a change in the propagation of lightwaves if in a moving, refracting medium a change of velocity occurs. I suppose the medium to have everywhere the same density and to be flowing with a velocity v parallel to the axis of X in a system of coordinates that is at rest with respect to the observer. In the direction of the Z axis a velocity gradient exists in such a way, that the velocity decreases with the distance to the X axis and becomes zero at the distance $z = \Delta$. If now the incident lightbeam (with a plane wave front) is parallel to the axis of X, the parts of the wave fronts which are near this axis will be more carried with the medium than those at a greater distance. The wave front will thus be rotated.

If the velocity decreases linearly in the direction of the Z axis the wavefront will remain plane. In a time t the angle of rotation, (supposed to be small) will be $\alpha = \frac{\varepsilon . v . t.}{\Delta}$, where ε is the FRESNEL coefficient and where v and Δ have the above mentioned meaning. More in general we may consider an element of the wave front and then write $\frac{dv}{dz}$ for $\frac{v}{\Delta}$. Moreover t may be expressed as a function of the velocity of light and the path through which the rays have travelled, so that we find

$$\alpha = \frac{\varepsilon l}{c/\mu} \frac{dv}{dz} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

In general this angle is infinitesimal, but it will take higher values if $-\frac{d\mu}{d\lambda}$ becomes very large. In the expression $\varepsilon_L = 1 - \frac{1}{\mu^2} - \frac{\lambda}{\mu}\frac{d\mu}{d\lambda}$ we only need to keep the last term, so that (1) becomes

$$a = -\frac{\lambda}{\mu}\frac{d\mu}{d\lambda} \cdot \frac{l}{c/\mu} \cdot \frac{dv}{dz} = -\lambda \frac{d\mu}{d\lambda}\frac{l}{c}\frac{dv}{dz} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the normal of the wave fronts forms an angle i with the direction in which the gradient of the velocity changes most, we find

$$a = -\lambda \frac{d\mu}{d\lambda}\frac{l}{c}\frac{dv}{dz} \sin i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation makes it possible to construct the path of the light ray starting from a given point in a given direction.

In order to show how great the influence of the dispersion term may become in different cases I give here some tables referring to water, carbonic disulphide and sodium vapour.

For water and carbonic disulphide we have calculated with the data from well known tables the values of μ for some values of λ (in Å.U.) In the third column the values of $-\frac{d\mu}{d\lambda}$ (λ in cm.) are given, while in the fourth the FRESNEL coefficient ε_L is found. The last column gives the value of the dispersion term separately.

For sodium I take the value of λ and μ_{obs} from WOOD's[1]) observations, made at 644° C.; now ε_L reduces to the dispersion term. The values of ε_L and of $-\frac{d\mu}{d\lambda}$ are only of interest as to the order of magnitude.

Water.

λ in Å.U.	μ	$-\frac{d\mu}{d\lambda}$	ε_L	$-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}$
4500	1.3393	650	0.464	0.021
4580	1.3388	615	0.463	0.021
5461	1.3346	390	0.454	0.015
6440	1.3314	270	0.449	0.013
6870	1.3308	216	0.447	0.012

Carbonic disulphide.

4358	1.6750	5000	0.774	0.130
5461	1.6370	1900	0.690	0.063
6870	1.6160	1200	0.668	0.051

[1]) Physical optics. p. 427. 1911

Sodium vapour.

λ *in* Å.U.	$\mu_{obs.}$	$-\frac{d\mu}{d\lambda}$	$-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda} = \varepsilon_L$.
5882	0,9908		
		$1{,}3.10^5$	7,8
5885	0,9870		
		$8{,}1.10^5$	48
5886,6	0,9740		
		$17.\ 10^5$	102
5888,4	0,9443		
		$280.\ 10^5$	2100
5889,6	0,614		

In the application we are going to make of equation (2) α is supposed small, so that we need not integrate over the path of the ray. We suppose in the sun a radially rising, selectively absorbing gas mass, in which a velocity gradient exists perpendicular to the radius. Even without the density gradients, which are necessary in the theory of JULIUS, there must be here a deflection of the light waves, especially for the wavelengths in the neighbourhood of the absorption lines.

If we try to work out quantitatively the idea, on which rests equation (2) we directly meet with the difficulty, that the necessary data are failing. Still we may derive a conclusion from (2), be it with little evidence, viz. that also with extremely small density of the considered vapour there may exist an observable influence of the FRESNEL coefficient on the light waves.

Let the radially ascending gas mass be found in the centre of the visible solar disc and suppose that an objective of e. g. 30 cm. diameter be used for observation. The light cone proceeding from the considered point of the sun has then (the distance of the earth to the sun being $1{,}5.10^{13}$ cm.), a value of $\frac{30}{1{,}5 \times 10^{13}} = 2.10^{-12}$ in radials. A ray deviating with half this amount from the line that connects the centre of the sun with the objective does not fall in the telescope. A ray to the rim of the objective however needs a deviation of the whole amount to fall beside the telescope.

For l we may take the depth of the "reversing layer", viz. a number of the order of 1000 k.m.

As to $\frac{d\mu}{d\lambda}$, according to the above mentioned observations of WOOD, this is in the neighbourhood of the sodium line and at 644° C. of the order 10^6. The density of sodium vapour is at 644° C. of the order 10^{-5}. This follows from a calculation, which Mr. C. M. HOOGENBOOM, assistent at the Physical Laboratory, made at my request, using the observations of HACKSPILL [1]).

As to the density of the metal vapours on the sun, which give rise to the finest lines in the solar spectrum, we may treat these according to LORENTZ [2]) as being very small. If p is the pressure in mm. mercury of the metal vapour, l the length of the layer that is traversed by the rays, LORENTZ finds at $T = 6000°$ $pl < 0{,}0015$ or $pl < 15000$ depending on the suppositions made. For $l = 10^8$ c.m. would follow $p < 0.00015$ mm. mercury in the latter case and $p < 0{,}00015 \times 10^{-7}$ mm. mercury in the former one. To the mentioned pressures correspond the densities 9.10^{-12} and 9.10^{-19}.

Let $-\frac{d\mu}{d\lambda}$ be proportional to the density, then we should find for a density 9.10^{-19} $-\frac{d\mu}{d\lambda} = \frac{10^6 . 9.10^{-19}}{10^{-5}} = 10^{-7}$ at 644° C. We shall suppose this number to be still valid at 6000°.

For $a = 10^{-12}$ and $\lambda = 6.10^{-5}$ c.m. we then roughly find from equation (2) $\frac{dv}{dz} = 50$. This number and therefore the velocity gradient becomes 10^7 times smaller, if we take 10^{-11} for the density of the metallic vapour and still smaller, if we assign a higher value to $-\frac{d\mu}{d\lambda}$ than we did above.

A few objections can be made to the application of the above given discussion to the explanation of solar phenomena. I shall mention these shortly.

Even if we confine ourselves to rays proceeding from one point of the sun, there seems to be a difficulty in the fact, that while rays of a definite wave length and definite direction are deflected away from the objective, there are other rays of the same wavelength and originally another direction, which are deflected towards the objective. This difficulty may be avoided by assuming a partition of velocities symmetrical with respect to the line connecting sun and objective. Then all rays that must be taken into consideration are deflected. If now we had to consider the light from one point of the sun only, we might directly conclude, that for the mentioned small velocity gradients the deflection of the light rays must give rise to observable phenomena. One of these phenomena would be the occurrence of complicated changes closely connected with the

[1]) HACKSPILL Ann. de Chim. et de Phys. (8) 28, 676 and 661. 1913.

[2]) H. A. LORENTZ. On the width of spectral lines. These proceedings, 23, 470. 1914.

dispersion bands of JULIUS, in the neighbourhood of the simple absorption line that would be observed in a gas mass at rest. If however instead of one point of the photosphere we consider a part of observable apparent area we only get a mean effect, which will be small.

Only a very special partition of the velocity may then give rise to a strong action.

Large velocity gradients will occur in the neighbourhood of surfaces of discontinuity; then $l\frac{dv}{dz}$ may become very large and α even of another order of magnitude. Ascending and descending currents may be found in neighbouring parts of space. Currents in these two directions may deflect the light, so that finally the light from a finite part of the photosphere may be deflected.

The aim of this communication is only to call the attention of astrophysicists to the fact, that under favourable circumstances the *simultaneous* existence of *velocity gradients* and *anomalous dispersion* in gases that are extremely rare and without density gradients, may give rise to a *deflection* of light.

Anatomy. — *"On the Relation between the Dentition of Marsupials and that of Reptiles and Monodelphians."* (*First Communication*). By Prof. L. BOLK.

(Communicated in the Meeting of May 29, 1915).

On the morphological significance of the dentition of Marsupials opinions have varied greatly in the course of time. The special characteristic of this dentition, the almost entire absence of a teeth-change, naturally gave rise to the question: with which of the two sets of teeth of the Monodelphian mammals does that of the Marsupials correspond, with the deciduous or with the permanent set? Older authors, more particularly led by comparative anatomical investigations, were generally of opinion that it must be considered as identical with the permanent set of the Monodelphian mammals. This was e.g. the opinion of OWEN, FLOWER, OLDFIELD THOMAS. With the Marsupials the milk-dentition would, according to them, remain undeveloped with the exception of a single tooth, namely the one immediately preceding the first molar. In fact with most Marsupials an existing tooth is here sooner or later expelled and replaced by